In the following, a quick derivation of the basic physics of electrons in strong magnetic fields is given, following Mészáros (1992). Note that this is rather advanced quantum mechanics, and while you should know the end results, the detailed derivation is beyond the scope of the lecture.

The (relativistic) equation of motion of an electron in a magnetic field is given by

$$\frac{d}{dt}(\gamma m \boldsymbol{v}) = \frac{q}{c}\boldsymbol{v}\times\boldsymbol{B} \tag{5.12}$$

where the Lorentz factor, γ, is

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \tag{5.13}$$

If we ignore the radiative losses caused by the acceleration of the electron, then $\gamma =$ const. and the equation of motion is

$$\frac{d\boldsymbol{v}_\parallel}{dt} = 0 \tag{5.14}$$

$$\frac{d\boldsymbol{v}_\perp}{dt} = \frac{q}{c}\boldsymbol{v}_\perp\times\boldsymbol{B} \tag{5.15}$$

where $\boldsymbol{v}_\parallel$ and $\boldsymbol{v}_\perp$ are the components of the velocity vector parallel and perpendicular to the magnetic field, respectively.

Since the force on $\boldsymbol{v}_\perp$ is perpendicular to $\boldsymbol{v}_\perp$, the motion of the electron will be described by a circular motion around the $\boldsymbol{B}$-field line together with constant motion parallel to the magnetic field.

It is easy to see (check for yourself!) that the frequency of motion around the magnetic field, the so-called *gyrofrequency* is given by

$$\omega_B = \frac{qB}{\gamma mc} = \frac{\omega_\mathrm{L}}{\gamma} \tag{5.16}$$

where the *Larmor frequency* is given by

$$\omega_\mathrm{L} = \frac{qB}{mc} \tag{5.17}$$

The Larmor frequency is the frequency of a non-relativistically gyrating electron.

Caveat emptor: many authors do not make a distinction between the Larmor frequency and the gyrofrequency. Be careful!

The radius of gyration, the *gyro radius*, is

$$r_g = \gamma v_\perp \frac{mc}{qB} = \gamma r_\mathrm{L} \tag{5.18}$$

and again, many authors do not distinguish between the gyro radius and the *Larmor radius*.

Finally, the cyclotron energy of the electron is

$$\hbar\omega_g = \hbar\frac{qB}{\gamma mc} \tag{5.19}$$

This classical approach performed so far is valid as long as the Larmor radius is large compared to the de Broglie wavelength of the electron,

$$\lambda_\mathrm{e} = \frac{\hbar}{p} = \frac{\hbar}{\gamma m v_\perp} \tag{5.20}$$

Therefore, QM effects are important once

$$\frac{\hbar}{\gamma m v_\perp} \ge \frac{v\gamma mc}{qB} \tag{5.21}$$

that is, for magnetic fields

$$B \ge \frac{m^2c^3\gamma^2}{q\hbar}\left(\frac{v_\perp}{c}\right)^2 = \gamma^2\beta_\perp^2\frac{m^2c^3}{e\hbar} = \gamma^2\beta_\perp^2 B_\mathrm{c} \tag{5.22}$$

here, $B_\mathrm{c} = m^2c^3/(q\hbar) \sim 4.4\times 10^{13}$ G is the *critical magnetic field*. This name derives from the fact that the nonrelativistic (!!!) cyclotron energy can be written as

$$\hbar\omega_\mathrm{L} = mc^2\frac{B}{B_\mathrm{c}} \tag{5.23}$$

that is, for $B = B_\mathrm{c}$ the nonrelativistic cyclotron energy equals the electron's rest mass. B_c is thus a natural quantum mechanical measure for the strength of magnetic fields.

Example: For $kT = 10$ keV, $\beta^2 = 5.6\times 10^{-2}$ and $\gamma\sim 1$, such that for $B\sim 10^{12}$ G quantum mechanics cannot be ignored.

