Wind Accretion

Early type stars (O, B, mass $\gtrsim 10\,M_\odot$):

- strong winds, driven by radiation pressure in absorption lines
- mass loss rates 10^{-10} to $10^{-6}\,M_\odot\,\mathrm{yr}^{-1}$
- Wind velocity

$$v(r) \sim v_\infty \left(1 - \frac{R_\star}{r}\right)^\beta \tag{4.54}$$

with $v_\infty \sim 2000\,\mathrm{km\,s^{-1}}$ and $\beta \sim 0.5 \ldots 1.0$

A fraction of the wind can be accreted by a compact object

$\Longrightarrow$ ~spherical accretion

$\Longrightarrow$ Bondi-Hoyle accretion

(Bondi & Hoyle, 1944)

The simplest case of wind accretion is spherically symmetric accretion.

For spherically symmetric accretion, we can derive the exact solution for the gas flow from the equations of gas dynamics:

Conservation of mass is described by the continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0 \tag{4.55}$$

while the conservation of momentum is described by the Euler equation

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} + \rho \boldsymbol{v} \cdot \nabla \boldsymbol{v} \cdot \nabla \boldsymbol{v} = -\nabla P + \boldsymbol{f} \tag{4.56}$$

where $\boldsymbol{f}$ is a force density (force per unit volume).

By definition, in the spherically symmetric case the flow has only a radial component. Furthermore, if the flow is steady, then all time derivatives vanish. This means that the equation of continuity now reads

$$\nabla \cdot (p \boldsymbol{v}) = \frac{1}{r^2} \frac{d}{dr} \left(r^2 \rho v\right) = 0 \tag{4.57}$$

and therefore

$$r^2 \rho v = \text{const.} = C \tag{4.58}$$

The constant is related to the mass accretion rate: Since the inward flux of mass is given by $\rho|v|$, the mass accretion rate is

$$\dot{M} = 4\pi r^2 \rho |v| \tag{4.59}$$

and therefore

$$C = \frac{\dot{M}}{4\pi} \tag{4.60}$$

To obtain the velocity profile, we use the Euler equation (Eq. 4.56). Because of Newton's law of gravitation

$$\boldsymbol{F} = \frac{GMm}{r^2} \frac{\boldsymbol{r}}{r} \tag{4.61}$$

the force density has a radial component only and is given by

$$f = -\frac{GM\rho}{r^2} \tag{4.62}$$

Inserting this into Euler's equation then results in

$$\rho v \frac{dv}{dr} = -\frac{dP}{dr} - \frac{GM\rho}{r^2} \tag{4.63}$$

which simplifies to

$$v \frac{dv}{dr} = -\frac{GM}{r^2} - \frac{1}{\rho} \frac{dP}{dr} \tag{4.64}$$

This differential equation can be solved under the boundary condition of some velocity at infinity. Furthermore, we need to know the equation of state, i.e., how the pressure relates to other quantities in the system. Here, we will be using the polytropic equation of state

$$P = K \rho^\gamma \tag{4.65}$$

where K is some constant. As shown in lectures on thermodynamics, if the gas is isothermal, then $\gamma = 1$, if the flow is adiabatic instead, then $\gamma = 5/3$ (γ is the ratio of specific heats).

With this equation of state, the speed of sound is

$$c_\mathrm{s}^2 = \frac{\partial P}{\partial \rho} = K \gamma \rho^{\gamma - 1} \tag{4.66}$$

We now insert the equation of state into Eq. (4.64):

$$v \frac{dv}{dr} = -\frac{GM}{r^2} - \frac{1}{\rho} \gamma K \rho^{-\gamma-1} \frac{d\rho}{dr} = -\frac{GM}{r^2} - c_\mathrm{s}^2 \frac{1}{\rho} \frac{d\rho}{dr} \tag{4.67}$$

But because of Eq. (4.57)

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \rho v\right) = 0 \tag{4.57}$$

we have

$$\frac{1}{r^2} \left(\frac{d\rho}{dr} \left(r^2 v\right) + \rho \frac{d}{dr} \left(r^2 v\right) \right) = 0 \iff \frac{1}{\rho} \frac{d\rho}{dr} = -\frac{1}{vr^2} \frac{d}{dr} \left(r^2 v\right) \tag{4.68}$$

Inserting this into Eq. (4.67) gives

$$v \frac{dv}{dr} = -\frac{GM}{r^2} - c_\mathrm{s}^2 \frac{1}{vr^2} \frac{d}{dr} \left(r^2 v\right) = -\frac{GM}{r^2} - c_\mathrm{s}^2 \left(-\frac{2}{r} - \frac{1}{v} \frac{dv}{dr} \right) \tag{4.69}$$

