Accretion in X-Ray Binaries

Introduction

X-ray binaries are powered by accretion
$\Longrightarrow$ need to look at accretion as a physical mechanism.

Unfortunately, this will have to be somewhat theoretical, but this cannot be avoided...

Structure of this chapter:

1. Accretion Luminosity: Eddington luminosity
2. Accretion Disks: Theory
3. Accretion Disks: Confrontation with observations

Literature

- J. Frank, A. King, D. Raine, 2002, Accretion Power in Astrophysics, 3rd edition, Cambridge Univ. Press

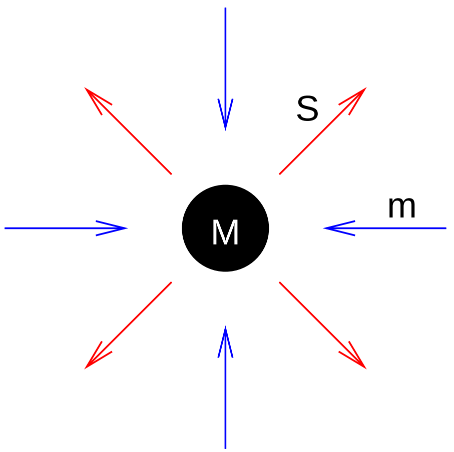
  The standard textbook on accretion, covering all relevant areas of the field.
- T. Padmanabhan, 2001, Theoretical Astrophysics, II. Stars and Stellar Systems, Cambridge Univ. Press

  See introduction to this lecture.
- J.E. Pringle, 1981, Accretion Disks in Astrophysics, Ann. Rev. Astron. Astrophys. **19**, 137

  Concise review of classical accretion disk theory.
- N.I. Shakura & R. Sunyaev, 1973, Black Holes in Binary Systems. Observational Appearance. Astron. Astrophys. **24**, 337 and J.E. Pringle & M. Rees, 1972, Accretion Disc Models for Compact X-Ray Sources, Astron. Astrophysi, 22(1), 1

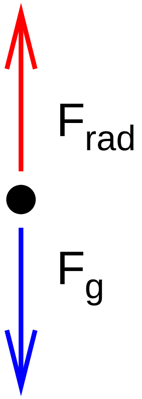
  The fundamental papers, which *really* started the field.

Eddington luminosity, IV



Assume mass M spherically symmetrically accreting ionized hydrogen gas.
At radius r, accretion produces energy flux S.
Important: Interaction between accreted material and radiation!

Eddington luminosity, IX



Accretion is only possible if gravitation dominates:

$$\frac{GMm_p}{r^2} > \frac{\sigma_T S}{c} = \frac{\sigma_T}{c} \cdot \frac{L}{4\pi r^2} \qquad (4.8)$$

and therefore

$$L < L_{Edd} = \frac{4\pi GMm_p c}{\sigma_T} \qquad (4.9)$$

or, in astronomically meaningful units

$$L < 1.3 \times 10^{38}\,\mathrm{erg\,s^{-1}} \cdot \frac{M}{M_\odot} \qquad (4.10)$$

where L_{Edd} is called the Eddington luminosity.

But remember the assumptions entering the derivation: spherically symmetric accretion of fully ionized pure hydrogen gas.

Eddington luminosity, X

Characterize accretion process through the accretion efficiency, η:

$$L = \eta \cdot \dot{M}c^2 \qquad (4.11)$$

where $\dot{M}$: mass accretion rate (e.g., $\mathrm{g\,s^{-1}}$ or $M_\odot\,\mathrm{yr^{-1}}$).

Therefore maximum accretion rate ("Eddington rate"):

$$\dot{m} = \frac{L_{Edd}}{\eta c^2} \sim 2 \times 10^{-8} \cdot \left(\frac{M}{1\,M_\odot}\right) M_\odot\,\mathrm{yr^{-1}} \qquad (4.12)$$

(for $\eta = 0.1$)

Emitted spectrum

Characterize photon by its radiation temperature, T_{rad}:

$$h\nu \sim kT_{rad} \implies T_{rad} = h\nu/k \qquad (4.19)$$

Optically thick medium: blackbody radiation

$$T_b = \left(\frac{L}{4\pi R^2 \sigma_{SB}}\right)^{1/4} \qquad (4.20)$$

Optically thin medium: L directly converted into radiation without further interactions $\Longrightarrow$ mean particle energy

