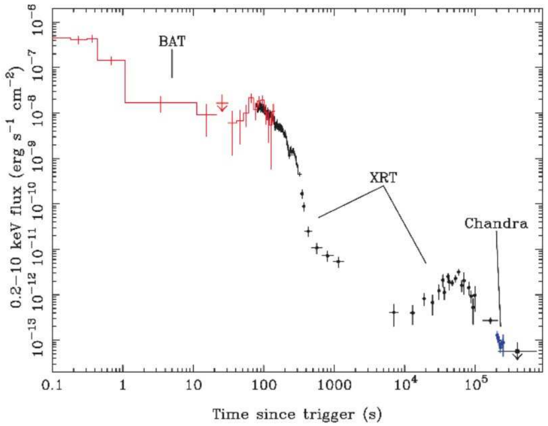
Short Burst GRBs

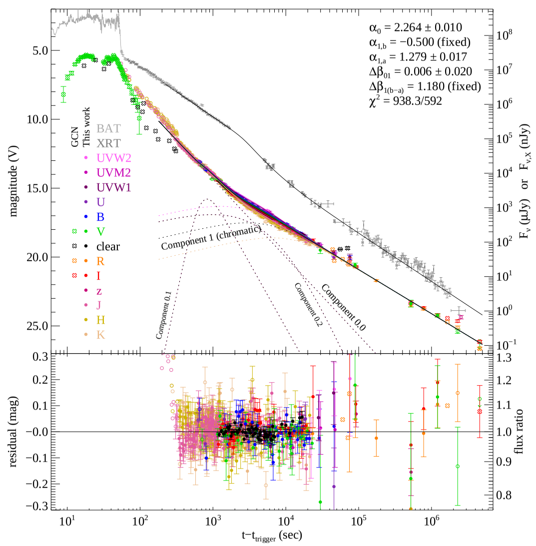


(GRB 050709 Barthelmy et al., 2005, Fig. 3)

Short bursts occur in regions of low star formation rate, afterglows are very different.
Smoking gun, e.g., GRB050709 in an elliptical galaxy
$\Longrightarrow$ low star formation rate
$\Longrightarrow$ SN origin very unlikely
Since isotropic energy of short bursts factor 100 to 1000 fainter than for long bursts
$\Longrightarrow$ different origin.

Currently discussed: coalescence of binary systems of neutron stars or neutron star–black hole pairs as progenitor

Long GRB Afterglows

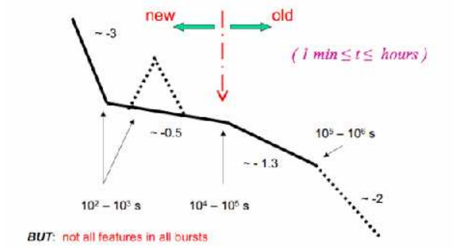


(Bloom et al., 2008, Fig. 1)

Long GRB Afterglows

Parameterize afterglow lightcurves and energy spectra as powerlaws:

$$F_X \propto t^{-\alpha_i} \quad f_\nu \propto \nu^{-\beta_i} \tag{7.1}$$



(Mészáros, 2006)

Overall features seen in the afterglow lightcurve (Mészáros, 2006):

1. $t \le t_1 = 300\dots500$ s: initial steep decay $3 \lesssim \alpha_1 \lesssim 5$ and $1 \lesssim \beta_1 \lesssim 2$
2. $t_1 < t \le t_2 = 10^3\dots10^4$ s: flatter decay $0.2 \lesssim \alpha_2 \lesssim 0.8$ and $0.7 \lesssim \beta_2 \lesssim 1.2$
3. $t_2 < t \le t_3 \sim 10^5$ s: "normal" decay $1.1 \lesssim \alpha_3 \lesssim 1.7$ and $0.7 \lesssim \beta_3 \lesssim 1.2$ (generally unchanged from previous stage),
4. $t_3 < t \le t_4 \sim$ few $\cdot\, 10^5$ s: sometimes steeper decay observed $2 \lesssim \alpha_4 \lesssim 3$.

In $\sim$ 50% of all afterglows, X-ray flares are seen, starting as early as 100 s after burst, and sometimes as late as 10^5 s.

Lightcurves contain further evidence for the relativistic fireball model of GRBs.

