Introduction

X-ray binaries are powered by accretion

$\Longrightarrow$ need to look at accretion as a physical mechanism.

Unfortunately, this will have to be somewhat theoretical, but this cannot be avoided...

Structure of this chapter:

1. Accretion Luminosity: Eddington luminosity
2. Accretion Disks: Theory
3. Accretion Disks: Confrontation with observations

Literature

- J. Frank, A. King, D. Raine, 2002, Accretion Power in Astrophysics, 3rd edition, Cambridge Univ. Press

  The standard textbook on accretion, covering all relevant areas of the field.

- T. Padmanabhan, 2001, Theoretical Astrophysics, II. Stars and Stellar Systems, Cambridge Univ. Press

  See introduction to this lecture.

- J.E. Pringle, 1981, Accretion Disks in Astrophysics, Ann. Rev. Astron. Astrophys. **19**, 137

  Concise review of classical accretion disk theory.

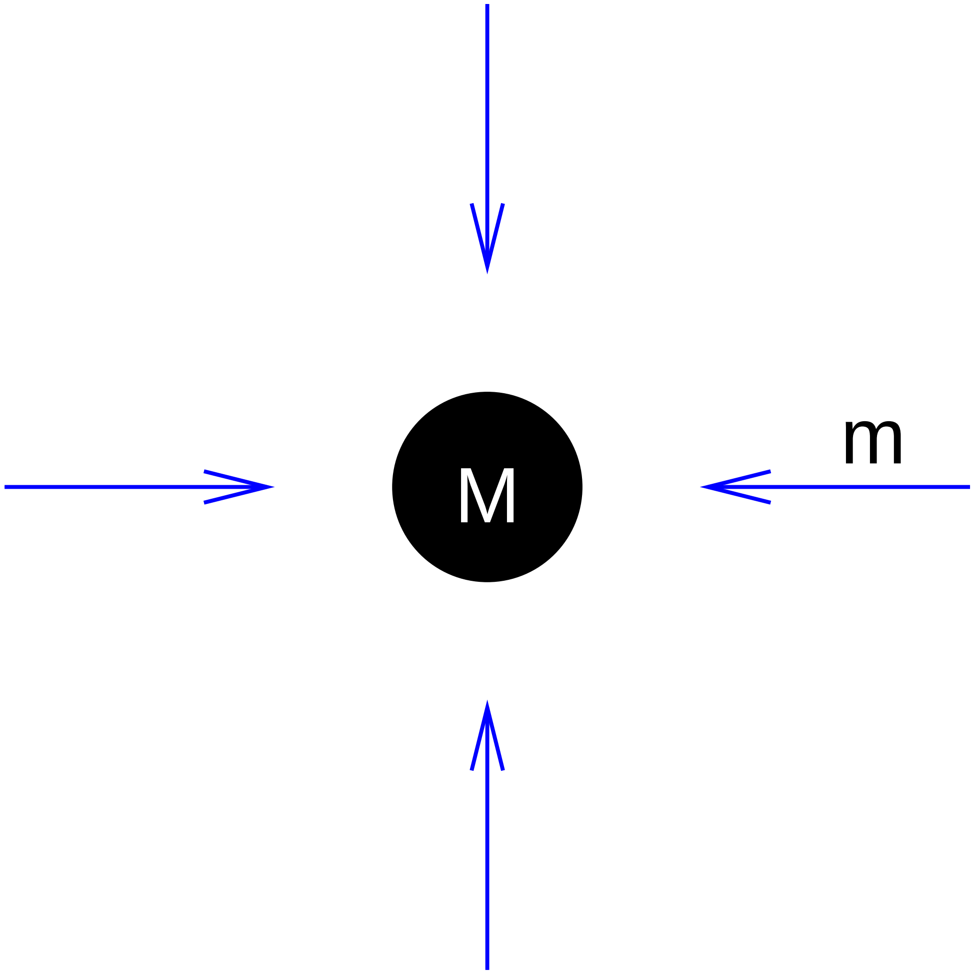
- N.I. Shakura & R. Sunyaev, 1973, Black Holes in Binary Systems. Observational Appearance. Astron. Astrophys. **24**, 337 and J.E. Pringle & M. Rees, 1972, Accretion Disc Models for Compact X-Ray Sources, Astron. Astrophysi, 22(1), 1

  The fundamental papers, which *really* started the field.

Eddington luminosity, I

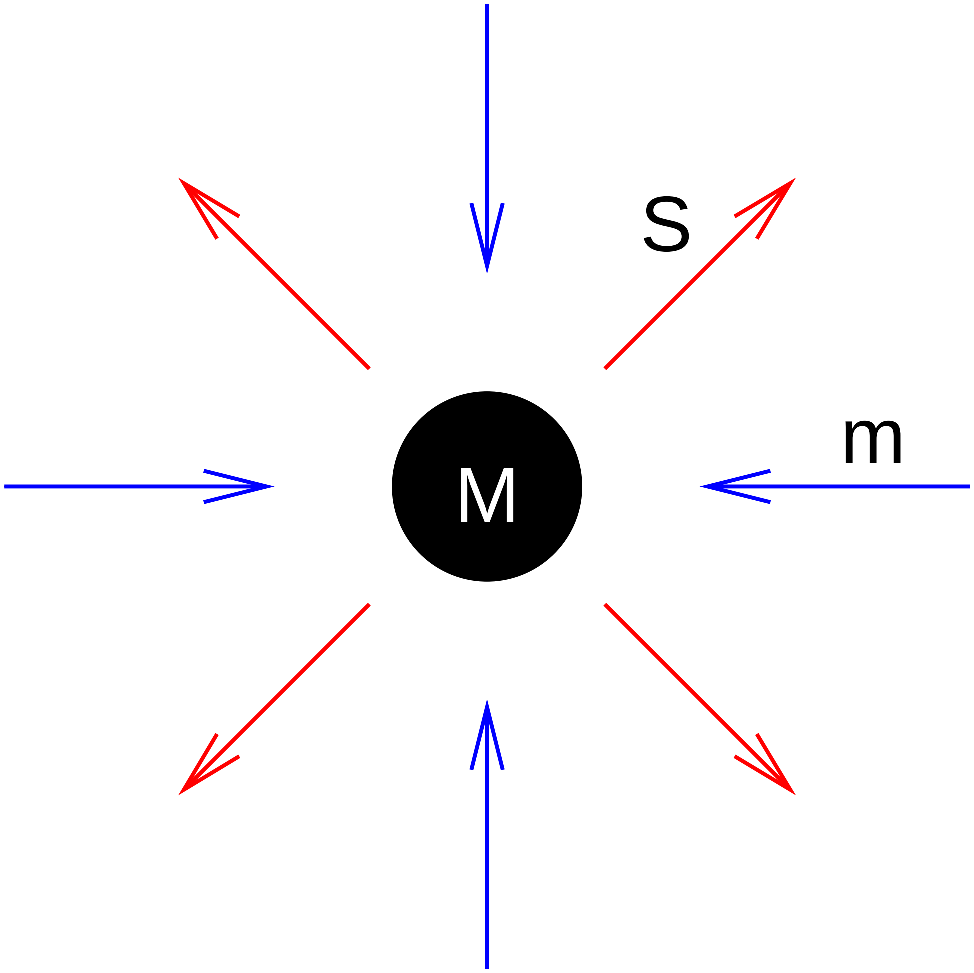
Assume mass M

Eddington luminosity, II



Assume mass M spherically symmetrically accreting ionized hydrogen gas.

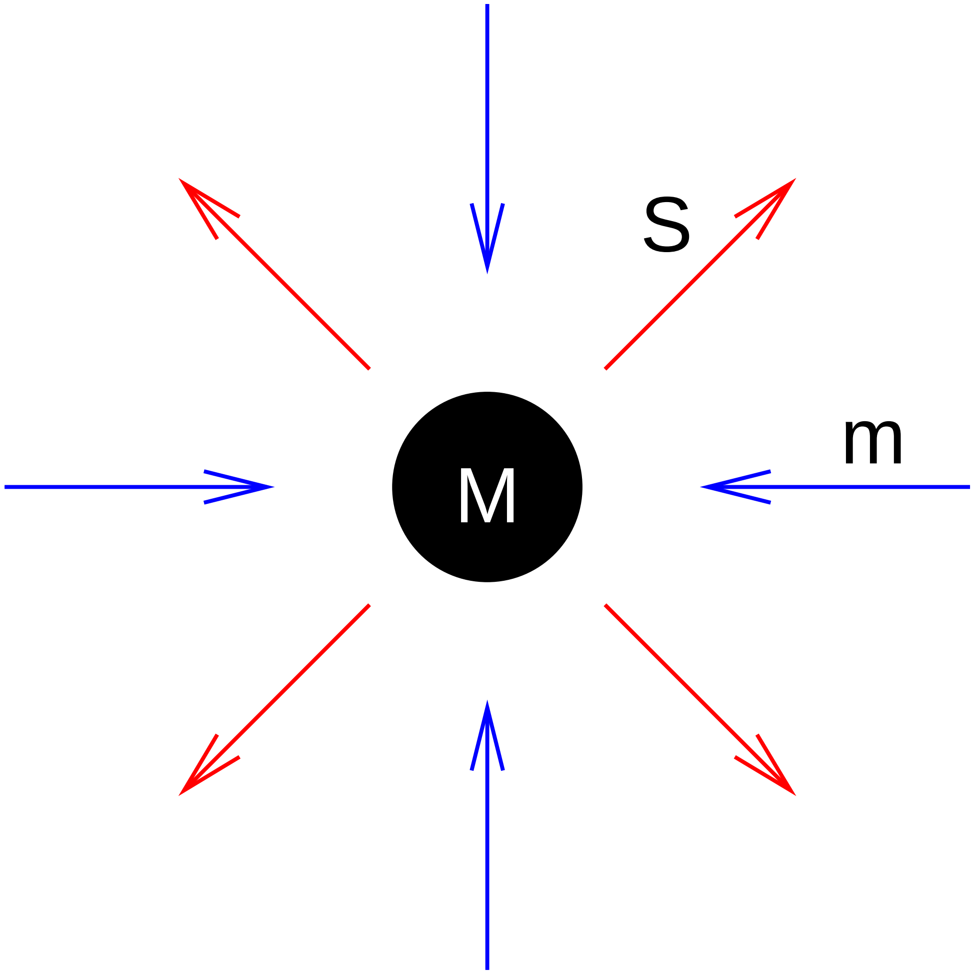
Eddington luminosity, III



Assume mass M spherically symmetrically accreting ionized hydrogen gas.