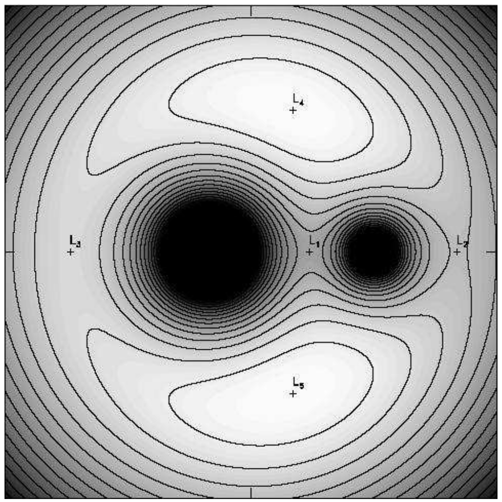
$$T_{th} = \frac{GMm_p}{3kR} \qquad (4.21)$$

Plugging in numbers for a typical solar mass compact object (NS/BH):

$$T_{rad} \sim 1\,\mathrm{keV} \quad \text{and} \quad T_{bb} \sim 50\,\mathrm{MeV} \qquad (4.22)$$

Accreting objects are broadband emitters in the X-rays and gamma-rays.

Roche Geometry, I



R. Hynes

Motion of gas in corotating frame around masses M_1, M_2 given by

$$\frac{d^2\boldsymbol{r}}{dt^2} + 2\boldsymbol{\omega} \times \frac{d\boldsymbol{r}}{dt} = -\frac{1}{\rho}\boldsymbol{\nabla}P - \boldsymbol{\nabla}\Phi$$

where the Roche potential:

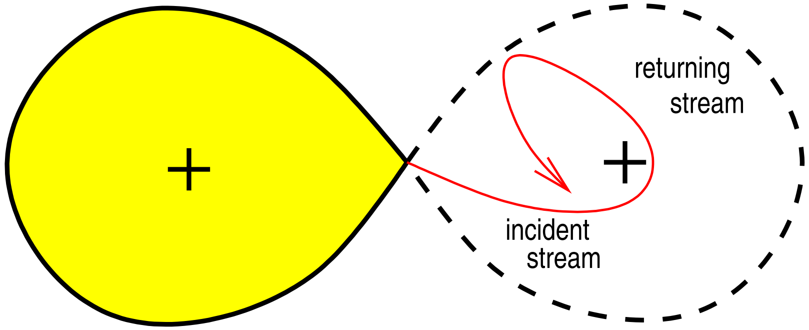
$$\Phi_R(\boldsymbol{r}) = -\frac{GM_1}{|\boldsymbol{r}-\boldsymbol{r}_1|} - \frac{GM_2}{|\boldsymbol{r}-\boldsymbol{r}_2|} - \frac{1}{2}(\boldsymbol{\omega}\times\boldsymbol{r})^2$$

and where

$$\boldsymbol{\omega} = \left(\frac{GM}{a^3}\right)^{1/2} \hat{e}$$

Matter comes from companion star
$\Longrightarrow$ accreted matter has angular momentum
$\Longrightarrow$ accretion disk forms.

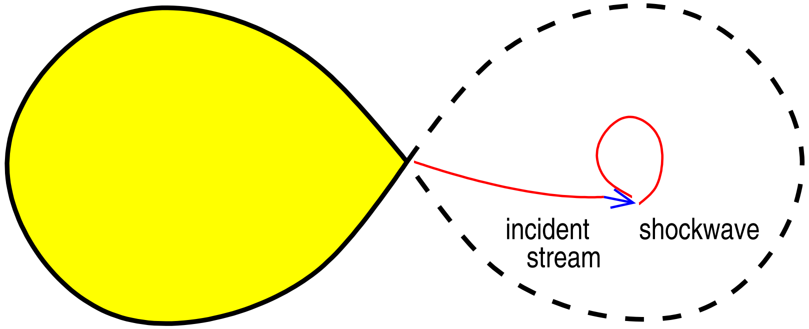
Roche Lobe Overflow, I



(after Lubow & Shu, 1975, Fig. 4)

Roche Lobe Accretion: Gas is transferred at inner Lagrange point.