Fireball Model

Initial hypernova explosion:

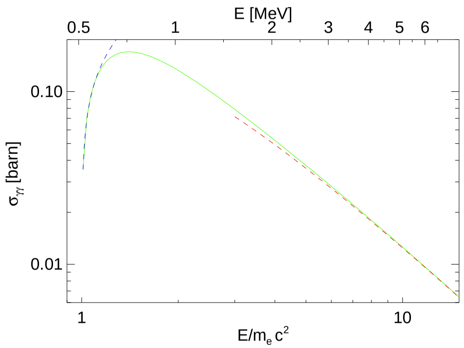
- $\sim 10^{53}$ erg of thermal $\nu_e\bar{\nu}_e$ with energies 10–30 MeV (unobserved)
- $\sim 10^{53}$ erg in gravitational waves with $\nu \sim 10^2$–10^3 Hz (unobserved)
- 10^{50}–10^{52} erg in high temperature fireball with $kT \gtrsim$ MeV consisting $e^{\rm pm}$, γ-rays, and baryons.

The GRB lightcurves are due to photons from the fireball.

Physics: Rees & Mészáros (1992), Mészáros (2006)

Relativistic Fireballs

Most photons from GRB observed at energies $\gg m_\mathrm{e}c^2$: photon-photon pair production, $\mathrm{e}^- + \mathrm{e}^+ \longrightarrow \gamma + \gamma$ must be important!



Cross section in center of mass system for $\epsilon = E/m_\mathrm{e}c^2 > 1$:

$$\sigma_{\gamma\gamma} = \frac{\pi r_0^2}{\epsilon^2}\Bigg(\left(2+\frac{2}{\epsilon^2}-\frac{1}{\epsilon^4}\right)\ln\left(\epsilon+\sqrt{\epsilon^2-1}\right)-\sqrt{1-\frac{1}{\epsilon^2}}\left(1+\frac{1}{\epsilon^2}\right)\Bigg) \tag{7.2}$$

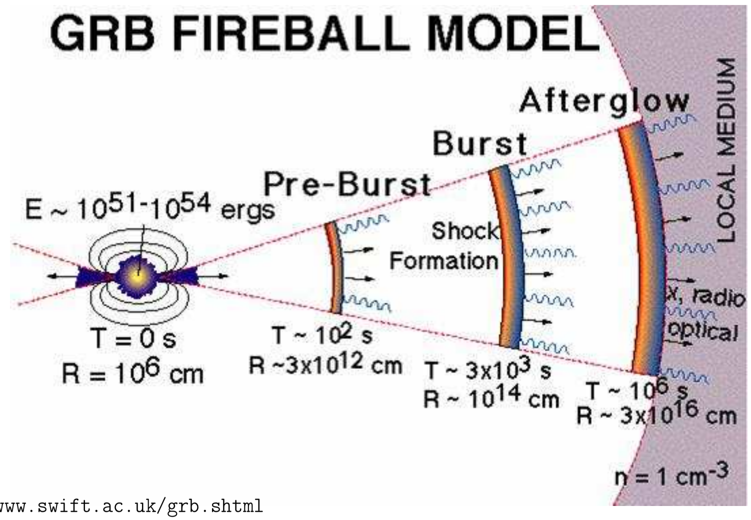
Non-relativistic fireball would have high optical depth and not be observable at high energies!

Relativistic fireball with gamma-factor Γ: threshold for pair production reduced since most photons will collide under angles $< \Gamma^{-1}$.

Better estimates: $\Gamma > 1$.

$\Longrightarrow$ Physics similar to blazar jets!

Relativistic Fireballs



`http://www.swift.ac.uk/grb.shtml`

GRB model: Relativistic fireball model (Rees & Mészáros, 1992)

Relativistic Fireballs

Evolution of fireball: relativistic shock propagating with gamma-factor Γ through medium of particle density n and B-field B.

Good model description: Sari et al. (1998)

Shock: power-law distributed electrons; B-field: synchrotron radiation:

Power: $P(\gamma_\mathrm{e}) = \frac{4}{3}\sigma_\mathrm{T}c\Gamma^2\gamma_\mathrm{e}^2\frac{B^2}{8\pi}$ char. frequency: $\nu(\gamma_\mathrm{e}) = \Gamma\gamma_\mathrm{e}^2\frac{q_\mathrm{e}B}{2\pi m_\mathrm{e}c}$ (7.3)

Spectral shape:

$$P_\nu = \begin{cases} \nu^{1/3} & \text{for } \nu < \nu(\gamma_\mathrm{e}) \\ \text{exp. cutoff} & \text{for } \nu > \nu(\gamma_\mathrm{e}) \end{cases} \tag{7.4}$$

Peak power emitted:

$$P_{\nu,\mathrm{max}} \approx \frac{P(\gamma_\mathrm{e})}{\nu(\gamma_\mathrm{e})} = \frac{m_\mathrm{e}c^2\sigma_T}{3q_\mathrm{e}}\Gamma B. \tag{7.5}$$

Note: $P_{\nu,\mathrm{max}}$ is *independent* of γ_e! Peak position is *dependent* γ_e!.