At radius r, accretion produces energy flux S.

Eddington luminosity, IV



Assume mass M spherically symmetrically accreting ionized hydrogen gas.

At radius r, accretion produces energy flux S.

Important: Interaction between accreted material and radiation!

Eddington luminosity, V

Force balance on accreted electrons and protons:

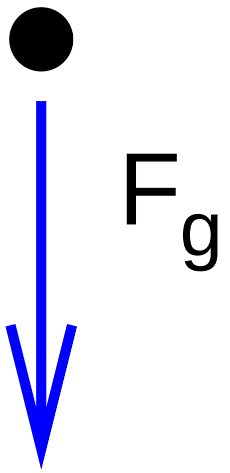
•

Eddington luminosity, VI

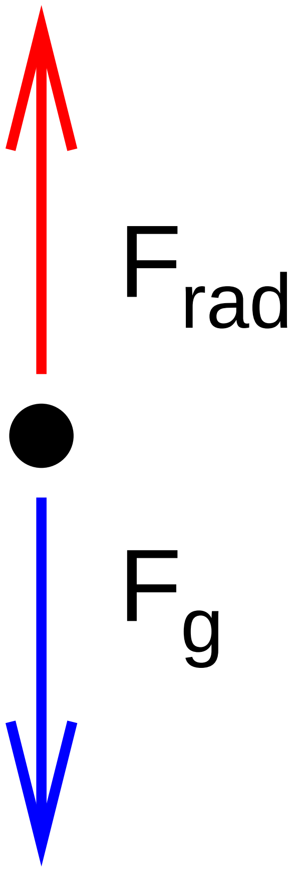
Force balance on accreted electrons and protons:

Inward force: gravitation:

$$F_{\mathrm{g}} = \frac{GMm_{\mathrm{p}}}{r^2} \quad (4.1)$$



Eddington luminosity, VII



Force balance on accreted electrons and protons:

Inward force: gravitation:

$$F_{\mathrm{g}} = \frac{GMm_{\mathrm{p}}}{r^2} \tag{4.2}$$

Outward force: radiation force:

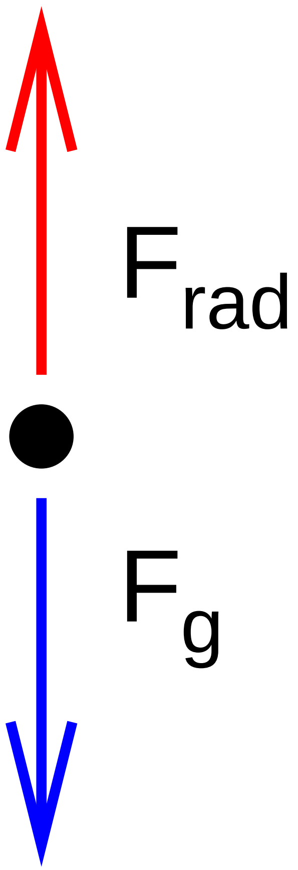
$$F_{\mathrm{rad}} = \frac{\sigma_{\mathrm{T}} S}{c} \tag{4.3}$$

where energy flux S is given by

$$S = \frac{L}{4\pi r^2} \tag{4.4}$$

where L: luminosity.

Eddington luminosity, VIII



Force balance on accreted electrons and protons:

Inward force: gravitation:

$$F_{\mathrm{g}} = \frac{GMm_{\mathrm{p}}}{r^2} \tag{4.5}$$

Outward force: radiation force:

$$F_{\mathrm{rad}} = \frac{\sigma_{\mathrm{T}} S}{c} \tag{4.6}$$

where energy flux S is given by

$$S = \frac{L}{4\pi r^2} \tag{4.7}$$

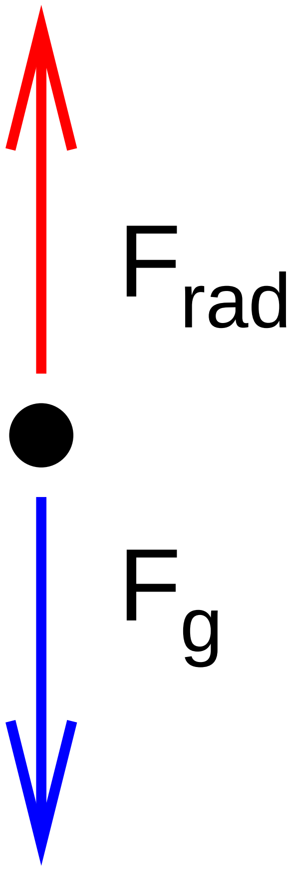
where L: luminosity.

Note: $\sigma_{\mathrm{T}} \propto (m_{\mathrm{e}}/m_{\mathrm{p}})^2$, so negligable for protons.

But: strong Coulomb coupling between electrons and protons

$\Longrightarrow F_{\mathrm{rad}}$ also has effect on protons!

Eddington luminosity, IX



Accretion is only possible if gravitation dominates:

$$\frac{GMm_{\mathrm{p}}}{r^2} > \frac{\sigma_{\mathrm{T}} S}{c} = \frac{\sigma_{\mathrm{T}}}{c} \cdot \frac{L}{4\pi r^2} \tag{4.8}$$

and therefore

$$L < L_{\mathrm{Edd}} = \frac{4\pi G M m_{\mathrm{p}} c}{\sigma_{\mathrm{T}}} \tag{4.9}$$

or, in astronomically meaningful units

$$L < 1.3 \times 10^{38}\,\mathrm{erg\,s^{-1}} \cdot \frac{M}{M_\odot} \tag{4.10}$$

where L_{Edd} is called the Eddington luminosity.

But remember the assumptions entering the derivation: spherically symmetric accretion of fully ionized pure hydrogen gas.

Eddington luminosity, X

Characterize accretion process through the accretion effi ciency, η:

$$L = \eta \cdot \dot{M} c^2 \tag{4.11}$$

where $\dot{M}$: mass accretion rate (e.g., $\mathrm{g\,s^{-1}}$ or $M_\odot\,\mathrm{yr}^{-1}$).

Therefore maximum accretion rate ("Eddington rate"):

$$\dot{m} = \frac{L_{\mathrm{Edd}}}{\eta c^2} \sim 2 \times 10^{-8} \cdot \left(\frac{M}{1\,M_\odot}\right)\,M_\odot\,\mathrm{yr}^{-1} \tag{4.12}$$

(for $\eta = 0.1$)

Emitted spectrum

Characterize photon by its radiation temperature, T_{rad}:

$$h\nu \sim kT_{\text{rad}} \implies T_{\text{rad}} = h\nu/k \tag{4.13}$$

Emitted spectrum

Characterize photon by its radiation temperature, T_{rad}:

$$h\nu \sim kT_{\text{rad}} \quad \Longrightarrow \quad T_{\text{rad}} = h\nu/k \tag{4.14}$$

Optically thick medium: blackbody radiation

$$T_{\text{b}} = \left(\frac{L}{4\pi R^2 \sigma_{\text{SB}}}\right)^{1/4} \tag{4.15}$$

Emitted spectrum

Characterize photon by its radiation temperature, T_{rad}:

$$h\nu \sim kT_{\mathrm{rad}} \quad \Longrightarrow \quad T_{\mathrm{rad}} = h\nu/k \tag{4.16}$$

Optically thick medium: blackbody radiation

$$T_{\mathrm{b}} = \left(\frac{L}{4\pi R^2 \sigma_{\mathrm{SB}}}\right)^{1/4} \tag{4.17}$$

Optically thin medium: L directly converted into radiation without further interactions $\Longrightarrow$ mean particle energy

$$T_{\mathrm{th}} = \frac{GMm_{\mathrm{p}}}{3kR} \tag{4.18}$$

Emitted spectrum

Characterize photon by its radiation temperature, T_{rad}:

$$h\nu \sim kT_{\text{rad}} \quad \Longrightarrow \quad T_{\text{rad}} = h\nu/k \tag{4.19}$$

Optically thick medium: blackbody radiation

$$T_{\text{b}} = \left(\frac{L}{4\pi R^2 \sigma_{\text{SB}}}\right)^{1/4} \tag{4.20}$$

Optically thin medium: L directly converted into radiation without further interactions $\Longrightarrow$ mean particle energy