Roche Lobe Overflow, II



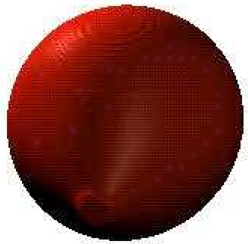
(after Lubow & Shu, 1975, Fig. 4)

Roche Lobe Accretion: Gas is transferred at inner Lagrange point.

Ballistic free fall towards compact object, forms elliptical orbit

Note: ellipse rotates because of Coriolis force!

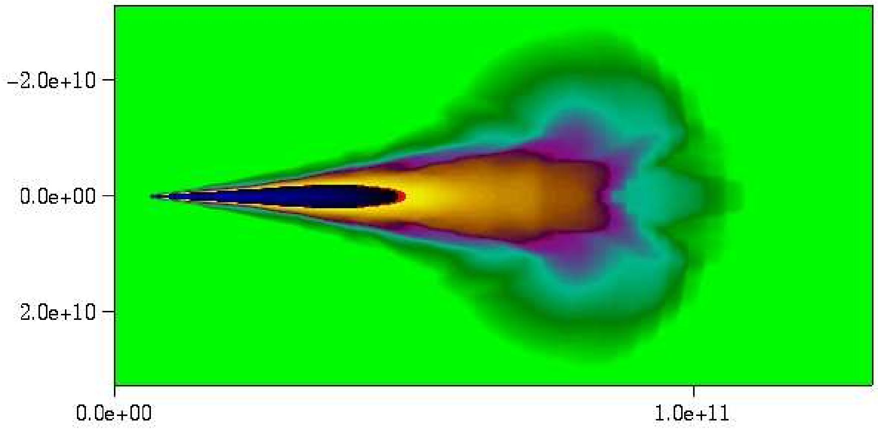
Stream intersects $\Longrightarrow$ shock $\Longrightarrow$ randomization $\Longrightarrow$ circular orbit forms.



Numerical simulation of disk formation by J. Blondin (NCSU)

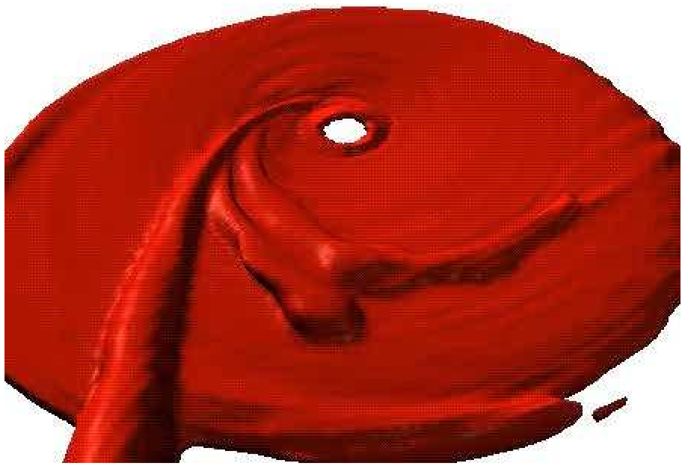
Stream is well described by ballistic motion, outer disk radius at ~0.5 Roche Lobe radius.

Roche Lobe Overflow, IV



J. Blondin

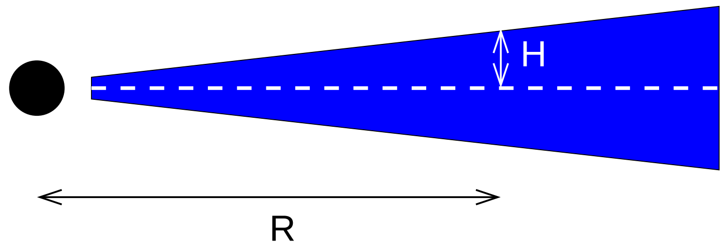
Disk is flared at outer radii due to accretion stream impact.



J. Blondin

Shock forms over large parts of the disk.