Relativistic Fireballs

Description on previous slide correct if electron does not loose significant power through radiation.

This is correct for

$$\gamma \gtrsim \gamma_\mathrm{c} \sim \frac{P_\mathrm{max}}{\Gamma m_\mathrm{e}c^2}t \overset{\text{exact theory}}{=} \frac{6\pi m_\mathrm{e}c}{\sigma_\mathrm{T}\gamma B^2 t} \tag{7.6}$$

where t time in observer's timeframe.

For $\gamma < \gamma_\mathrm{c}$: effectively no power loss.

Energy with initial Lorentz factor $\gamma \gg \gamma_\mathrm{c}$: Electron cools down to γ_c in time t.

Result: Peak frequency of emission varies as $\nu \propto \gamma^2$, spectral power varies as $\nu^{-1/2}$ above $\nu(\gamma_\mathrm{c})$

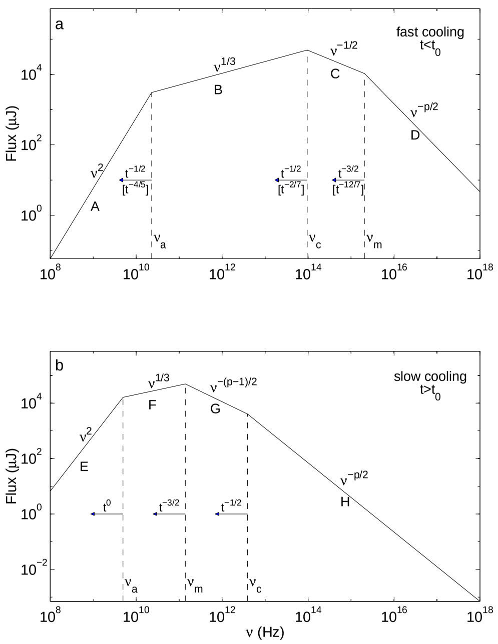
Below γ_c: no further emission possible $\Longrightarrow$ Spectrum $\propto \nu^{1/3}$

To obtain full spectrum, integrate over power-law electron distribution.

Once electron population has evolved: only low energy electrons with large cooling timescales have survived

$\Longrightarrow$ classical $\nu^{-(p-1)/2}$ synchrotron spectrum

Plus self-absorption in all cases.



Synchrotron spectrum of a relativistic shock.

Fast cooling: Early phase of GRB

Slow cooling: Later phase of GRB

Sari et al. (1998, Fig. 1)

Relativistic Fireballs

To derive lightcurves of afterglow, look at time evolution of shock.

Physics to be looked at similar to evolution of supernova remnants (but obviously much faster!)

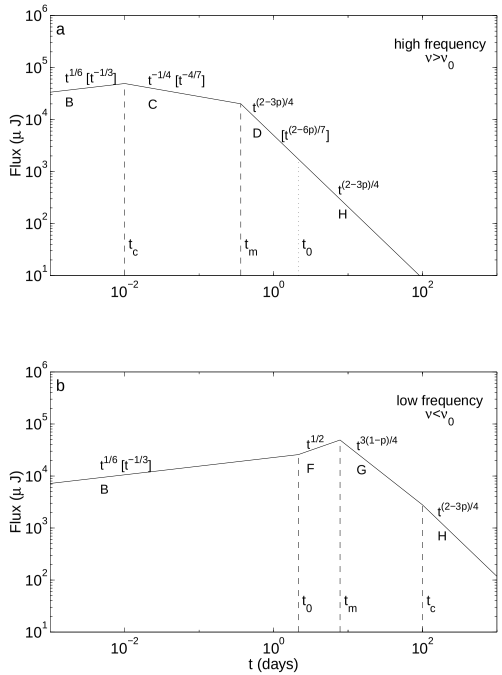
Two cases:

1. Radiative evolution: energy going into electrons is large, fast cooling. Shock energy varies as $E \propto \Gamma$, where $\Gamma \sim (R/\mathcal{R})^{-3}$ and where $\mathcal{R} = [17M/(16\pi m_p n)]^{1/3}$ is the radius where the mass swept up from the ejecta = initial mass M of ejecta
2. Otherwise: Adiabatic evolution, i.e., energy of spherical shock is constant, $E = 16\pi\gamma^2 R^3 n m_\mathrm{p} c^2/17$

Use these assumptions to determine afterglow evolution.