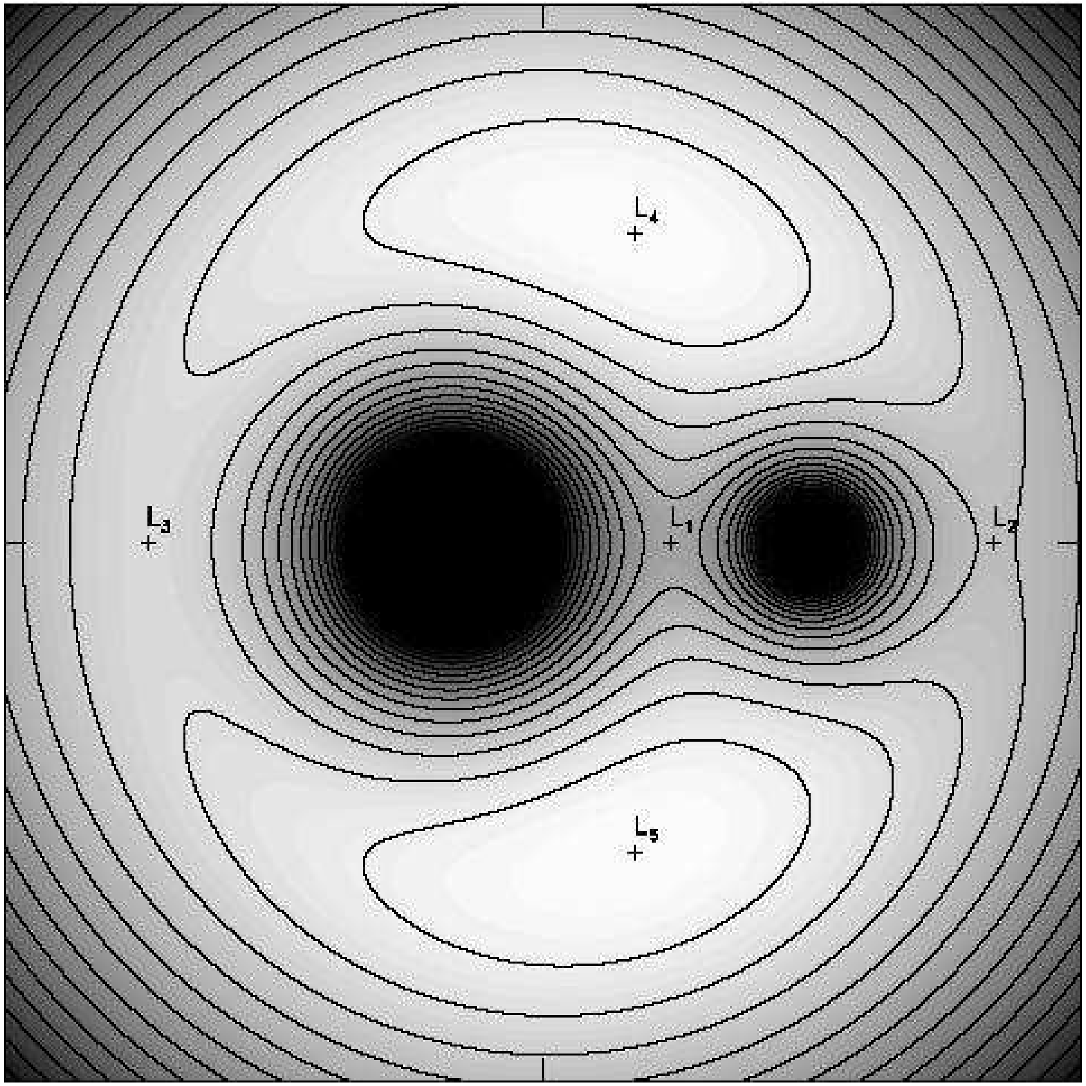
$$T_{\text{th}} = \frac{GMm_{\text{p}}}{3kR} \tag{4.21}$$

Plugging in numbers for a typical solar mass compact object (NS/BH):

$$T_{\text{rad}} \sim 1\,\text{keV} \quad \text{and} \quad T_{\text{bb}} \sim 50\,\text{MeV} \tag{4.22}$$

Accreting objects are broadband emitters in the X-rays and gamma-rays.

Roche Geometry, I



R. Hynes

Motion of gas in corotating frame around masses M_1, M_2 given by

$$\frac{d^2\boldsymbol{r}}{dt^2} + 2\boldsymbol{\omega} \times \frac{d\boldsymbol{r}}{dt} = -\frac{1}{\rho}\nabla P - \nabla\Phi$$

where the Roche potential:

$$\Phi_{\mathrm{R}}(\boldsymbol{r}) = -\frac{GM_1}{|\boldsymbol{r}-\boldsymbol{r}_1|} - \frac{GM_2}{|\boldsymbol{r}-\boldsymbol{r}_2|} - \frac{1}{2}(\boldsymbol{\omega}\times\boldsymbol{r})^2$$

and where

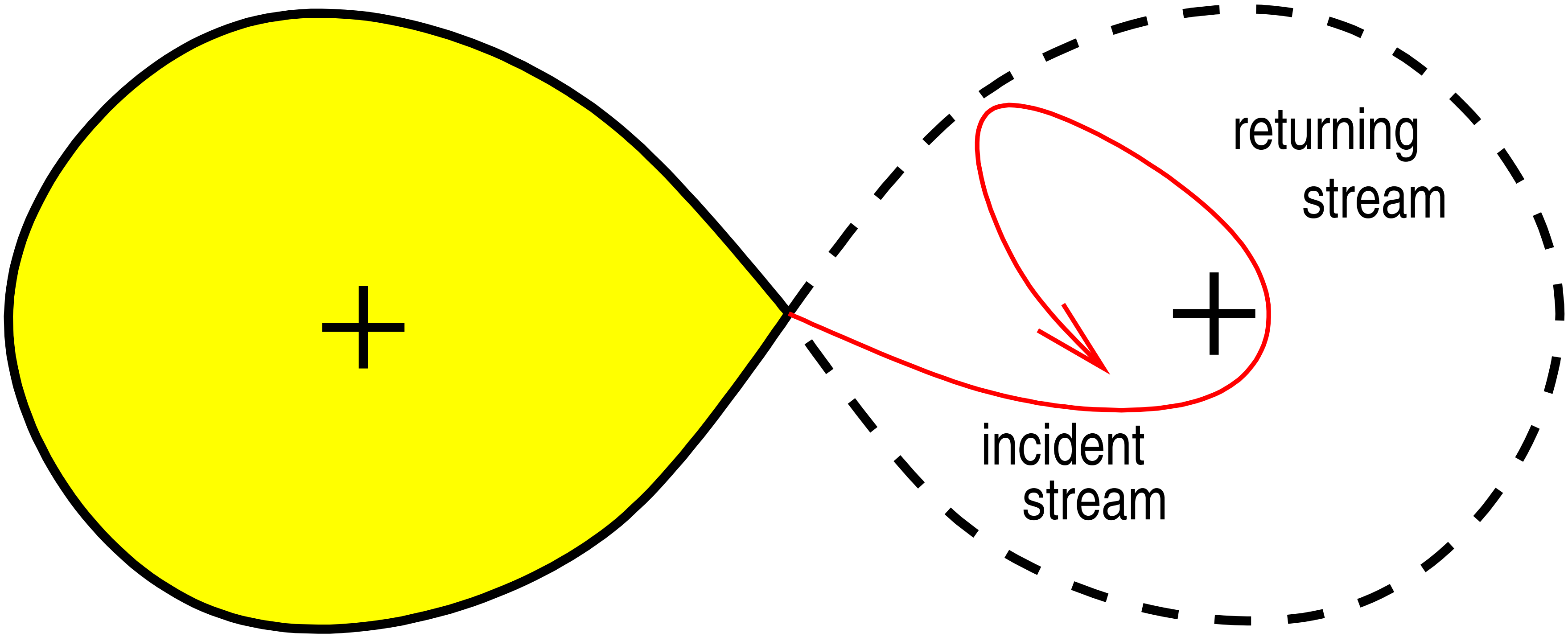
$$\boldsymbol{\omega} = \left(\frac{GM}{a^3}\right)^{1/2}\hat{e}$$

Matter comes from companion star

$\Longrightarrow$ accreted matter has angular momentum

$\Longrightarrow$ accretion disk forms.

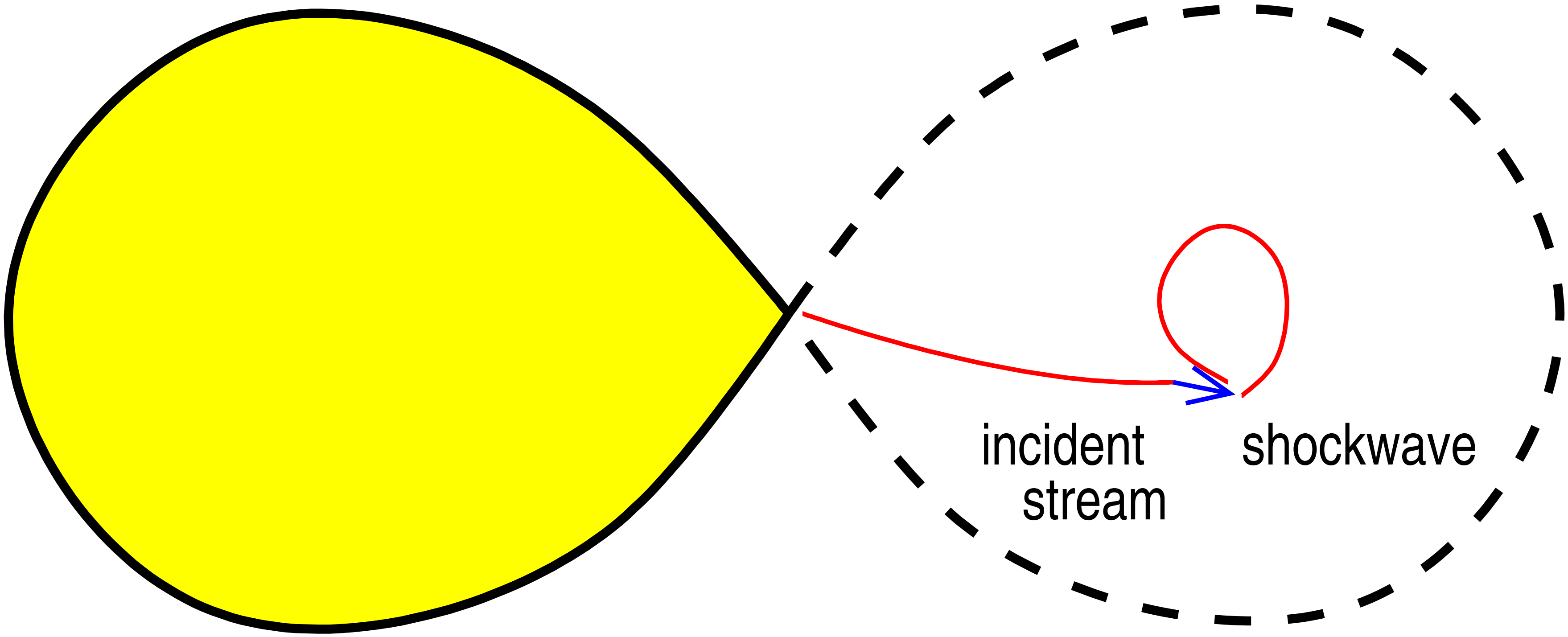
Roche Lobe Overfbw, I



(after Lubow & Shu, 1975, Fig. 4)

Roche Lobe Accretion: Gas is transferred at inner Lagrange point.