Thin Disks, II



Thin disk: no radiation pressure

$\Longrightarrow$ gas pressure must support disk vertically against gravitation:

$$\frac{GM}{R^2}\frac{H}{R} = \frac{1}{\rho}\left|\frac{\partial P}{\partial z}\right| \sim \frac{P_c}{\rho_c H} \tag{4.24}$$

where P_c characteristic pressure, ρ_c characteristic density.

Thin Disks, III

The speed of sound is given by

$$c_s^2 = \frac{P}{\rho} \tag{4.25}$$

therefore the condition for vertical support can be written as

$$\frac{GM}{R^2}\frac{H}{R} \sim \frac{P_c}{\rho_c H} = \frac{c_s^2}{H} \tag{4.26}$$

such that

$$c_s^2 = \frac{GM}{R}\frac{H^2}{R^2} = v_\phi^2 \cdot \frac{H^2}{R^2} \tag{4.27}$$

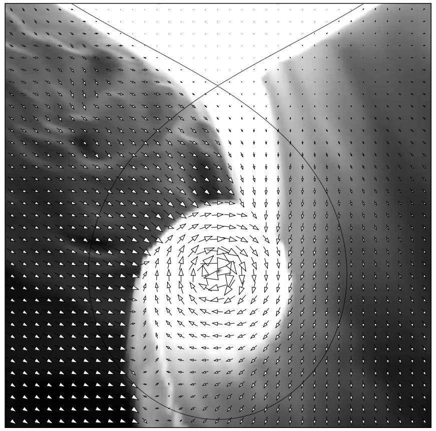
where $v_\phi = \sqrt{GM/R} = 1.2 \times 10^{10}\,(M/M_\odot)(R/10^6\,\mathrm{cm})^{-1}\,\mathrm{cm\,s^{-1}}$: Kepler speed.

Since $H/R \ll 1$:

$$c_s \ll v_\phi \tag{4.28}$$

Thin accretion disks are highly supersonic.

Thin Disks: Radial Structure



J. Blondin (priv. comm.)

Radial acceleration due to pressure:

$$\frac{1}{\rho}\frac{\partial P}{\partial R} \sim \frac{P_\mathrm{c}}{\rho_\mathrm{c} R} \sim \frac{c_\mathrm{s}^2}{R} \sim \frac{GM}{R^2}\frac{H^2}{R^2} \ll \frac{GM}{R^2} \qquad (4.29)$$

$\Longrightarrow$ radial acceleration due to pressure negligable compared to gravitational acceleration

Thin disk: fluid motion is Keplerian to very high degree of precision.

$\Longrightarrow$ for the radial velocity, v_R: $v_R \ll v_\phi$

Thin Disks: Vertical Structure and Mass Conservation

Amount of mass crossing radius R:

$$\dot{M} = -2\pi R \cdot \Sigma \cdot v_R \qquad (4.30)$$

where Σ: surface density of disk,

$$\Sigma(R) = \int n(r)dz \qquad (4.31)$$

and where $\dot{M}$: mass accretion rate

Since acceleration $\perp z$

$$F_z \propto \frac{GM}{R^2}\frac{z}{R} \propto z \qquad (4.32)$$

vertical density profile

$$n(z) \propto \exp\left(-\frac{z}{H}\right) \qquad (4.33)$$

where H: scale height (depends on details of accretion disk theory).

Thin Disks: Angular Momentum Transport, I

Most important question: angular momentum transport

Angular velocity in Keplerian disk:

$$\omega(R) = \left(\frac{GM}{R^3}\right)^{1/2} \qquad (4.34)$$

("differential rotation")

$\Longrightarrow$ angular momentum per mass ("specific angular momentum"):

$$\mathcal{L} = R \cdot v = R \cdot R\omega(R) = R^2\,\omega(R) \propto R^{1/2} \qquad (4.35)$$

$\Longrightarrow$ decreases with decreasing R!