Electrodynamics shows that the generalized momentum for the motion of particles in a magnetic field is

$$\boldsymbol{p} = m\boldsymbol{v} - \frac{e}{c}\boldsymbol{A} \tag{5.24}$$

where $\boldsymbol{A} = \nabla\times\boldsymbol{B}$, and (in the *Landau gauge*)

$$\boldsymbol{A} = \frac{1}{2}\boldsymbol{B}\times\boldsymbol{r} = \frac{1}{2}Br\hat{\varphi} \tag{5.25}$$

where $\hat{\varphi}$ is an unit vector along the azimuthal angle coordinate. The classical equation of motion for these particles is

$$\frac{mv^2}{r} = \frac{qv}{c}B \tag{5.26}$$

and the quantization condition is (similarly to the Bohr atom!)

$$\boldsymbol{p}\times\boldsymbol{r} = n\hbar\hat{B} \tag{5.27}$$

where $\hat{B}$ is an unit vector in the direction of the magnetic field.

Inserting the generalized momentum into the quantization condition gives after some algebra

$$mvr - \frac{1}{2}\frac{eB}{c}r^2 = n\hbar \tag{5.28}$$

After dividing by m and noticing that $v = \omega_g r$ one finds

$$\frac{1}{2}\omega_g r^2 = \frac{n\hbar}{m} \tag{5.29}$$

Using Eq. (5.26), the kinetic energy can then be found from

$$E_n = \frac{1}{2}mv^2 = \frac{evB}{2c}r = \frac{1}{2}\frac{eB}{c}\frac{2n\hbar}{m} = n\hbar\omega_g \tag{5.30}$$

Similarly one obtains for the orbital radii

$$r_n = \sqrt{\frac{2n\hbar c}{qB}} = 2.6\,\text{Å}\cdot\sqrt{\frac{2n}{B_{12}}} \tag{5.31}$$

where $B_{12} = B/10^{12}$ G.

The proper derivation of the Landau levels starts from the Schrödinger equation for the motion of a charged particle in an electromagnetic field. Using again the Landau gauge, the nonrelativistic Hamiltonian is

$$\hat{H} = \frac{1}{2m}\left(\hat{p} - \frac{q}{c}\hat{A}\right)^2 - \frac{q\hbar}{2mc}\hat{\sigma}\cdot\boldsymbol{B} \tag{5.32}$$

where $\hat{\sigma}$ is the Pauli spin operator.

To solve for the wave function ψ, insert the Hamiltonian into the Schrödinger equation

$$\hat{H}\psi = E\psi \tag{5.33}$$

and perform the ansatz

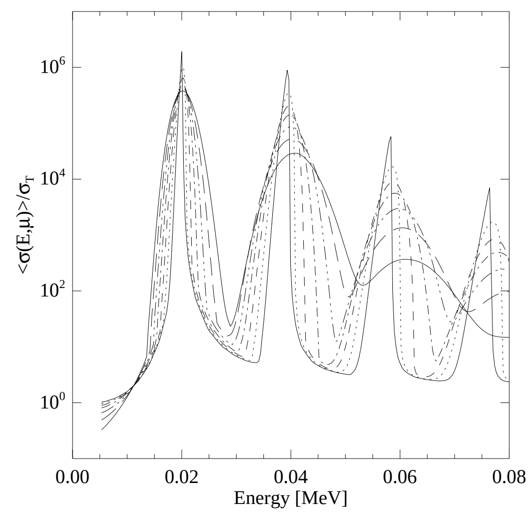
$$\psi = \exp\left(\frac{i}{\hbar}(p_x x + p_z z)\right)\chi(y) \tag{5.34}$$

After some rather tedious algebra one can then show that the resulting differential equation for χ has the solution

$$E_n = \left(n + \frac{1}{2} + \sigma\right)\hbar\omega_\mathrm{L} + \frac{p_z^2}{2m} \tag{5.35}$$

where $\sigma = \pm 1/2$. This is Eq. (5.10). The exact equation, Eq. (5.8) is obtained from solving the Dirac equation in the presence of a magnetic field. This is beyond the scope of this lecture.