Important timescale: switch from radiative to adiabatic evolution:

$$t_0 = 4.6\epsilon_B^{7/5}\epsilon_e^{7/5}E_{52}^{4/5}\gamma_2^{-4/5}n_1^{3/5}\,\mathrm{days} \tag{7.7}$$



Time evolution of an afterglow

Sari et al. (1998, Fig. 2)

Open Questions

- Nature of X-Ray Flashes and their relationship to short GRBs

  Geometry? E.g., we're not looking directly into jet? or different formation mechanism such as neutron star - neutron star coalescence?

- Nature of jets and shocks
- Further understanding of GRB and afterglow lightcurves

  Picture shown here is very simplified, this is an active field and there are many counterexamples, see Willingale et al. (2007)

- Discovery of GRBs in Gravitational Waves and Neutrinos

  $\Longrightarrow$ Multi-Messenger Astronomy

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AGN Evolution

Introduction

Result of previous lectures:

> AGN produce large amounts of energy over timescales of $\gtrsim 10^8$ years and they strongly interact with their environment.

Questions:

- What galaxies harbor AGN?
- Are these galaxies different from others?
- How do galaxies with AGN evolve?
- How do AGN form?

To answer these questions, we need to study statistical properties of AGN and their hosts, both among morphological type and with time: AGN surveys

But first, we need to talk about the basics of doing science in an expanding universe.

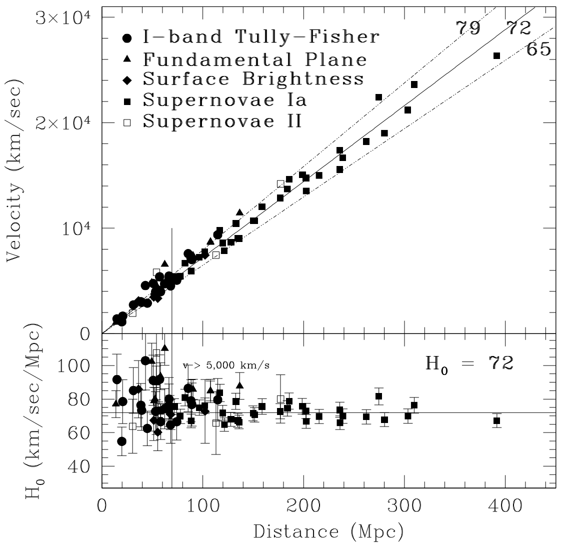
Basic Facts

Observations show that there are *four major facts* about the universe as a whole:

> The universe is:
> - expanding,
> - isotropic,
> - and homogeneous.

That the universe is isotropic and homogeneous is called the *cosmological principle*.

Expansion



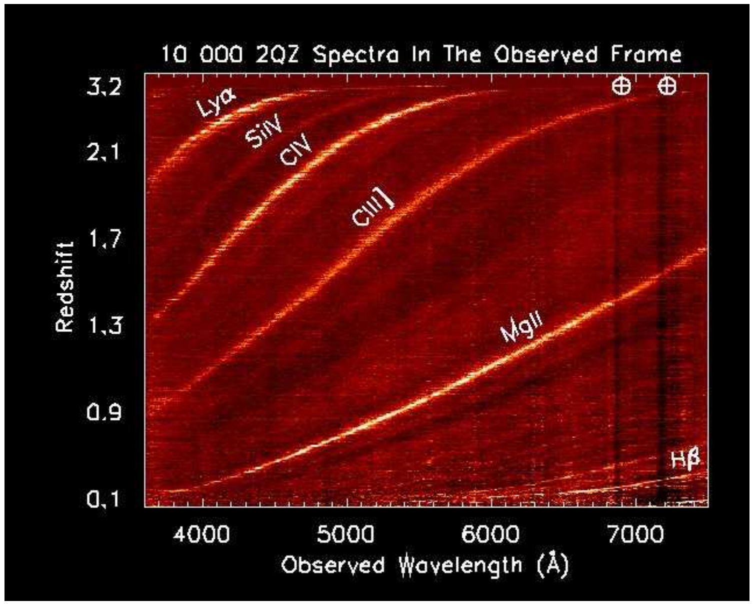
Hubble (1929): The "velocity", v, of a galaxy depends linearly from its distance, d: $v(r) = H_0 d$

where $v/c = \Delta\lambda/\lambda$ and where H_0: Hubble constant or Hubble parameter.