Multiplying by v then results in

$$v^2 \frac{dv}{dr} = -\frac{GMv}{r^2} + \frac{2v}{r} c_\mathrm{s}^2 + c_\mathrm{s}^2 \frac{dv}{dr} \tag{4.70}$$

and therefore

$$\left(v^2 - c_\mathrm{s}^2\right) \frac{dv}{dr} = v \left(\frac{2c_\mathrm{s}^2}{r} - \frac{GM}{r^2} \right) \tag{4.71}$$

Bondi-Hoyle Accretion, I

Spherical symmetric accretion:

$$\left(v^2 - c_\mathrm{s}^2\right) \frac{dv}{dr} = v \left(\frac{2c_\mathrm{s}^2}{r} - \frac{GM}{r^2} \right) \tag{4.71}$$

For r large: right hand side is positive.

Since $dv/dr < 0$ for accretion, this means that for large r: $v < c_\mathrm{s}$.

Similarly, for small r: $v > c_\mathrm{s}$

$\Longrightarrow$ sonic point for $v = c_\mathrm{s}$ at

$$r_\mathrm{sonic} = \frac{GM}{2c_\mathrm{s}^2} \tag{4.72}$$

$\Longrightarrow$ *If* the flow goes supersonic, it does so at $r = r_\mathrm{sonic}$

Note that c_s depends on r, several other solutions are possible, but the above one is the most common one for the objects we're looking at. See Holzer & Axford (1970) for details.

Bondi-Hoyle Accretion, II

To finish the discussion of Bondi-Hoyle accretion, we now explicitly integrate Euler's equation

$$v\frac{dv}{dr} + \frac{GM}{r^2} + \frac{1}{\rho}\frac{dP}{dr} = 0 \tag{4.64}$$

over r:

$$\int v\frac{dv}{dr}\,dr + \int \frac{GM}{r^2}\,dr + \int \frac{dP}{\rho} = 0 \tag{4.73}$$

inserting $dP = K\gamma\rho^{\gamma-1}d\rho$ and integrating then gives the Bernoulli integral

$$\frac{1}{2}v^2 + \frac{\gamma}{\gamma-1}K\rho^{\gamma-1} - \frac{GM}{r} = \text{const.} \tag{4.74}$$

which obviously is related to energy conservation and can be written as

$$\frac{1}{2}v^2 + \frac{c_\mathrm{s}^2}{\gamma-1} - \frac{GM}{r} = \text{const.} = \frac{c_{\mathrm{s},\infty}}{\gamma-1} \tag{4.75}$$

where $c_{\mathrm{s},\infty}$ is the speed of sound at $r = \infty$.

This follows since $v(r \to \infty) = 0$.

Bondi-Hoyle Accretion, III

From Eq. (4.75) we can now determine the speed of sound at the sonic point

$$c_\mathrm{s}^2(r_\mathrm{sonic}) = c_{\mathrm{s},\infty}\left(\frac{2}{5-3\gamma}\right)^{1/2} \tag{4.76}$$

and the mass accretion rate is

$$\dot{M} = 4\pi r^2\rho|v| = 4\pi r_\mathrm{sonic}^2\rho(r_\mathrm{sonic})c_\mathrm{s}(r_\mathrm{sonic}) \tag{4.77}$$

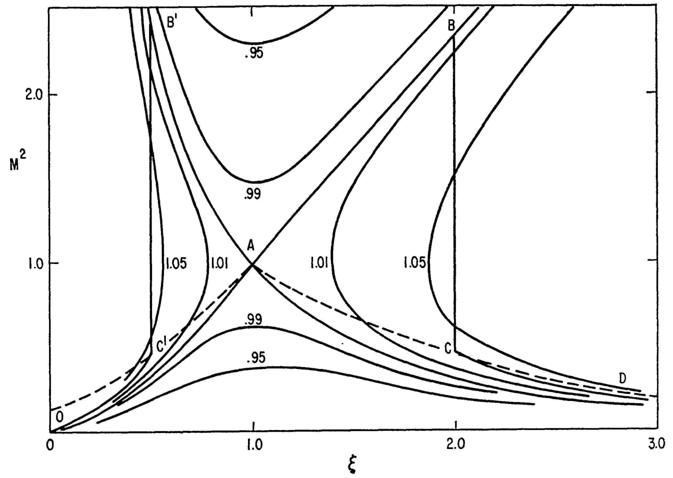
Since $c_\mathrm{s}^2 \propto \rho^{\gamma-1}$,

$$\rho(r_\mathrm{sonic}) = \rho_\infty\left(\frac{c_\mathrm{s}(r_\mathrm{sonic})}{c_{\mathrm{s},\infty}}\right)^{2/(\gamma-1)} \tag{4.78}$$