Doppler Broadening



Now look at diagnostics of cyclotron lines in detail

Hot plasma

$\Longrightarrow$ thermal broadening:

- Lines narrow perpendicular to B-field
- Lines broad for motion along B-field

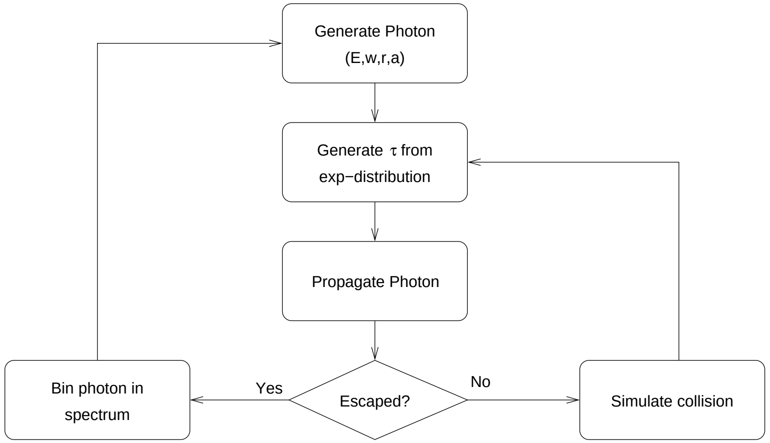
expected line width (Mészáros, 1992)

$$\frac{\Delta E_\mathrm{FWHM}}{E_\mathrm{cyc}} \sim \sqrt{kT_\mathrm{e}}|\cos\theta|$$

($\sim$ 6 keV for $kT_\mathrm{e} = 40$ keV)

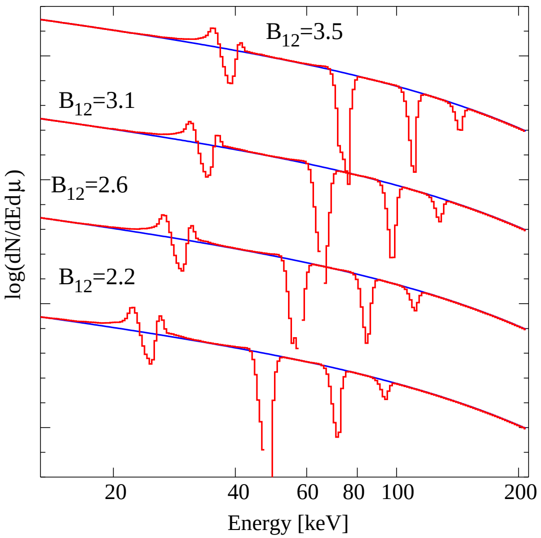
$B = 1.7\times 10^{12}$ G, $kT = E_\mathrm{cyc}/4$, θ: angle between B-field and photon direction; Schönherr et al. (2007), after Araya & Harding (1999)

Monte Carlo Simulations: Approach



Here: use Monte Carlo approach.

Theoretical results, I

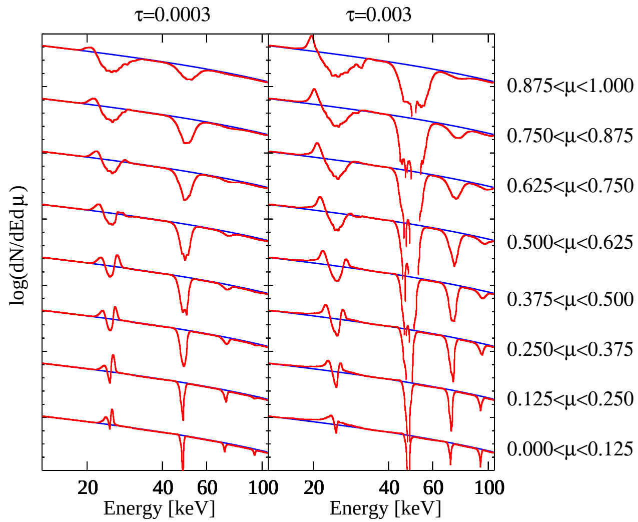


Dependence on B-field.