Currently accepted value:

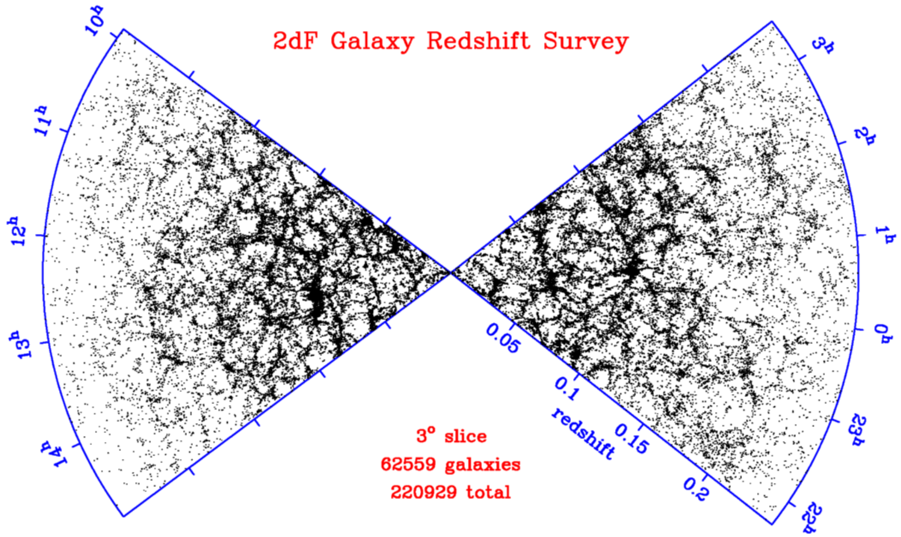
$$H_0 = 72 \pm 8\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \tag{8.1}$$

Freedman et al. (2001, Fig. 4)



courtesy 2dF QSO Redshift survey

As a consequence of the cosmological redshift, for different z different parts of the spectrum of a distant source are visible.



2dF Survey, ~220000 galaxies total

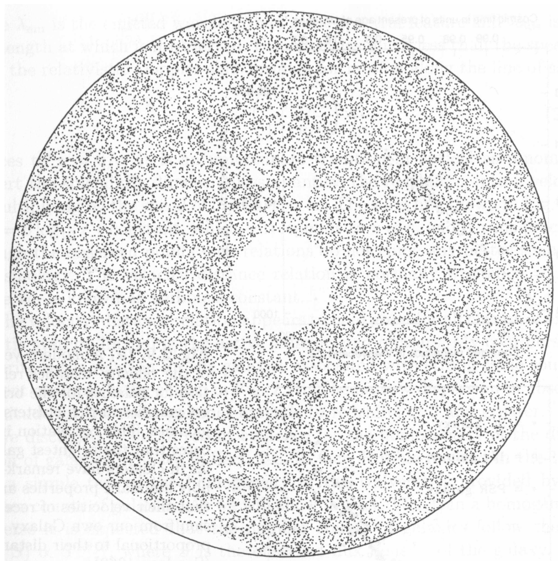
The universe is homogeneous $\Longleftrightarrow$ The universe looks the same everywhere in space

Testable by observing spatial distribution of galaxies.

On scales $\gg$ 100 Mpc the universe looks indeed the same. Below that: structure.

Structures seen are galaxy clusters (gravitationally bound) and superclusters (larger structures, not [yet] gravitationally bound).

Isotropy



The universe is isotropic $\Longleftrightarrow$ The universe looks the same in all directions

Radio galaxies are mainly quasars $\Longrightarrow$ Sample large space volume ($z \gtrsim 1$) $\Longrightarrow$ Clear isotropy.

Anisotropy in the image: galactic plane, exclusion region around Cyg A, Cas A, and the north celestial pole.

Peebles (1993): Distribution of 31000 objects at λ =6 cm from the Greenbank Catalogue.

World Models

World Model: theoretical framework describing a world governed by the cosmological principle.

Use combination of

- General Relativity
- Thermodynamics
- Quantum Mechanics

$\Longrightarrow$ Complicated!

For 99% of the work, the above points can be dealt with separately:

1. Define metric obeying cosmological principle.
2. Obtain equation for evolution of universe using Einstein field equations.
3. Use thermo/QM to obtain equation of state.
4. Solve equations.

World Models

Before we can start to think about universe: Brief introduction to assumptions of general relativity.

$\Longrightarrow$ See theory lectures for the gory details, or check with the literature (Weinberg or MTW).

Assumptions of GRT:

- Space is 4-dimensional, might be curved
- Matter (=Energy) modifies space (Einstein field equation).
- Covariance: physical laws must be formulated in a coordinate-system independent way.
- Strong equivalence principle: There is no experiment by which one can distinguish between free falling coordinate systems and inertial systems.
- At each point, space is locally Minkowski (i.e., locally, SRT holds).