Roche Lobe Overflow, II



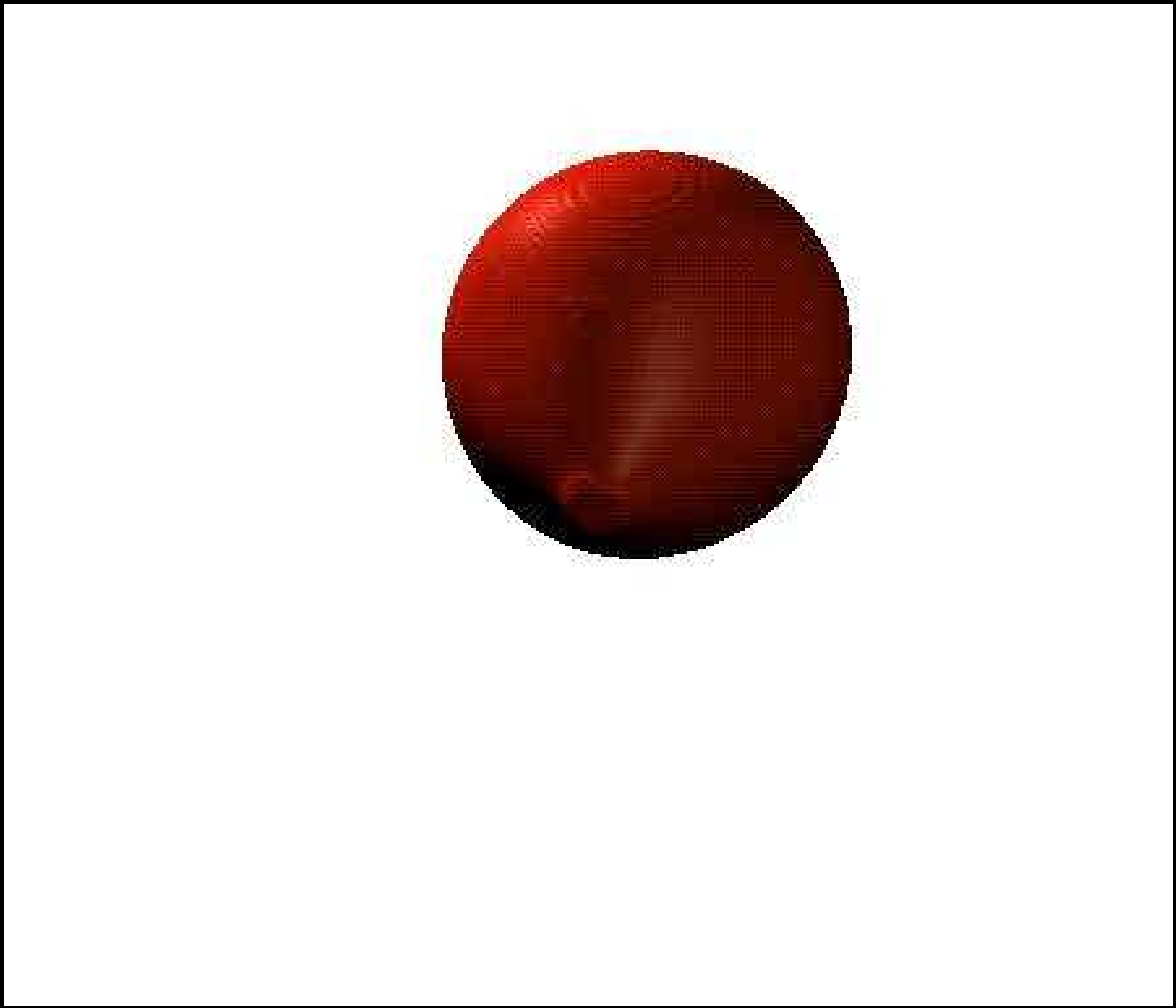
(after Lubow & Shu, 1975, Fig. 4)

Roche Lobe Accretion: Gas is transferred at inner Lagrange point.

Ballistic free fall towards compact object, forms elliptical orbit

Note: ellipse rotates because of Coriolis force!

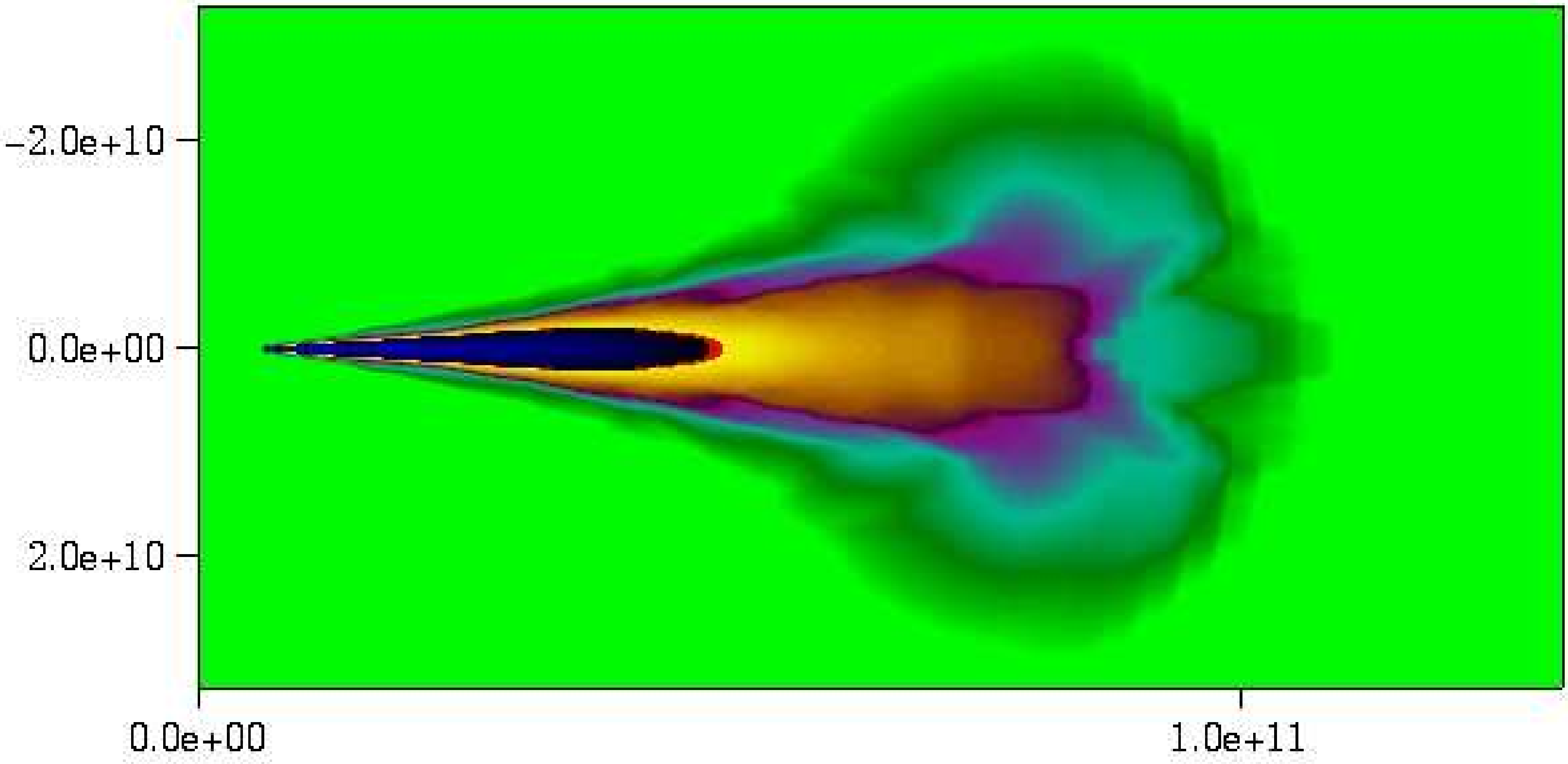
Stream intersects $\Longrightarrow$ shock $\Longrightarrow$ randomization $\Longrightarrow$ circular orbit forms.



Numerical simulation of disk formation by J. Blondin (NCSU)

Stream is well described by ballistic motion, outer disk radius at ∼0.5 Roche Lobe radius.