- Note emission wings
- Note much deeper 1st harmonic

(Schönherr et al., 2007)

Theoretical results, II

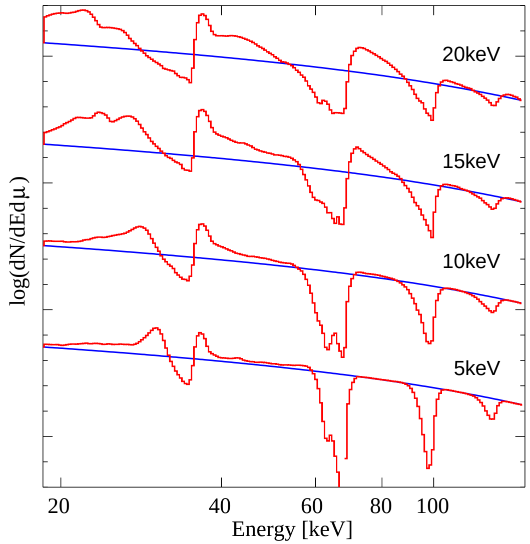


Dependence on optical depth and angle.

Note: resonance optical depth $\sim 10^5 \cdot \tau_{\mathrm{T}}$!

$B = 1.76 \times 10^{12}$ G, $kT_{\mathrm{e}} = 3$ keV; (Schönherr et al., 2007)

Theoretical results, III

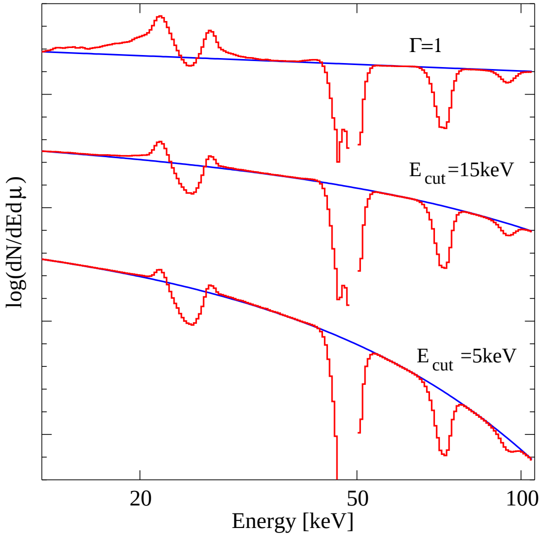


Dependence on electron temperature.

Asymmetry: relativistic Maxwellian.

(Schönherr et al., 2007)

Theoretical results, IV



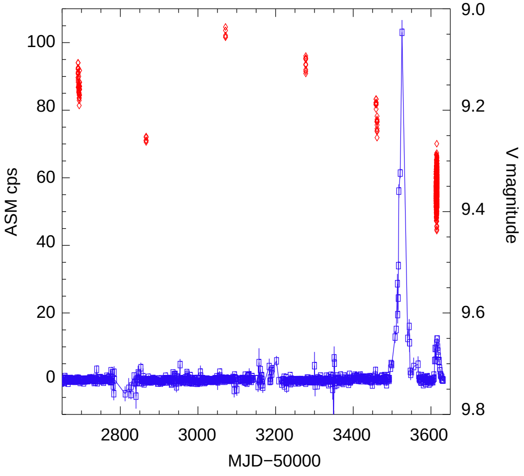
Dependence on continuum shape.