Therefore

$$\dot{M} = \pi G^2M^2\frac{\rho_\infty}{c_{\mathrm{s},\infty}^3}\left(\frac{2}{5-3\gamma}\right)^{(5-3\gamma)/2(\gamma-1)} \tag{4.79}$$

Bondi-Hoyle Accretion, IV



Mach number ($M = v(r)/c_\mathrm{s}(r)$) as a function of radial distance, $\xi = r/r_\mathrm{sonic}$, for all possible solutions of the spherical accretion problem.

(Holzer & Axford, 1970, Fig. 1)

Bondi-Hoyle Accretion, V

Taking $\gamma = 5/3$, Eq. (4.79) becomes

$$\begin{aligned}\dot{M} &= \pi G^2M^2\frac{\rho_\infty}{c_{\mathrm{s},\infty}^3} \\ &= \pi\left(\frac{GM}{c_{\mathrm{s},\infty}^2}\right)^2\rho_\infty c_{\mathrm{s},\infty} \\ &= \pi r_\mathrm{acc}^2\rho_\infty c_{\mathrm{s},\infty}\end{aligned} \tag{4.80}$$

where the accretion radius is defined as

$$r_\mathrm{acc} = \frac{GM}{c_{\mathrm{s},\infty}^2} \tag{4.81}$$

Often, r_acc is defined as $r_\mathrm{acc} = 2GM/c_\mathrm{s}$, see next slide for the reason why.

r_acc defines the approximate radius of influence of an accreting body.

Wind accretion, I

If the ambient medium is not at rest: **wind accretion**. In principle, we can do a similar calculation as for Bondi-Hoyle accretion, however, this would take too long, so let's do an approximate treatment here.

Let the wind's velocity be v_∞. The material in the wind is captured once

$$\frac{1}{2}v_\infty^2 = \frac{GM}{r_\text{acc}} \tag{4.82}$$

such that the accretion radius for wind accretion is

$$r_\text{acc} = \frac{2GM}{v_\infty^2} \tag{4.83}$$

...explaining why many people like to have a factor 2 also in the definition of r_acc for Bondi-Hoyle accretion.

Therefore, analoguously to Eq. (4.80),

$$\dot{M} = \pi r_\text{acc}^2 \rho_\infty v_\infty = \frac{4\pi G^2 M^2 \rho_\infty}{v_\infty^3} \tag{4.84}$$

Wind accretion, II

To estimate the typical parameters of a wind accretor, we need to estimate v_∞ for a compact object at a distance a from the donor star

The typical velocity consists of two contributions:

1. The stellar wind velocity profile

$$v_\text{wind}(a) \sim v_{\text{wind},\infty}\left(1 - \frac{R_\star}{a}\right)^\beta \tag{4.54}$$

2. The orbital velocity of the compact object

$$v_\text{compact}(a) = \sqrt{\frac{GM}{a}} \tag{4.85}$$

Therefore

$$v_\infty^2 \sim v_\text{wind}^2 + v_\text{compact}^2 = \frac{GM}{a} + v_{\text{wind},\infty}^2\left(1 - \frac{R_\star}{a}\right)^{2\beta} \sim \frac{GM}{a} + v_{\text{wind},\infty}^2 \tag{4.86}$$

the last is true assuming that the compact object is outside of the wind acceleration zone