Roche Lobe Overflow, IV

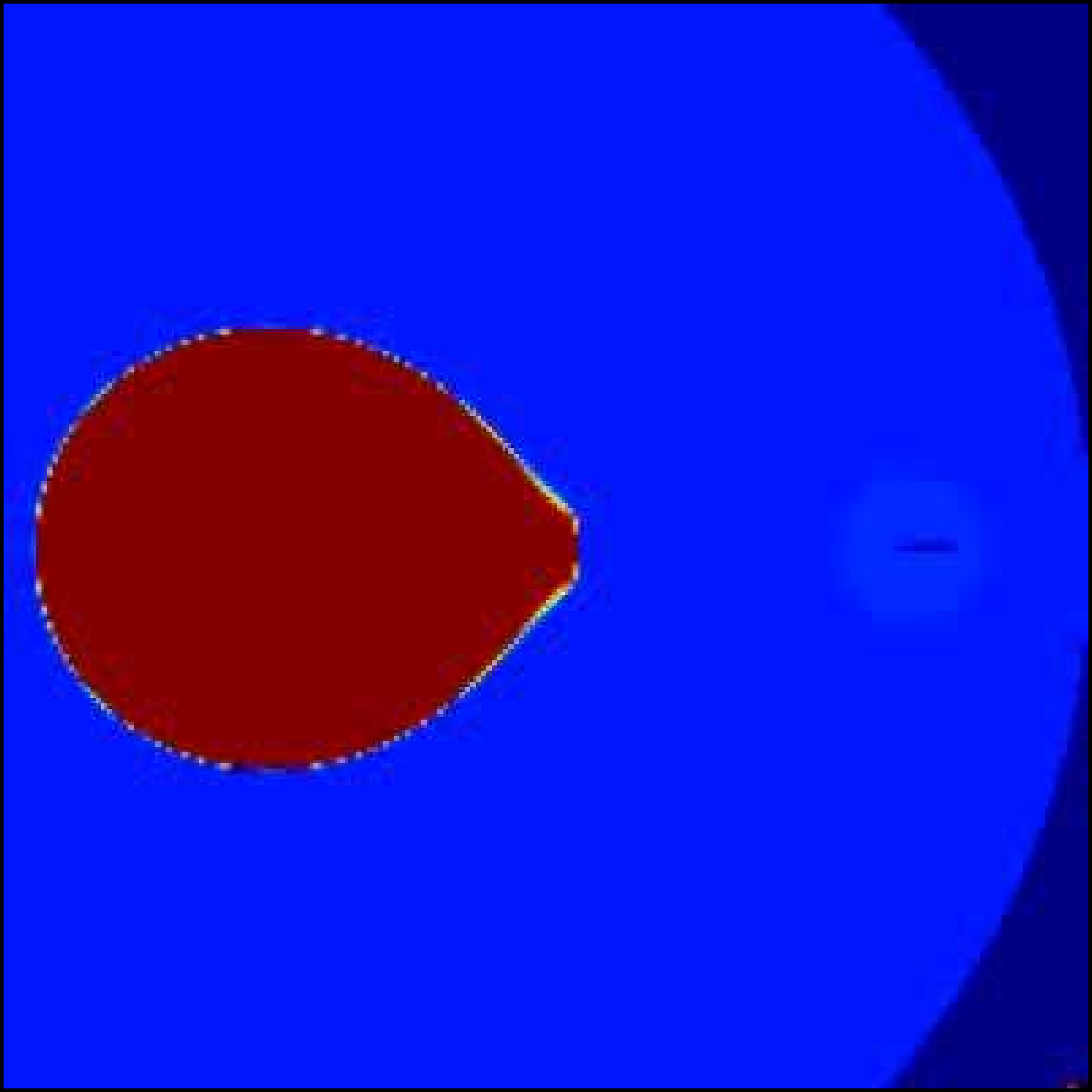


J. Blondin

Disk is flared at outer radii due to accretion stream impact.

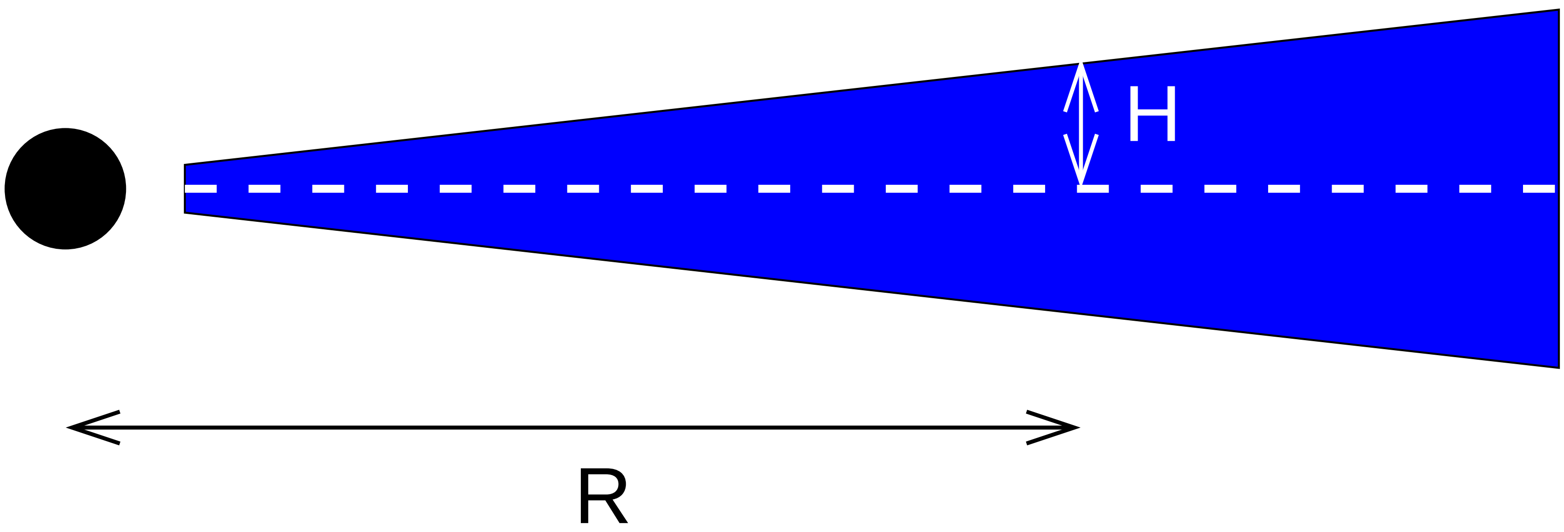
J. Blondin

Shock forms over large parts of the disk.



courtesy J. Blondin

Thin Disks, I



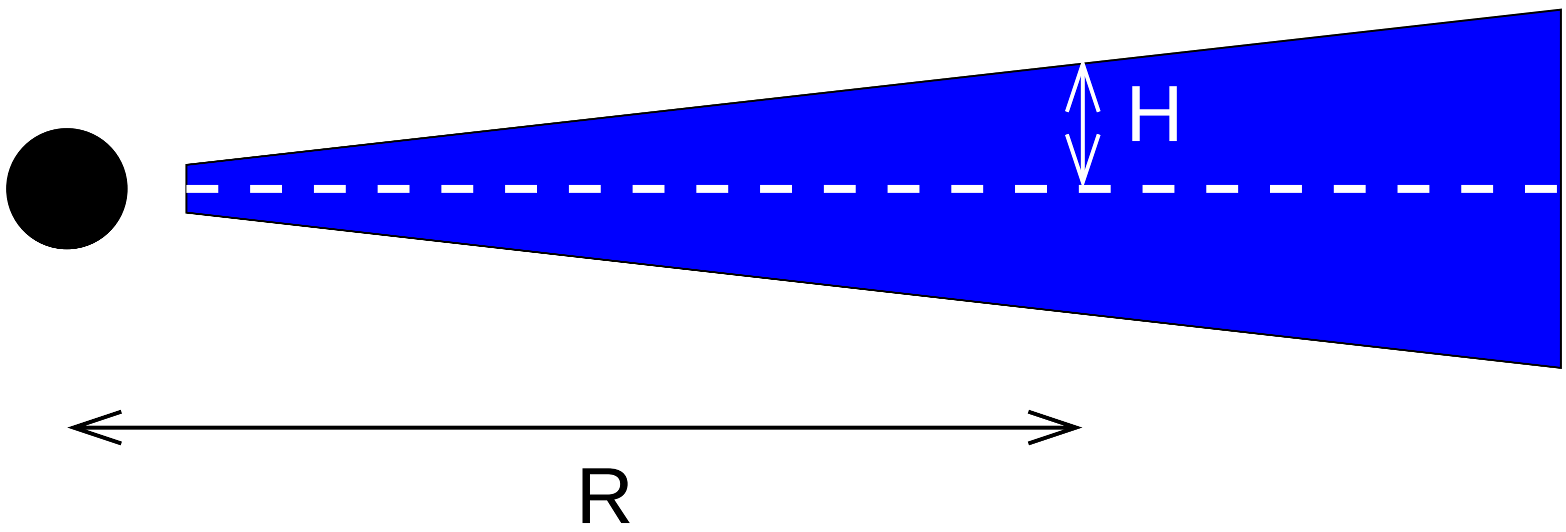
Most important case: thin accretion disks, i.e., vertical thickness, H, much smaller than radius R:

$$H \ll R \tag{4.23}$$

$\Longrightarrow$ Requires that radiation pressure is negligable

$\Longrightarrow$ $L \ll L_{\text{Edd}}$

Thin Disks, II



Thin disk: no radiation pressure

$\Longrightarrow$ gas pressure must support disk vertically against gravitation:

$$\frac{GM}{R^2}\frac{H}{R} = \frac{1}{\rho}\left|\frac{\partial P}{\partial z}\right| \sim \frac{P_c}{\rho_c H} \tag{4.24}$$

where P_c characteristic pressure, ρ_c characteristic density.