(Schönherr et al., 2007)

Cyclotron Line Sources

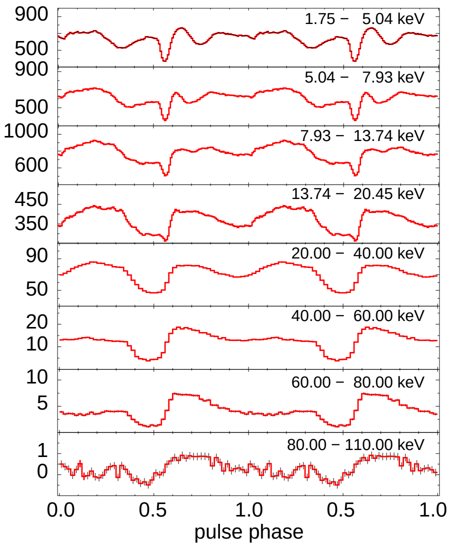
Source	E_{cyc} (keV)	P_{puls} (s)	P_{orb} (d)	companion	discovery
4U 0115+63	14, 24, 36, 48, 62	3.6	24.31	Be	HEAO-1 (Wheaton, '79) RXTE, SAX (Heindl '99, Sant.,'99)
4U 1907+09	18, 38	438	8.38	B2 III–IV	SAX (Cusumano, '98)
4U 1538−52	20	530	3.73	B0I	Ginga (Clark, '90)
Vela X-1	24, 52	283	8.96	B0.5Ib	Mir-HEXE (Kendziorra, '92), RXTE (Kreykenbohm, '02)
V 0332+53	27	4.37	34.25	Be	Ginga (Makishima, '90)
Cep X-4	28	66.25	>23	B1	Ginga (Mihara, '91)
Cen X-3	29	4.8	2.09	O6.5II	SAX (Santangelo, '98) RXTE (Heindl, '98)
X Per	29	837	250.3	B0 III–Ve	RXTE (Coburn, '01)
XTE J1946+274	36	15.8	169.2	B0-1V-IVe	RXTE (Heindl, '01)
OAO 1657−415	36?	37.7	10.4	B0-B6Ia-Iab	SAX (Orlandini, '99)
4U 1626−67	37	7.66	0.028	WD?	SAX (Orlandini, '98) RXTE (Heindl, '98)
GX 301−2	37	690	41.5	B1.2Ia	Ginga (Mihara, '95)
Her X-1	41	1.24	1.7	A9-B	Ballon-HEXE (Trümper, '78)
A0535+26	50, 110	105	110.58	Be	HEXE (Kendziorra, '92, '94), CGRO (Maisack, '97)
LMC X-4	100?	13.5	1.41	O7IV	SAX (LaBarbera, '01)

A0535+26, I



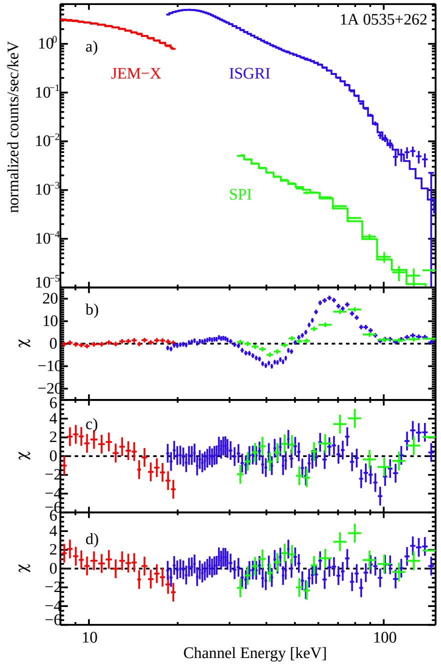
- transient O9.7IIIe/NS binary, 111 d orbit, outbursts in 1975, 1980, 1983, 1989, 1994, 2005
- giant outburst in May/June: over 1 Crab, but: too close to the Sun
- second weaker outburst: ~300 mCrab in August/September, ToO observations by *INTEGRAL* and *RXTE*

A0535+26, II



- Pulse period: 103.3920(4) s
- narrow feature at low energies: accretion stream?