$\Longrightarrow$ Understanding of geometry of space necessary to understand physics.

RW Metric

- Cosmological principle + expansion $\Longrightarrow$ $\exists$ freely expanding cosmical coordinate system.
  - Observers =: fundamental observers
  - Time =: cosmic time

  This is the coordinate system in which the 3K radiation is isotropic, clocks can be synchronized, e.g., by adjusting time to the local density of the universe.

  $\Longrightarrow$ Metric has temporal and spatial part.

  This also follows directly from the equivalence principle.
- *Homogeneity and isotropy* $\Longrightarrow$ spatial part is spherically symmetric:

$$\mathrm{d}\psi^2 := \mathrm{d}\theta^2 + \sin^2\theta \;\; \mathrm{d}\phi^2 \tag{8.2}$$

- *Expansion:* $\exists$ scale factor, $R(t)$ $\Longrightarrow$ measure distances using comoving coordinates.

RW Metric

A metric based on these points looks like

$$\mathrm{d}s^2 = c^2 \;\mathrm{d}t^2 - R^2(t)\left[f^2(r)\;\mathrm{d}r^2 + g^2(r)\;\mathrm{d}\psi^2\right] \tag{8.3}$$

where $f(r)$ and $g(r)$ are arbitrary.

Metrics of the form of eq. (8.3) are called Robertson-Walker (RW) metrics (1935), but have been previously also studied by Friedmann and Lemaître.

One common choice is

$$\mathrm{d}s^2 = c^2 \;\mathrm{d}t^2 - R^2(t)\left[\mathrm{d}r^2 + S_k^2(r)\;\mathrm{d}\psi^2\right] \tag{8.4}$$

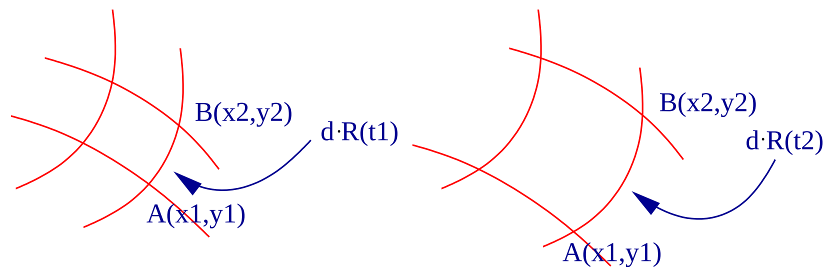
where $R(t)$: scale factor, containing the physics, t: cosmic time, r, θ, ϕ: comoving coordinates, and where

$$S_k(\theta) = \begin{cases} \sin\theta & \text{for } k = +1 \\ \theta & \text{for } k = \;\;\,0 \\ \sinh\theta & \text{for } k = -1 \end{cases} \tag{8.5}$$

Remark: θ and ϕ describe directions on sky, as seen from the arbitrary center of the coordinate system (=us), r can be interpreted as a radial coordinate.

RW Metric

RW metric: defines universal coordinate system tied to the expansion of space:



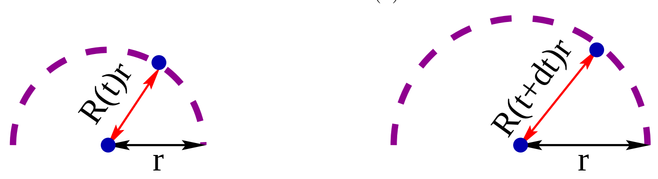
Scale factor $R(t)$ describes evolution of universe.

- d is called the comoving distance.
- $D(t) := d \cdot R(t)$ is called the proper distance.

(note that R is unitless, i.e., d and $d \cdot R(t)$ are measured in Mpc)

Hubble's Law

Hubble's Law follows from the variation of $R(t)$:



Small scales $\Longrightarrow$ Euclidean geometry. Proper distance between two observers:

$$D(t) = d \cdot R(t) \tag{8.6}$$

Expansion $\Longrightarrow$ proper separation changes:

$$\frac{\Delta D}{\Delta t} = \frac{R(t+\Delta t)d - R(t)d}{\Delta t} \quad \text{with} \lim_{\Delta t \to 0}: \quad v = \frac{\mathrm{d}D}{\mathrm{d}t} = \dot{R}\, d = \frac{\dot{R}}{R} D =: H\, D \tag{8.7}$$

$\Longrightarrow$ Identify local Hubble "constant" with

$$H = \dot{R}/R = \dot{a}(t) \quad \text{where} \quad a(t) = R(t)/R(\text{today}) \tag{8.8}$$

Note that $R = R(t) \Longrightarrow H$ is time-dependent!