Thin Disks, III

The speed of sound is given by

$$c_s^2 = \frac{P}{\rho} \tag{4.25}$$

therefore the condition for vertical support can be written as

$$\frac{GM}{R^2}\frac{H}{R} \sim \frac{P_c}{\rho_c H} = \frac{c_s^2}{H} \tag{4.26}$$

such that

$$c_s^2 = \frac{GM}{R}\frac{H^2}{R^2} = v_\phi^2 \cdot \frac{H^2}{R^2} \tag{4.27}$$

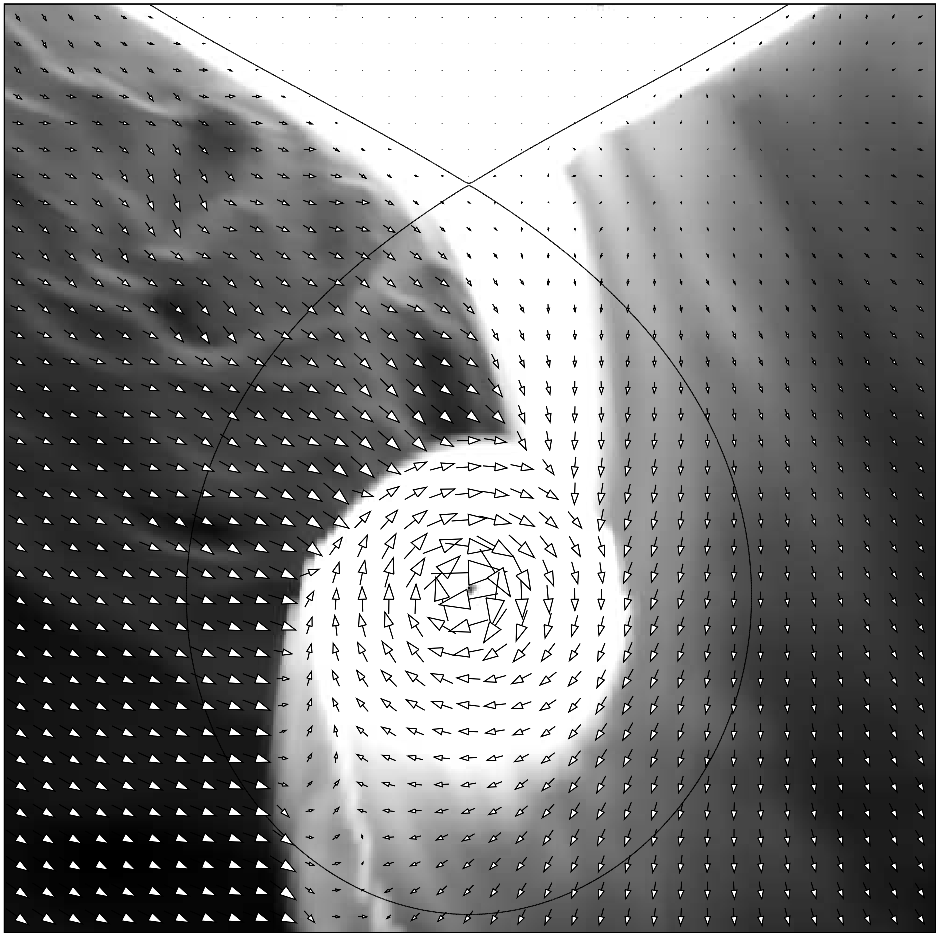
where $v_\phi = \sqrt{GM/R} = 1.2 \times 10^{10}\,(M/M_\odot)(R/10^6\,\mathrm{cm})^{-1}\,\mathrm{cm\,s^{-1}}$: Kepler speed.

Since $H/R \ll 1$:

$$c_s \ll v_\phi \tag{4.28}$$

Thin accretion disks are highly supersonic.

Thin Disks: Radial Structure



J. Blondin (priv. comm.)

Radial acceleration due to pressure:

$$\frac{1}{\rho}\frac{\partial P}{\partial R} \sim \frac{P_\mathrm{c}}{\rho_\mathrm{c} R} \sim \frac{c_\mathrm{s}^2}{R} \sim \frac{GM}{R^2}\frac{H^2}{R^2} \ll \frac{GM}{R^2} \quad (4.29)$$

$\Longrightarrow$ radial acceleration due to pressure negligable compared to gravitational acceleration

Thin disk: fluid motion is Keplerian to very high degree of precision.

$\Longrightarrow$ for the radial velocity, v_R: $v_R \ll v_\phi$

Thin Disks: Vertical Structure and Mass Conservation

Amount of mass crossing radius R:

$$\dot{M} = -2\pi R \cdot \Sigma \cdot v_R \tag{4.30}$$

where Σ: surface density of disk,

$$\Sigma(R) = \int n(z)dz \tag{4.31}$$

and where $\dot{M}$: mass accretion rate

Since acceleration $\perp z$

$$F_z \propto \frac{GM}{R^2}\frac{z}{R} \propto z \tag{4.32}$$

vertical density profi le

$$n(z) \propto \exp\left(-\frac{z}{H}\right) \tag{4.33}$$

where H: scale height (depends on details of accretion disk theory).

Thin Disks: Angular Momentum Transport, I

Most important question: angular momentum transport

Angular velocity in Keplerian disk:

$$\omega(R) = \left(\frac{GM}{R^3}\right)^{1/2} \tag{4.34}$$

("differential rotation")

$\Longrightarrow$ angular momentum per mass ("specifi c angular momentum"):

$$\mathcal{L} = R \cdot v = R \cdot R\omega(R) = R^2\,\omega(R) \propto R^{1/2} \tag{4.35}$$

$\Longrightarrow$ decreases with decreasing R!

Total angular momentum lost when mass moves in unit time from $R + dR$ to R:

$$\frac{dL}{dR} = \dot{M} \cdot \frac{d(R^2\omega(R))}{dR} \tag{4.36}$$

Thin Disks: Angular Momentum Transport, II

Since L changes: accreting matter needs to lose angular momentum. This is done by viscous forces excerting torques:

Force due to viscosity per unit length:

$$\mathcal{F} = \nu\Sigma \cdot \Delta v = \nu\Sigma \cdot R\frac{d\omega}{dR} \tag{4.37}$$

where ν: coeffi cient of kinematic viscosity

Therefore total torque

$$G(R) = 2\pi R\mathcal{F} \cdot R = \nu\Sigma 2\pi R^3 \left(\frac{d\omega}{dR}\right) \tag{4.38}$$

and the net torque acting on a ring is

$$\frac{dG}{dR}dR \tag{4.39}$$

$\Longrightarrow$ This net torque needs to balance change in specifi c angular momentum in disk.

Thin Disks: Angular Momentum Transport, III

Balancing net torque and angular momentum loss gives:

$$\dot{M}\frac{d(R^2\omega)}{dR} = -\frac{d}{dR}\left(\nu\Sigma 2\pi R^3 \frac{d\omega}{dR}\right) \tag{4.40}$$

Insert $\omega(R) = (GM/R^3)^{1/2}$ and integrate:

$$\nu\Sigma R^{1/2} = \frac{\dot{M}}{3\pi}R^{1/2} + \text{const.} \tag{4.41}$$

const. obtained from no torque boundary condition at inner edge of disk at $R = R_*$: $dG/dR(R_*) = 0$, such that

$$\nu\Sigma = \frac{\dot{M}}{3\pi}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right] \tag{4.42}$$

Therefore the viscous dissipation rate per unit area is

$$D(R) = \nu\Sigma\left(R\frac{d\omega}{dR}\right)^2 = \frac{3GM\dot{M}}{4\pi R^3}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right] \tag{4.43}$$

Thin Disks: Temperature Profile, I

The viscous dissipation rate was

$$D(R) = \nu\Sigma \left(R\frac{d\omega}{dR}\right)^2 = \frac{3GM\dot{M}}{4\pi R^3}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right] \qquad (4.43)$$

If disk is optically thick: Thermalization of dissipated energy

$\Longrightarrow$ Temperature from Stefan-Boltzmann-Law:

$$2\sigma_{\mathrm{SB}}T^4 = D(R) \qquad (4.44)$$

(disk has *two* sides!) and therefore

$$T(R) = \left\{\frac{3GM\dot{M}}{8\pi R^3\sigma_{\mathrm{SB}}}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right]\right\}^{1/4} \qquad (4.45)$$

Thin Disks: Temperature Profile, II

Inserting astrophysically meaningful numbers:

$$T(R) = \left\{ \frac{3GM\dot{M}}{8\pi R^3 \sigma_{\text{SB}}} \left[1 - \left(\frac{R_*}{R} \right)^{1/2} \right] \right\}^{1/4}$$

$$= 6.8 \times 10^5\,\text{K} \cdot \eta^{-1/4} \left(\frac{L}{L_{\text{Edd}}} \right)^{1/2} L_{46}^{-1/4} \mathcal{R}^{1/4} x^{-3/4}$$

where $\eta = L_{\text{Edd}}/\dot{M}_{\text{Edd}} c^2$, $x = c^2 R/2GM$, $\mathcal{R} = (1 - (R_*/R)^{1/2})$.

Thin Disks: Temperature Profile, III

Inserting astrophysically meaningful numbers:

$$T(R) = \left\{ \frac{3GM\dot{M}}{8\pi R^3 \sigma_{\mathrm{SB}}} \left[1 - \left(\frac{R_*}{R} \right)^{1/2} \right] \right\}^{1/4}$$

$$= 6.8 \times 10^5\,\mathrm{K} \cdot \eta^{-1/4} \left(\frac{L}{L_{\mathrm{Edd}}} \right)^{1/2} L_{46}^{-1/4} \mathcal{R}^{1/4} x^{-3/4}$$

where $\eta = L_{\mathrm{Edd}}/\dot{M}_{\mathrm{Edd}} c^2$, $x = c^2 R/2GM$, $\mathcal{R} = (1 - (R_*/R)^{1/2})$.

Radial dependence of T:

$$T(R) \propto R^{-3/4}$$

Thin Disks: Temperature Profile, IV

Inserting astrophysically meaningful numbers:

$$T(R) = \left\{ \frac{3GM\dot{M}}{8\pi R^3 \sigma_{\mathrm{SB}}} \left[1 - \left(\frac{R_*}{R} \right)^{1/2} \right] \right\}^{1/4}$$

$$= 6.8 \times 10^5\,\mathrm{K} \cdot \eta^{-1/4} \left(\frac{L}{L_{\mathrm{Edd}}} \right)^{1/2} L_{46}^{-1/4} \mathcal{R}^{1/4} x^{-3/4}$$

where $\eta = L_{\mathrm{Edd}}/\dot{M}_{\mathrm{Edd}} c^2$, $x = c^2 R/2GM$, $\mathcal{R} = (1 - (R_*/R)^{1/2})$.