(Caballero et al., 2007)

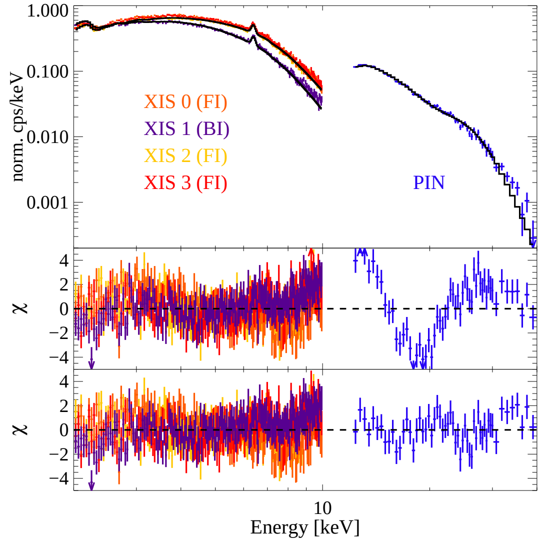


- 2 CRSFs
  - $E_{\text{cyc},1} \sim 45$ keV, $\tau_1 \sim 0.5$
  - $E_{\text{cyc},2} \sim 100$ keV, $\tau_2 \sim 0.6$
- *RXTE* and *INTEGRAL* consistent!

and confirming earlier claims for a lower line

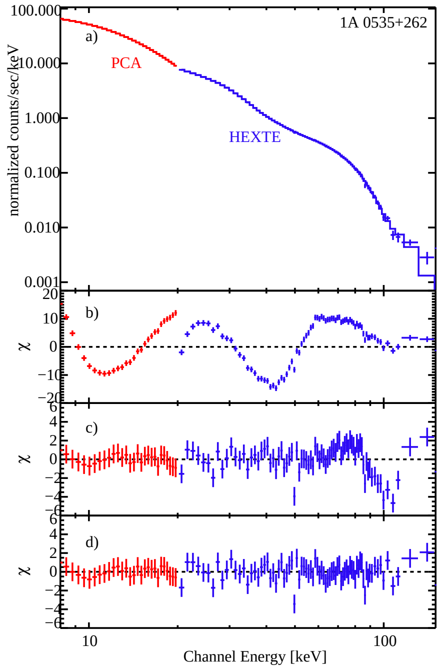
(Caballero et al., 2007)

4U1907+09



4U1907+09, 2004 December outburst: $E_{\text{cyc}} = 19.7(4)$ keV, $\sigma = 3.3(3)$ keV, in agreement with earlier results (e.g., *INTEGRAL*).

Pottschmidt et al. (2007)

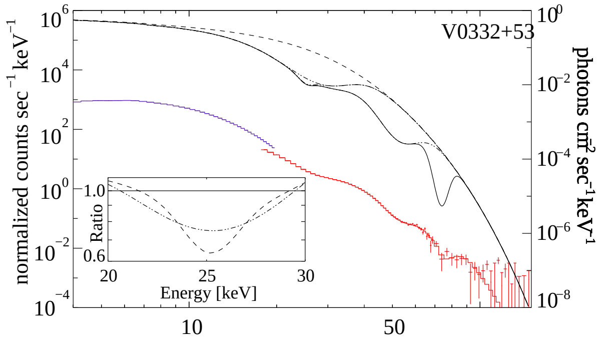


- 2 CRSFs
  - $E_{\text{cyc},1} \sim 45$ keV, $\tau_1 \sim 0.5$
  - $E_{\text{cyc},2} \sim 100$ keV, $\tau_2 \sim 0.6$
- *RXTE* and *INTEGRAL* consistent!

and confirming earlier claims for a lower line

(Caballero et al., 2007)