Hubble's Law

The cosmological redshift is a consequence of the expansion of the universe:

Since the comoving distance is constant:

$$d = \frac{D(t = \text{today})}{R(t = \text{today})} = \frac{D(t)}{R(t)} = \text{const.} \tag{8.9}$$

Set $a(t) = R(t)/R(t = \text{today})$, then Eq. (8.9) implies

$$\lambda_{\text{obs}} = \frac{\lambda_{\text{emit}}}{a_{\text{emit}}} \iff z = \frac{\lambda_{\text{obs}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}} = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}} - 1 \tag{8.10}$$

(z: observed redshift, λ_{obs}: observed wavelength, λ_{emit}: emitted wavelength)

$$1 + z = \frac{1}{a_{\text{emit}}} = \frac{R(t = \text{today})}{R(t)} \tag{8.11}$$

Light emitted at $z = 1$ was emitted when the universe was half as big as today!

z: measure for *relative size* of universe at time the observed light was emitted.

Hubble's Law

For light, $d = c\Delta t$. Therefore

$$\frac{c\,\Delta t_{\text{e}}}{R(t_{\text{emit}})} = \frac{c\,\Delta t_{\text{obs}}}{R(t_{\text{obs}})} \quad \text{such that} \quad \frac{\mathrm{d}t}{R(t)} = \text{const.} \tag{8.12}$$

This means that

$$\frac{\mathrm{d}t_{\text{obs}}}{\mathrm{d}t_{\text{emit}}} = \frac{R(t_{\text{obs}})}{R(t_{\text{emit}})} = 1 + z \tag{8.13}$$

$\Longrightarrow$ Time dilatation of events at large z.

This cosmological time dilatation has been observed in the light curves of supernova outbursts.

Expansion and Spectra

The total number of photons in a box $\mathrm{d}A \cdot c\,\mathrm{d}t$ and in a frequency range ν to $\nu + \mathrm{d}\nu$ is

$$N = n_\nu(\nu)\ \mathrm{d}A\ \mathrm{d}\nu\, c\ \mathrm{d}t \tag{8.14}$$

This number is conserved during the expansion of the universe:

$$n_\nu(\nu_{\text{emit}})\ \mathrm{d}A\ \mathrm{d}\nu_{\text{emit}}\, c\ \mathrm{d}t_{\text{emit}} = n_\nu(\nu_{\text{obs}})\frac{d\nu_{\text{emit}}}{1+z}\ \mathrm{d}A\, c\ \mathrm{d}t_{\text{emit}}(1+z) \tag{8.15}$$

$$n_\nu(\nu_{\text{obs}})\ \mathrm{d}A\ \mathrm{d}\nu_{\text{obs}}\, c\ \mathrm{d}t_{\text{obs}} \tag{8.16}$$

but: arrival time differs $\Longrightarrow$ energy flux density changes:

$$F_\nu(\nu_{\text{obs}}) = h\nu_{\text{obs}} n_\nu(\nu_{\text{obs}}) = h\frac{\nu_{\text{emit}}}{1+z}\nu_n(\nu_{\text{emit}}) = \frac{F_\nu(\nu_{\text{emit}})}{1+z} \tag{8.17}$$

and consequently the total flux in a certain energy band changes as well:

$$F_{\text{obs}} = \int F_\nu(\nu_{\text{obs}})\ \mathrm{d}\nu_{\text{obs}} = \int \frac{F_\nu(\nu_{\text{emit}})}{1+z}\cdot\frac{\mathrm{d}\nu_{\text{emit}}}{1+z} = \frac{F_{\text{emit}}}{(1+z)^2} \tag{8.18}$$

One power of $1+z$ from decreased photon energy, one from decreased arrival rate.

For wavelength based flux densities, since $F_\lambda = F_\nu c/\lambda^2$ one finds $F_\lambda(\lambda_{\text{obs}}) = F_\lambda(\lambda_{\text{emit}})/(1+z)^3$.

Luminosity Distance

For AGN studies at high z, we need to take into account cosmological effects: How to convert a measured flux into luminosity.

Assume source with luminosity L at comoving coordinate r.