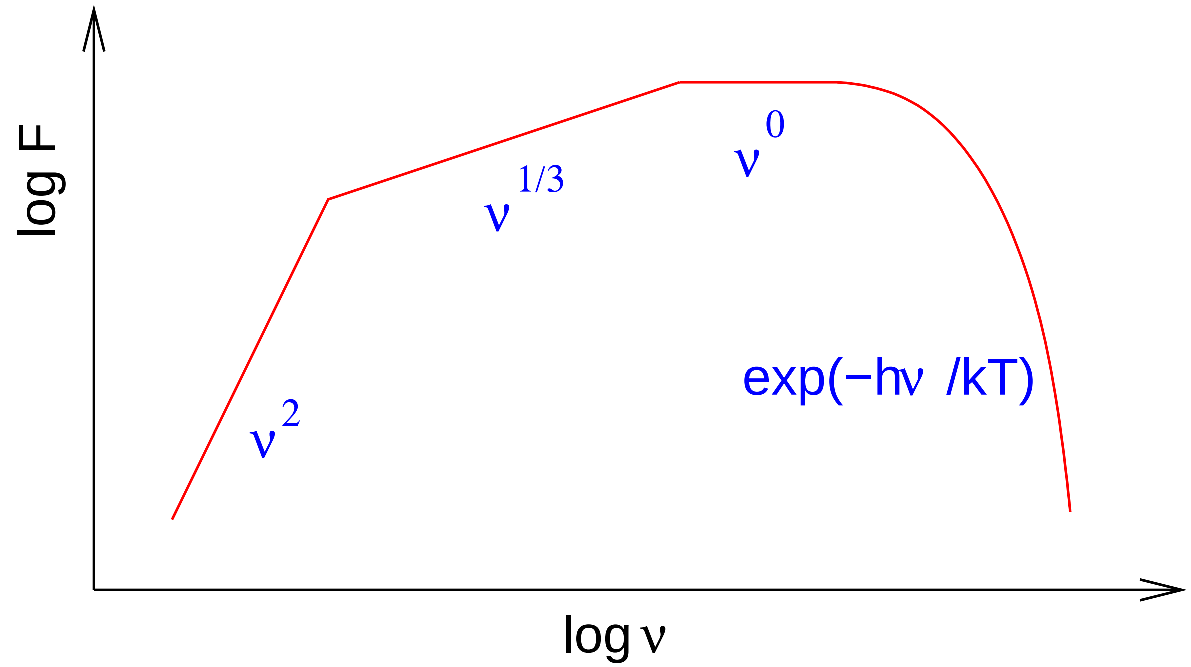
Radial dependence of T:

$$T(R) \propto R^{-3/4}$$

Dependence on mass (note: for NS/BH inner radius $R_* \propto M$!):

inner disk temperature $T_{\mathrm{in}} \propto (\dot{M}/M^2)^{1/4}$

Thin Disks: Emitted Spectrum, I



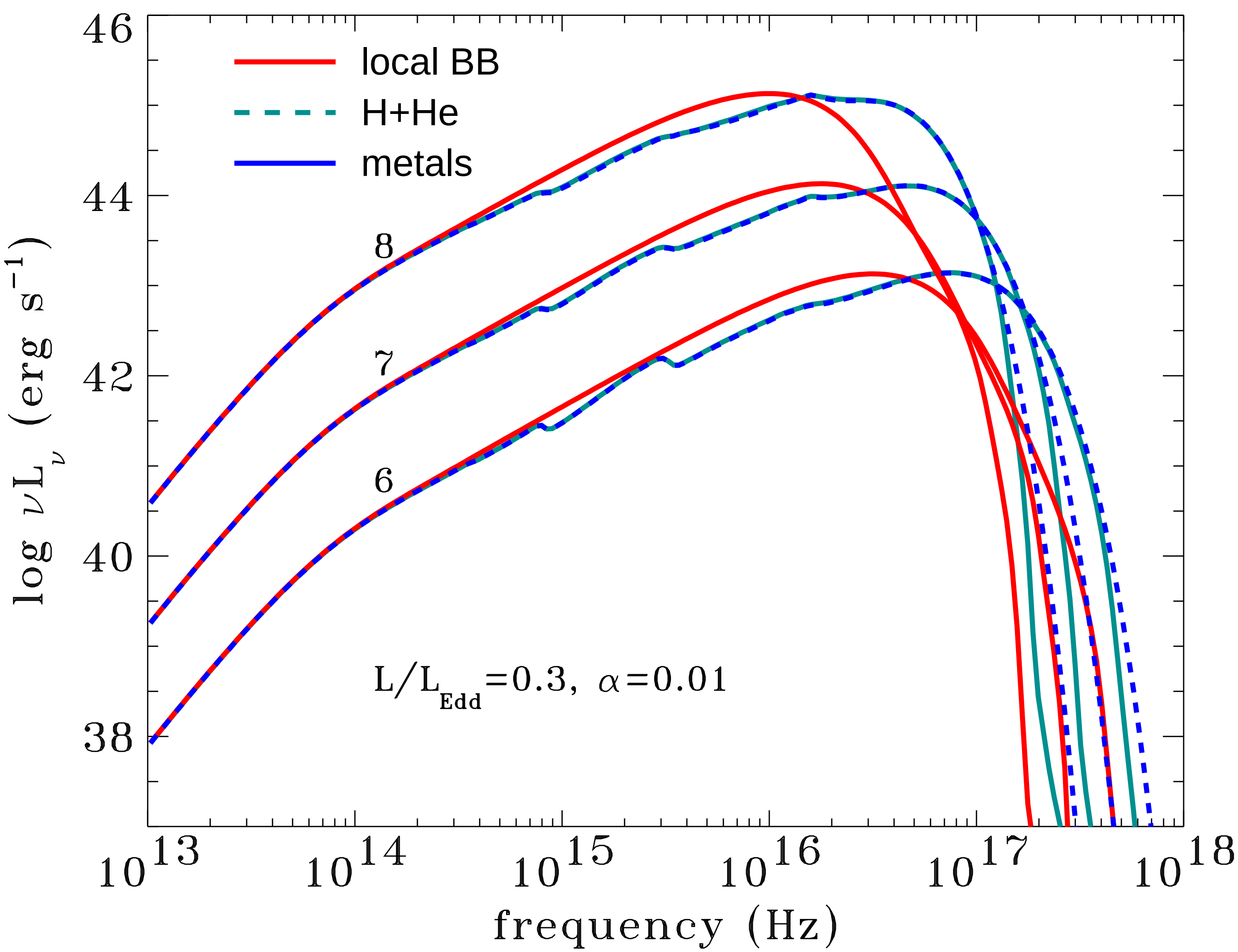
If disk is optically thick, then locally emitted spectrum is black body.

Total emitted spectrum obtained by integrating over disk

$$F_\nu = \int_{R_*}^{R_{\text{out}}} B(T(R))\, 2\pi R\, dR \qquad (4.46)$$

Resulting spectrum looks essentially like a stretched black body.

Thin Disks: Emitted Spectrum, II



Hubeny et al., 2001, Fig. 13

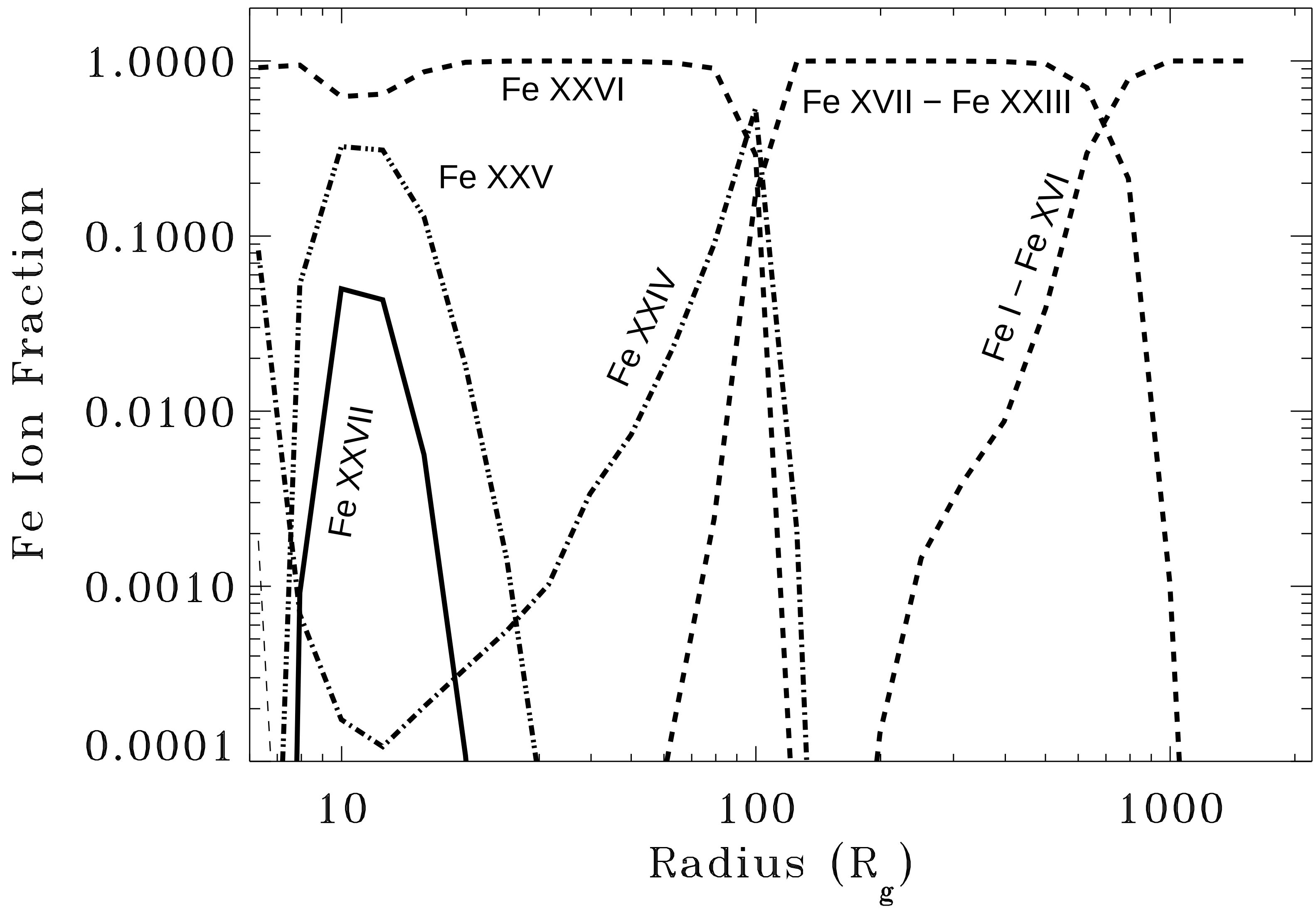
In reality: accretion disk spectrum depends on