Total angular momentum lost when mass moves in unit time from $R + dR$ to R:

$$\frac{dL}{dR} = \dot{M} \cdot \frac{d(R^2\omega(R))}{dR} \qquad (4.36)$$

Thin Disks: Angular Momentum Transport, II

Since L changes: accreting matter needs to lose angular momentum. This is done by viscous forces excerting torques:

Force due to viscosity per unit length:

$$\mathcal{F} = \nu\Sigma \cdot \Delta v = \nu\Sigma \cdot R\frac{d\omega}{dR} \qquad (4.37)$$

where ν: coefficient of kinematic viscosity

Therefore total torque

$$G(R) = 2\pi R\mathcal{F} \cdot R = \nu\Sigma 2\pi R^3 \left(\frac{d\omega}{dR}\right) \qquad (4.38)$$

and the net torque acting on a ring is

$$\frac{dG}{dR}dR \qquad (4.39)$$

$\Longrightarrow$ This net torque needs to balance change in specific angular momentum in disk.

Thin Disks: Angular Momentum Transport, III

Balancing net torque and angular momentum loss gives:

$$\dot{M}\frac{d(R^2\omega)}{dR} = -\frac{d}{dR}\left(\nu\Sigma 2\pi R^3 \frac{d\omega}{dR}\right) \tag{4.40}$$

Insert $\omega(R) = (GM/R^3)^{1/2}$ and integrate:

$$\nu\Sigma R^{1/2} = \frac{\dot{M}}{3\pi} R^{1/2} + \text{const.} \tag{4.41}$$

const. obtained from no torque boundary condition at inner edge of disk at $R = R_*$: $dG/dR(R_*) = 0$, such that

$$\nu\Sigma = \frac{\dot{M}}{3\pi}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right] \tag{4.42}$$

Therefore the viscous dissipation rate per unit area is

$$D(R) = \nu\Sigma\left(R\frac{d\omega}{dR}\right)^2 = \frac{3GM\dot{M}}{4\pi R^3}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right] \tag{4.43}$$

Thin Disks: Temperature Profile, I

The viscous dissipation rate was

$$D(R) = \nu\Sigma\left(R\frac{d\omega}{dR}\right)^2 = \frac{3GM\dot{M}}{4\pi R^3}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right] \tag{4.43}$$

If disk is optically thick: Thermalization of dissipated energy

$\Longrightarrow$ Temperature from Stefan-Boltzmann-Law:

$$2\sigma_{\text{SB}} T^4 = D(R) \tag{4.44}$$

(disk has *two* sides!) and therefore

$$T(R) = \left\{\frac{3GM\dot{M}}{8\pi R^3 \sigma_{\text{SB}}}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right]\right\}^{1/4} \tag{4.45}$$

Thin Disks: Temperature Profile, IV

Inserting astrophysically meaningful numbers:

$$T(R) = \left\{\frac{3GM\dot{M}}{8\pi R^3 \sigma_{\text{SB}}}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right]\right\}^{1/4}$$
$$= 6.8 \times 10^5\,\text{K} \cdot \eta^{-1/4}\left(\frac{L}{L_{\text{Edd}}}\right)^{1/2} L_{46}^{-1/4}\mathcal{R}^{1/4}x^{-3/4}$$

where $\eta = L_{\text{Edd}}/\dot{M}_{\text{Edd}}c^2$, $x = c^2R/2GM$, $\mathcal{R} = (1 - (R_*/R)^{1/2})$.

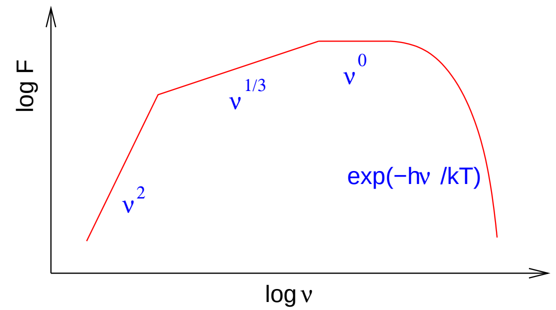
Radial dependence of T:

$$T(R) \propto R^{-3/4}$$

Dependence on mass (note: for NS/BH inner radius $R_* \propto M$!):