When light has reached us, then it has spread over sphere of area

$$A = 4\pi(R_0 r)^2 \tag{8.19}$$

R_0: today's scale factor

such that the flux measured in the same reference frame is

$$F_{\text{ref}} = \frac{L}{4\pi(R_0 r)^2} \tag{8.20}$$

and the measured flux is (correcting for Doppler effect):

$$F = \frac{F_{\text{ref}}}{(1+z)^2} = \frac{L}{4\pi(1+z)^2(R_0 r)^2} \tag{8.21}$$

Friedmann Equations

General relativistic approach: Insert metric into Einstein equation to obtain differential equation for $R(t)$:

Einstein equation:

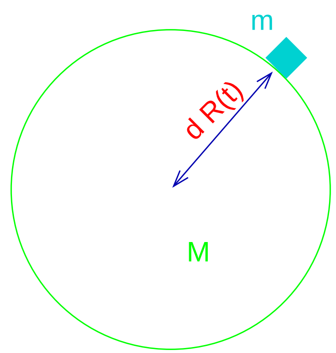
$$\underbrace{R_{\mu\nu} - \frac{1}{2}\mathscr{R} g_{\mu\nu}}_{G_{\mu\nu}} = \frac{8\pi G}{c^4}T_{\mu\nu} + \Lambda g_{\mu\nu} \tag{8.22}$$

where

$g_{\mu\nu}$: Metric tensor ($\mathrm{d}s^2 = g_{\mu\nu}\ \mathrm{d}x^\mu\ \mathrm{d}x^\nu$)
$R_{\mu\nu}$: Ricci tensor (function of $g_{\mu\nu}$)
$\mathscr{R}$: Ricci scalar (function of $g_{\mu\nu}$)
$G_{\mu\nu}$: Einstein tensor (function of $g_{\mu\nu}$)
$T_{\mu\nu}$: Stress-energy tensor, describing curvature of space due to fields present (matter, radiation,…)
Λ: Cosmological constant

$\Longrightarrow$ Messy, but doable

Friedmann Equations



Here, Newtonian derivation of Friedmann equations: Dynamics of a mass element on the surface of sphere of density $\rho(t)$ and comoving radius d, i.e., proper radius $d \cdot R(t)$ (after McCrea & Milne, 1934).

Mass of sphere:

$$M = \frac{4\pi}{3}(d\,R)^3\rho(t) = \frac{4\pi}{3}d^3\rho_0 \quad \text{where} \quad \rho(t) = \frac{\rho_0}{R(t)^3} \tag{8.23}$$

Force on mass element:

$$m\frac{\mathrm{d}^2}{\mathrm{d}t^2}(d\,R(t)) = -\frac{GMm}{(dR(t))^2} = -\frac{4\pi G}{3}\frac{d\rho_0}{R^2(t)}m \tag{8.24}$$

Canceling $m \cdot d$ gives the momentum equation:

$$\ddot{R} = -\frac{4\pi G}{3}\frac{\rho_0}{R^2} = -\frac{4\pi G}{3}\rho(t)R(t) \tag{8.25}$$

Friedmann Equations

Multiplying

$$\ddot{R} = -\frac{4\pi G}{3}\frac{\rho_0}{R^2} = -\frac{4\pi G}{3}\rho(t)R(t) \tag{8.25}$$

with $\dot{R}$ and integrating, or alternatively considering energy conservation yields the energy equation,

$$\begin{aligned}\frac{1}{2}\dot{R}^2 &= +\frac{4\pi G}{3}\frac{\rho_0}{R(t)} + \text{const.}\\ &= +\frac{4\pi G}{3}\rho(t)R^2(t) + \text{const.}\end{aligned} \tag{8.26}$$

where the constant can only be obtained from GR.

[redacted]

Note: derivation implicitly assumes $r_{\text{cloud}} < \infty$, which violates the cosmological principle, and assumes that the particle moves through space, which violates SRT. However, since GR $\sim$ Newton on small scales and mass densities, there is a scale invariance on Mpc scales and Newton is valid in the classical limit of GR.

Friedmann Equations

The exact GR derivation of Friedmanns equation gives:

$$\begin{aligned}\ddot{R} &= -\frac{4\pi G}{3}R\left(\rho + \frac{3p}{c^2}\right) + \left[\frac{1}{3}\Lambda R\right]\\ \dot{R}^2 &= +\frac{8\pi G\rho}{3}R^2 - kc^2 + \left[\frac{1}{3}\Lambda c^2 R^2\right]\end{aligned} \tag{8.27}$$

Notes:

1. For $k = 0$: Eq. (8.27) $\longrightarrow$ Eq. (8.26).
2. $k \in \{-1, 0, +1\}$ determines the curvature of space.
3. The density, ρ, includes the contribution of all different kinds of energy (remember mass-energy equivalence!).
4. There is energy associated with the vacuum, parameterized by the parameter Λ.

The evolution of the Hubble parameter is ($\Lambda = 0$):

$$\left(\frac{\dot{R}}{R}\right)^2 = H^2(t) = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2} \tag{8.28}$$