V0332+53



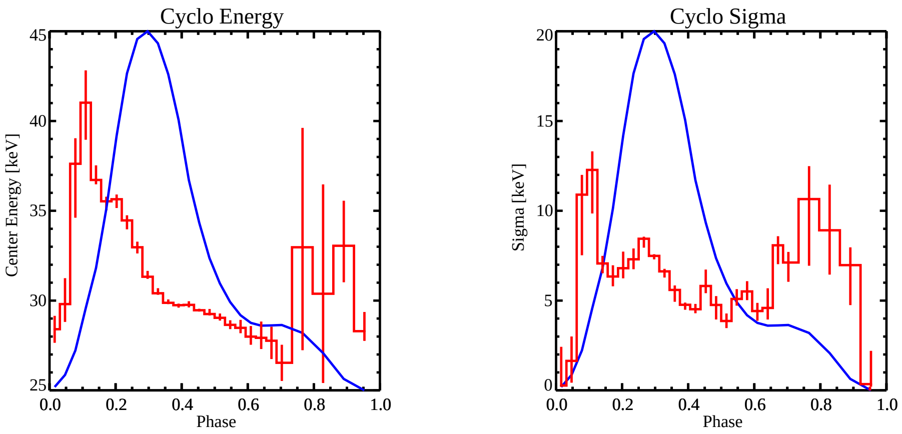
(Pottschmidt et al., 2005)

V0332+53: Cyclotron lines at 27, 51, and 74 keV; complex fundamental.

2nd source after 4U 0115+63 with more than 2 lines.

Line ratios $\neq 2$, agrees with QED prediction; also require scattering angle of $\gtrsim 60^\circ$, in agreement with expectation from resonant cross-section.

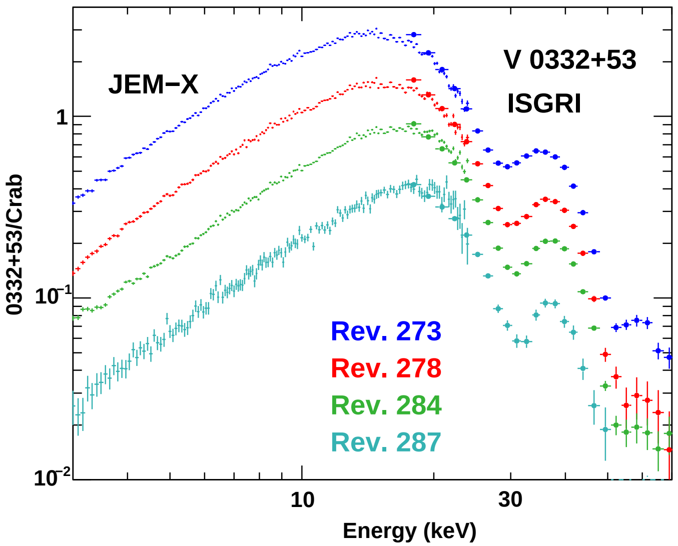
Cen X-3



Cen X-3 (Suchy et al., in prep)

Many cyclotron sources show pulse phase dependence of line parameters.
$\Longrightarrow$ effect of viewing angle / height in accretion column?

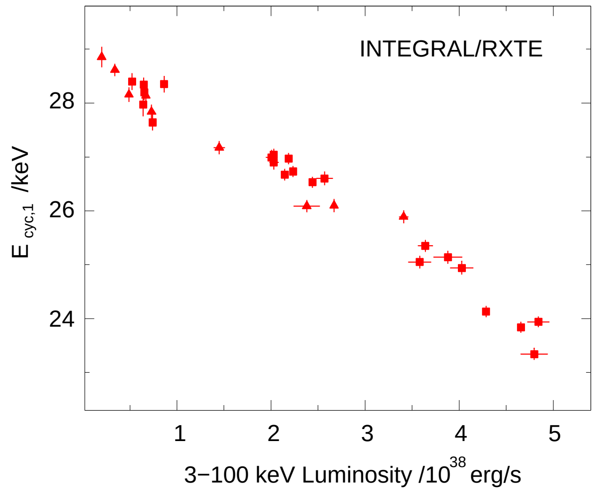
Luminosity Dependence



V0332+53: Energy of fundamental cyclotron line changes over outburst

(Mowlavi et al., 2006)

Luminosity Dependence



V0332+53: Cyclotron line energy depends on luminosity
$\Longrightarrow$ change of height of accretion column with $\dot{M}$

(Tsygankov et al., 2006)