inner disk temperature $T_{\text{in}} \propto (\dot{M}/M^2)^{1/4}$

Thin Disks: Emitted Spectrum, I



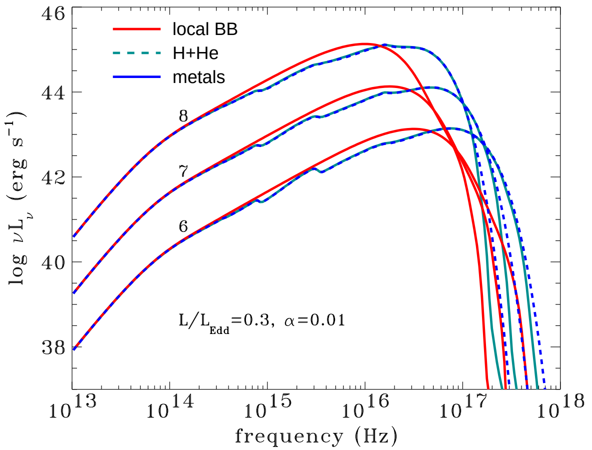
If disk is optically thick, then locally emitted spectrum is black body.

Total emitted spectrum obtained by integrating over disk

$$F_\nu = \int_{R_*}^{R_{\text{out}}} B(T(R))\, 2\pi R\, dR \tag{4.46}$$

Resulting spectrum looks essentially like a stretched black body.

Thin Disks: Emitted Spectrum, II



In reality: accretion disk spectrum depends on

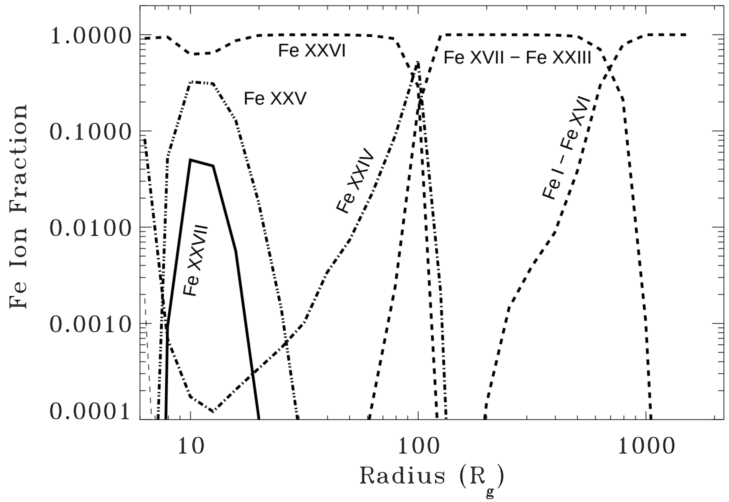
- elemental composition ("metallicity")
- viscosity ("α-parameter")
- ionization state and luminosity of disk ($\dot{M}$)
- properties of compact object

and many further parameters

Until today: no really satisfactory disk model available.

Hubeny et al., 2001, Fig. 13

Thin Disks: Emitted Spectrum, III



Fe species in a disk around a Galactic BH (Davis et al., 2005, Fig. 6)

Viscosity

Most important unknown in accretion disk theory: viscosity

even though it dropped out of $T(R)$!

Earth: viscosity of fluids typically due to molecular interactions (molecular viscosity).

Kinematic viscosity:

$$\nu_{\text{mol}} \sim \lambda_{\text{mfp}} c_{\text{s}} \tag{4.47}$$

where the mean free path

$$\lambda_{\text{mfp}} \sim \frac{1}{n\sigma} \sim 6.4 \times 10^4 \left(\frac{T^2}{n}\right) \text{cm} \tag{4.48}$$

and the speed of sound

$$c_{\text{s}} \sim 10^4 T^{1/2}\,\text{cm}\,\text{s}^{-1} \tag{4.49}$$

such that

$$\nu_{\text{mol}} \sim 6.4 \times 10^8\, T^{5/2} n^{-1}\,\text{cm}^2\,\text{s}^{-1} \tag{4.50}$$

Viscosity

Viscosity important when Reynolds number small ("laminar flow"), where

$$\text{Re} = \frac{\text{inertial force}}{\text{viscous force}} \sim \frac{\rho R v}{\rho \nu} = \frac{Rv}{\nu} \tag{4.51}$$

Follows from Navier-Stokes Equations

Using typical accretion disk parameters:

$$\text{Re}_{\text{mol}} \sim 2 \times 10^{14} \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{R}{10^{10}\,\text{cm}}\right)^{1/2} \left(\frac{n}{10^{15}\,\text{cm}^{-3}}\right) \left(\frac{T}{10^4\,\text{K}}\right)^{-5/2} \tag{4.52}$$