Wind accretion, III

Finally, making use of the fact that the wind density is

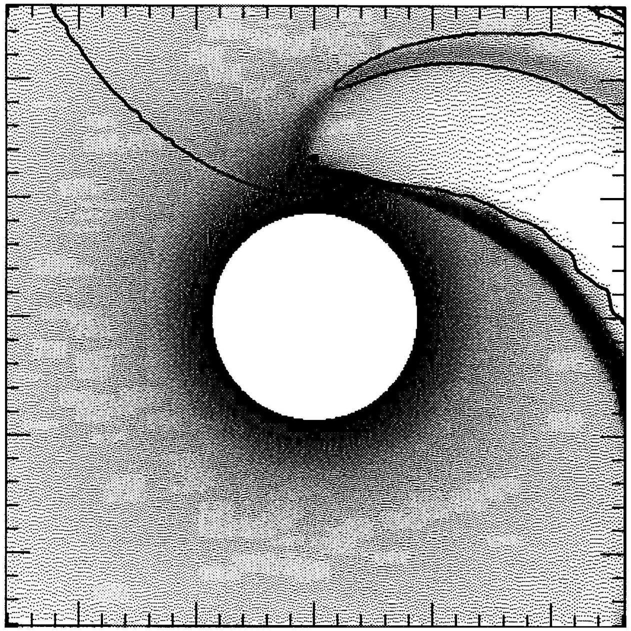
$$\rho_\infty = \frac{\dot{M}_\text{W}}{4\pi a^2 v_{\text{wind},\infty}} \tag{4.87}$$

where $\dot{M}_\text{W}$ is the wind loss rate of the donor.

Therefore, the accretion rate of the compact object is

$$\begin{aligned}\dot{M} &= \frac{G^2M^2}{a^2 v_{\text{wind},\infty}\left(\frac{GM}{a} + v_{\text{wind},\infty}^2\right)^{3/2}}\dot{M}_\text{W} \\ &= \begin{cases}\left(\frac{GM}{a v_{\text{wind},\infty}^2}\right)^{1/2}\dot{M}_\text{W} & \text{for } v_\text{orbit} \gg v_{\text{wind},\infty} \\ \frac{G^2M^2}{a^2 v_{\text{wind},\infty}^4}\dot{M}_\text{W} & \text{for } v_\text{orbit} \ll v_{\text{wind},\infty}\end{cases}\end{aligned} \tag{4.88}$$

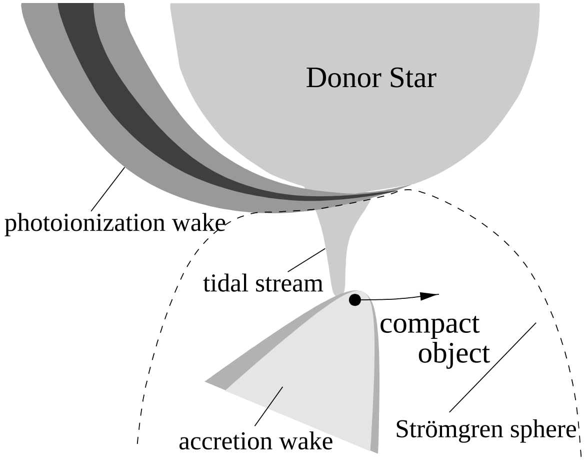
So, for $M = 1.44\,M_\odot$, $v_{\text{wind},\infty} = 500\,\text{km}\,\text{s}^{-1}$, $a = 10^7\,\text{km}$, $\dot{M} = 6 \times 10^{-3}\dot{M}_\text{W}$, i.e., the Eddington rate ($\dot{M}_\text{Edd} = 2.9 \times 10^{-8}\,M_\odot\,\text{yr}^{-1}$ for $1.44\,M_\odot$) is reached for $\dot{M}_\text{W} = 4.8 \times 10^{-6}\,M_\odot\,\text{yr}^{-1}$, which is very realistic.



(Blondin 1994, Fig. 4)

Realistic hydrodynamical computations are difficult (asymmetry of accretion process, ionization of wind, large range of length-scales involved,...).

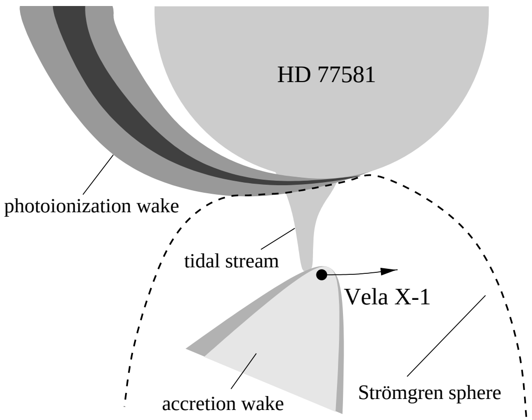
Accretion in HMXB



Principal components for wind-accretion:

- Ionized Strömgren region (wind ionized by X-rays from compact object).
- Accretion shock around compact object (since $v_{\mathrm{orb}} > c_{\mathrm{s}}$).
- Ionization wake where material is overdense.