5–59

Araya, R. A., & Harding, A. K., 1999, ApJ, 517, 334

Arons, J., Klein, R. I., & Lea, S. M., 1987, ApJ, 312, 666

Basko, M. M., & Sunyaev, R. A., 1976, MNRAS, 175, 395

Becker, P. A., 1998, ApJ, 498, 790

Becker, P. A., & Wolff, M. T., 2005a, ApJ, 621, L45

Becker, P. A., & Wolff, M. T., 2005b, ApJ, 630, 465

Becker, P. A., & Wolff, M. T., 2007, ApJ, 654, 435

Bildsten, L., et al., 1997, ApJS, 113, 367

Braun, A., & Yahel, R. Z., 1984, A&A, 278, 349

Caballero, I., et al., 2007, A&A, 465, L21

Canuto, V., 1970, ApJ, 160, L153

Canuto, V., Lodenquai, J., & Ruderman, M., 1971, Phys. Rev. D, 3, 2303

Charles, P. A., & Seward, F. D., 1995, Exploring the X-Ray Universe, (Cambridge: Cambridge Univ. Press)

Davidson, K., & Ostriker, J. P., 1973, ApJ, 179, 585

Fritz, S., Kreykenbohm, I., Wilms, J., Staubert, R., Bayazit, F., Rodriguez, J., & Santangelo, A., 2006, A&A, 458, 885

Ghosh, P., & Lamb, F. K., 1979, ApJ, 232, 239

Gnedin, Y. N., & Sunyaev, R. A., 1973, A&A, 25, 233

Gonthier, P. L., Harding, A. K., Baring, M. G., Costello, R. M., & Mercer, C. L., 2000, ApJ, 540, 907

Harding, A. K., 1994, in The Evolution of X-Ray Binaries, ed. S. S. Holt, C. S. Day, (Washington: AIP), 429

Heindl, W. A., Rothschild, R. E., Coburn, W., Staubert, R., Wilms, J., Kreykenbohm, I., & Kretschmar, P., 2004, in AIP Conf. Proc. 714: X-ray Timing 2003: Rossi and Beyond, ed. P. Kaaret, F. K. Lamb, J. H. Swank, 323

Inoue, H., 1975, PASJ, 27, 311

Kreykenbohm, I., Kretschmar, P., Wilms, J., Staubert, R., Kendziorra, E., Gruber, D., & Rothschild, R., 1999, A&A, 341, 141

Langer, S. H., & Rappaport, S., 1982, ApJ, 257, 733

Mészáros, P., 1984, Space Sci. Rev., 38, 325

Mészáros, P., 1992, High-energy radiation from magnetized neutron stars, (Chicago: Chicago Univ. Press)

Mészáros, P., & Nagel, W., 1985a, ApJ, 298, 147

Mészáros, P., & Nagel, W., 1985b, ApJ, 299, 138

Mowlavi, N., et al., 2006, A&A, 451, 817

Nagel, W., 1981a, ApJ, 251, 278

Nagel, W., 1981b, ApJ, 251, 288

Ostriker, J. P., & Davidson, K., 1973, in X- and Gamma-Ray Astronomy, ed. H. Bradt, R. Giacconi, 143

Pottschmidt, K., et al., 2005, ApJ, 634, L97

Pringle, J. E., & Rees, M. J., 1972, A&A, 21, 1

Rappaport, S., & Joss, P. C., 1977, Nature, 266, 683

Schönherr, G., Wilms, J., Kretschmar, P., Kreykenbohm, I., Santangelo, A., Rothschild, R. E., Staubert, R., & Coburn, W., 2007, A&A, submitted

Trümper, J., Pietsch, W., Reppin, C., Voges, W., Staubert, R., & Kendziorra, E., 1978, ApJ, 219, L105

Tsygankov, S. S., Lutovinov, A. A., Churazov, E. M., & Sunyaev, R. A., 2006, MNRAS, 371, 19

Ventura, J., 1979, Phys. Rev. D, 19, 1684