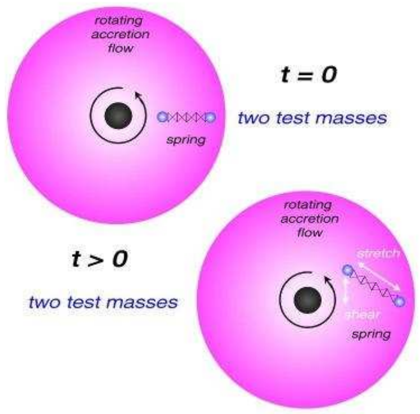
$\Longrightarrow$ Molecular viscosity is irrelevant for astrophysical disks!

since Re $\gtrsim 10^3$: turbulence $\Longrightarrow$ Shakura & Sunyaev posit turbulent viscosity

$$\nu_{\text{turb}} \sim v_{\text{turb}} \ell_{\text{turb}} \sim \alpha\, c_{\text{s}} \cdot H \tag{4.53}$$

where $\alpha \lesssim 1$ and $\ell_{\text{turb}} \lesssim H$ typical size for turbulent eddies.

Viscosity



R. Müller

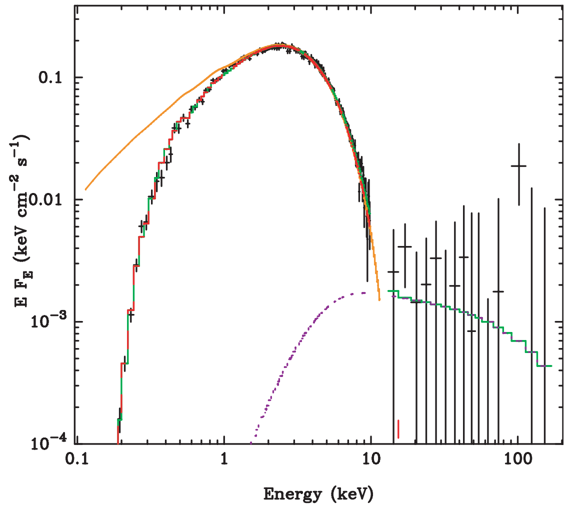
Mechanical analogy of MRI: spring in differentially rotating medium.

Physics of turbulent viscosity is unknown, however, α prescription yields good agreement between theory and observations.

Possible origin: Magnetorotational instability (MRI): MHD instability amplifying B-field inhomogeneities caused by small initial radial displacements in accretion disk $\Longrightarrow$ angular momentum transport

(Balbus & Hawley 1991, going back to Velikhov 1959 and Chandrasekhar (1961).

Galactic Black Holes

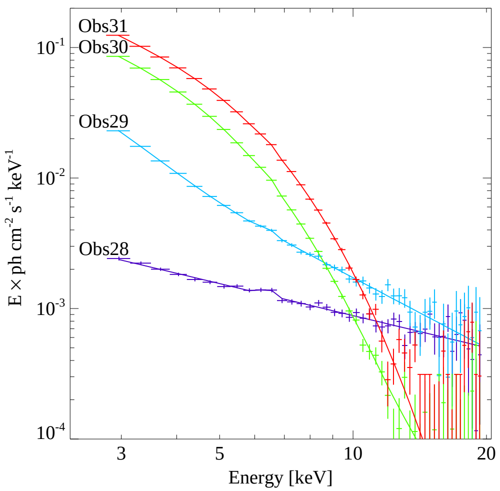


(Davis, Done & Blaes, 2006)

Comparison of self-consistent accretion disk model with LMC X-3 data $\Longrightarrow$ good agreement, although values of α smaller than expected (fits find $0.01 < \alpha < 0.1$ instead of 0.1–0.8).

Top red line: inferred accretion disk spectrum without interstellar absorption.

Galactic Black Holes



LMC X-3, (Wilms et al., 2001)

Problem with AGN: peak of disk in UV

$\Longrightarrow$ Galactic Black Holes: T is higher

Find ok agreement between accretion disk models and theory.

In general: models with just $T \propto r^{-3/4}$ and no additional (atomic) physics seem to work best?!?