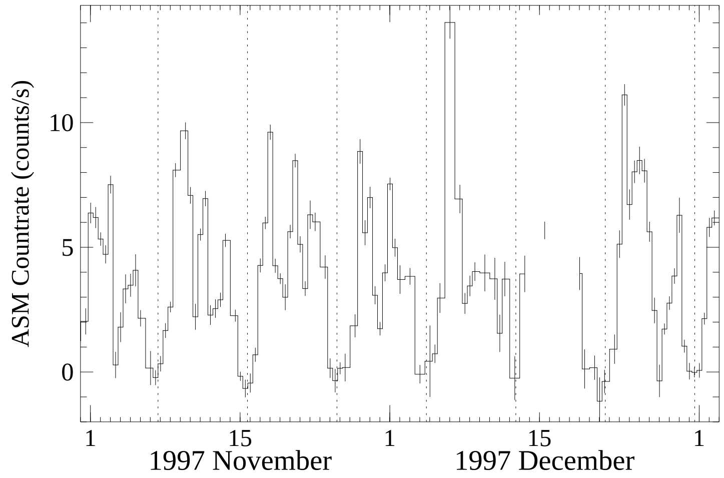
Accretion in HMXB



In realistic HMXB, because the accreted material still has some angular momentum, a small accretion disk still forms.

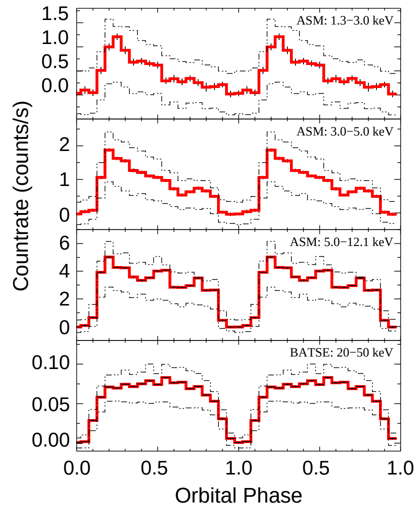
J. Blondin: "The disk is being BASHED by the stellar wind, BATTERED by the tidal stream, and BLASTED by X-rays"

Accretion in HMXB

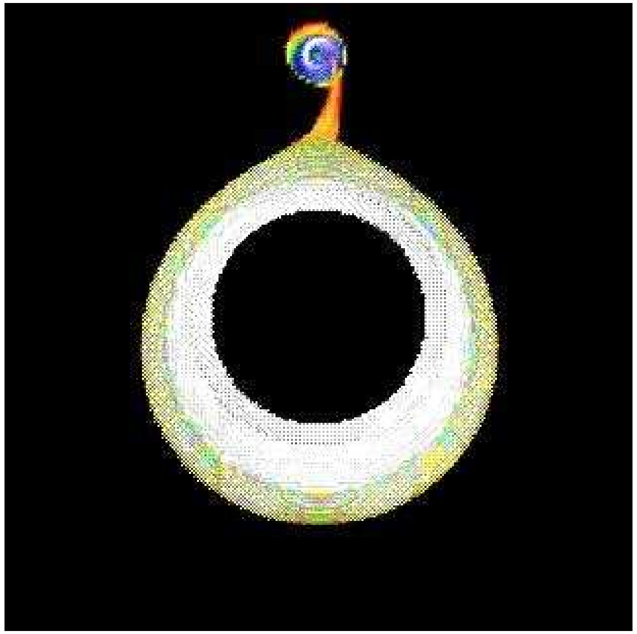


RXTE-ASM 2–10 keV lightcurve of the HMXB Vela X-1

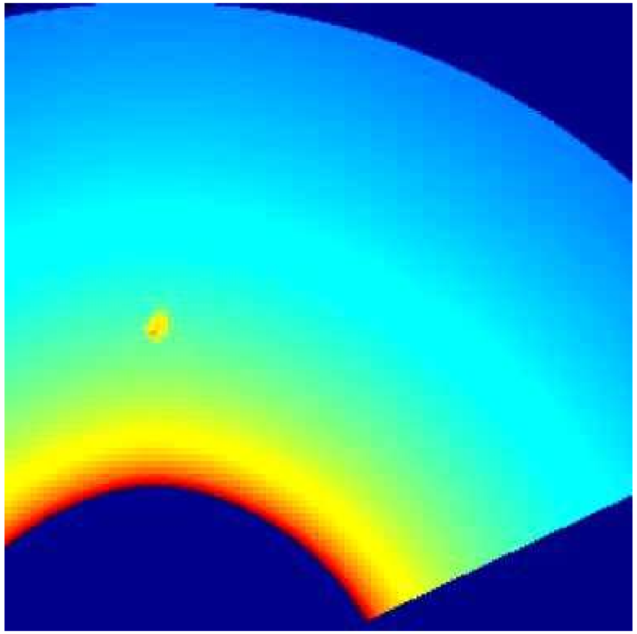
Accretion in HMXB



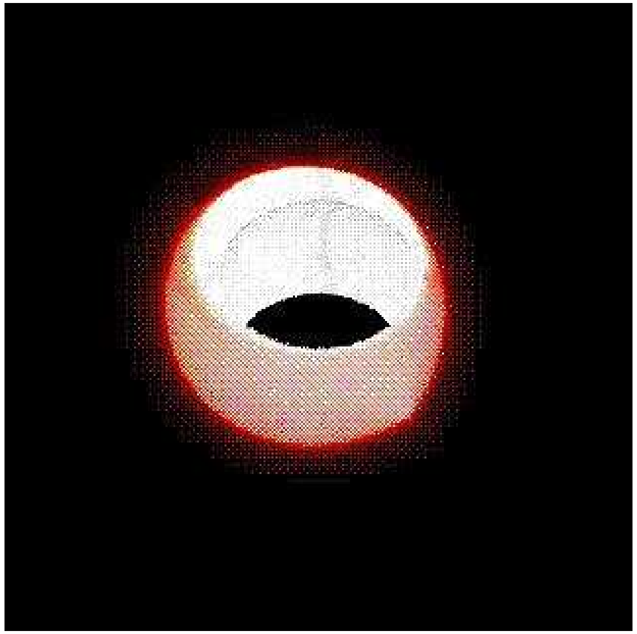
Orbit averaged, energy resolved lightcurves of Vela X-1



courtesy J. Blondin



courtesy J. Blondin



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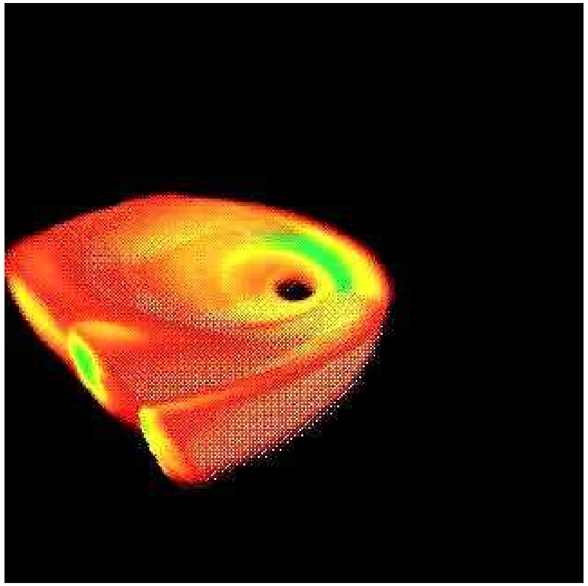

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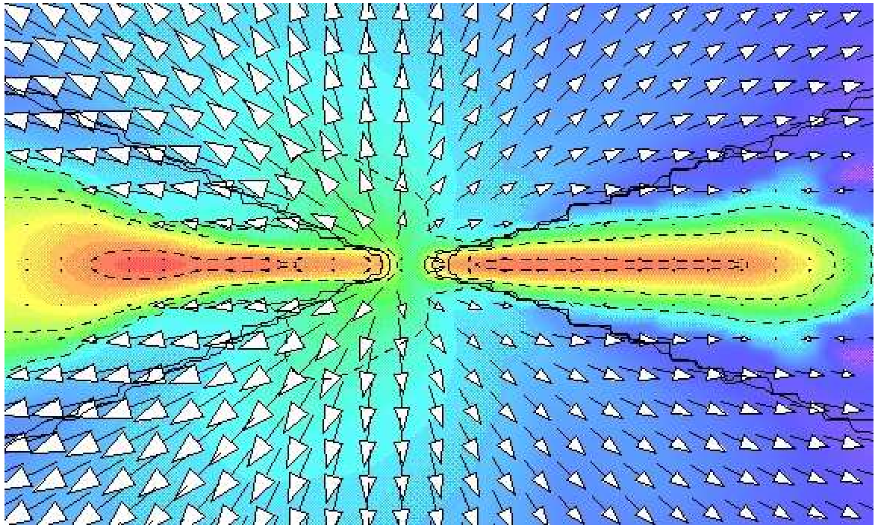

courtesy J. Blondin

X-rays from central source heat disk surface, drive a strong wind.

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