- elemental composition ("metallicity")
- viscosity ("α-parameter")
- ionization state and luminosity of disk ($\dot{M}$)
- properties of compact object

and many further parameters

Until today: no really satisfactory disk model available.

Thin Disks: Emitted Spectrum, III



Fe species in a disk around a Galactic BH (Davis et al., 2005, Fig. 6)

Viscosity

Most important unknown in accretion disk theory: viscosity

even though it dropped out of $T(R)$!

Earth: viscosity of fluids typically due to molecular interactions (molecular viscosity).

Kinematic viscosity:

$$\nu_{\text{mol}} \sim \lambda_{\text{mfp}} c_{\text{s}} \tag{4.47}$$

where the mean free path

$$\lambda_{\text{mfp}} \sim \frac{1}{n\sigma} \sim 6.4 \times 10^4 \left(\frac{T^2}{n}\right) \text{ cm} \tag{4.48}$$

and the speed of sound

$$c_{\text{s}} \sim 10^4 T^{1/2} \text{ cm s}^{-1} \tag{4.49}$$

such that

$$\nu_{\text{mol}} \sim 6.4 \times 10^8 \, T^{5/2} n^{-1} \text{ cm}^2 \text{ s}^{-1} \tag{4.50}$$

Viscosity

Viscosity important when Reynolds number small ("laminar flow"), where

$$\mathrm{Re} = \frac{\text{inertial force}}{\text{viscous force}} \sim \frac{\rho R v}{\rho \nu} = \frac{Rv}{\nu} \tag{4.51}$$

Follows from Navier-Stokes Equations

Using typical accretion disk parameters:

$$\mathrm{Re}_{\mathrm{mol}} \sim 2 \times 10^{14} \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{R}{10^{10}\,\mathrm{cm}}\right)^{1/2} \left(\frac{n}{10^{15}\,\mathrm{cm}^{-3}}\right) \left(\frac{T}{10^4\,\mathrm{K}}\right)^{-5/2} \tag{4.52}$$

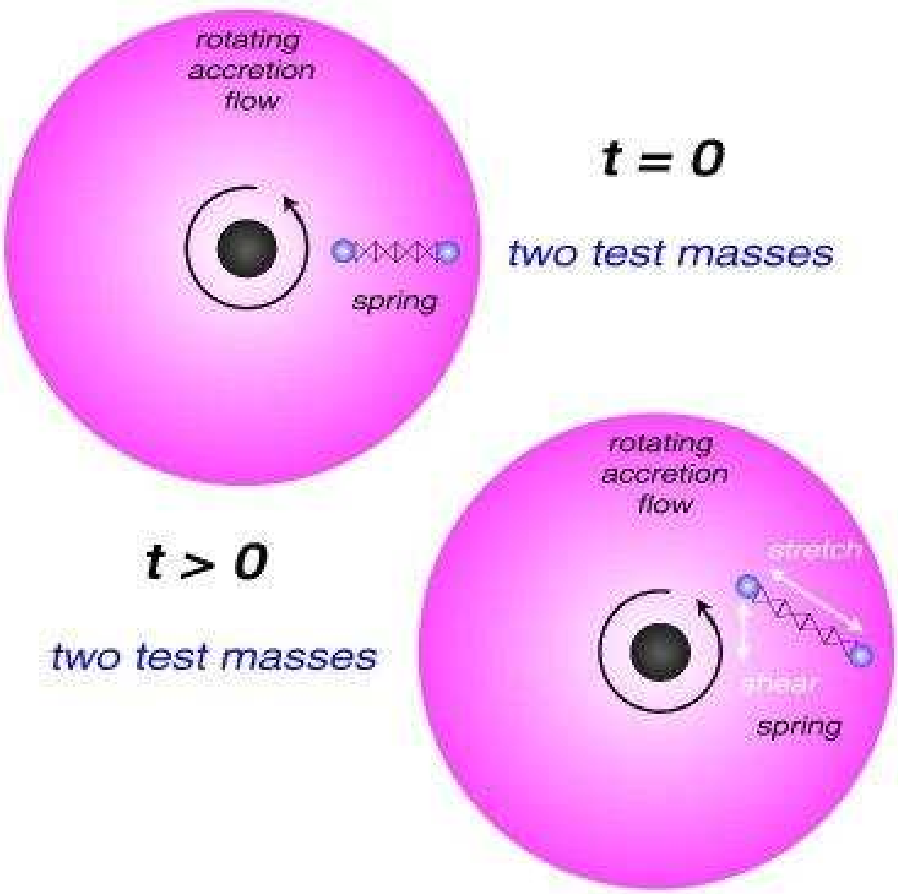
$\Longrightarrow$ Molecular viscosity is irrelevant for astrophysical disks!

since $\mathrm{Re} \gtrsim 10^3$: turbulence $\Longrightarrow$ Shakura & Sunyaev posit turbulent viscosity

$$\nu_{\mathrm{turb}} \sim v_{\mathrm{turb}} \ell_{\mathrm{turb}} \sim \alpha\, c_{\mathrm{s}} \cdot H \tag{4.53}$$

where $\alpha \lesssim 1$ and $\ell_{\mathrm{turb}} \lesssim H$ typical size for turbulent eddies.

Viscosity



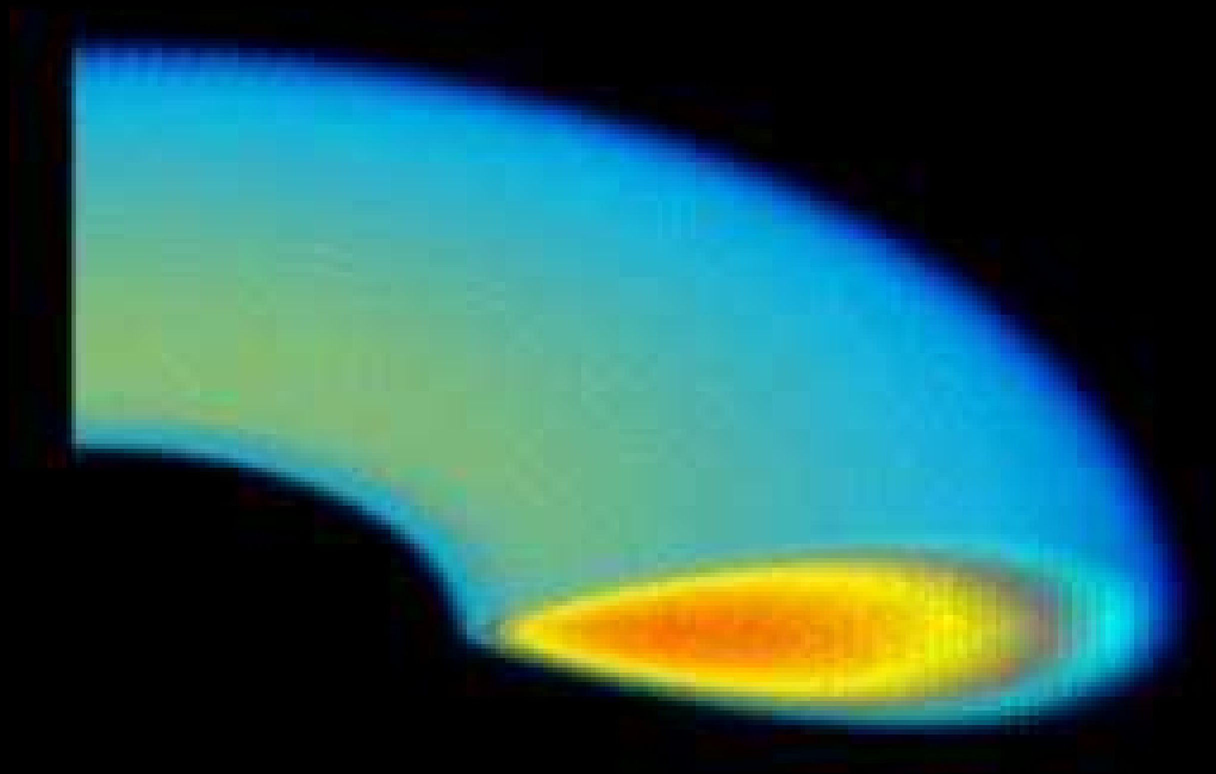
R. Müller

Mechanical analogy of MRI: spring in differentially rotating medium.

Physics of turbulent viscosity is unknown, however, α prescription yields good agreement between theory and observations.

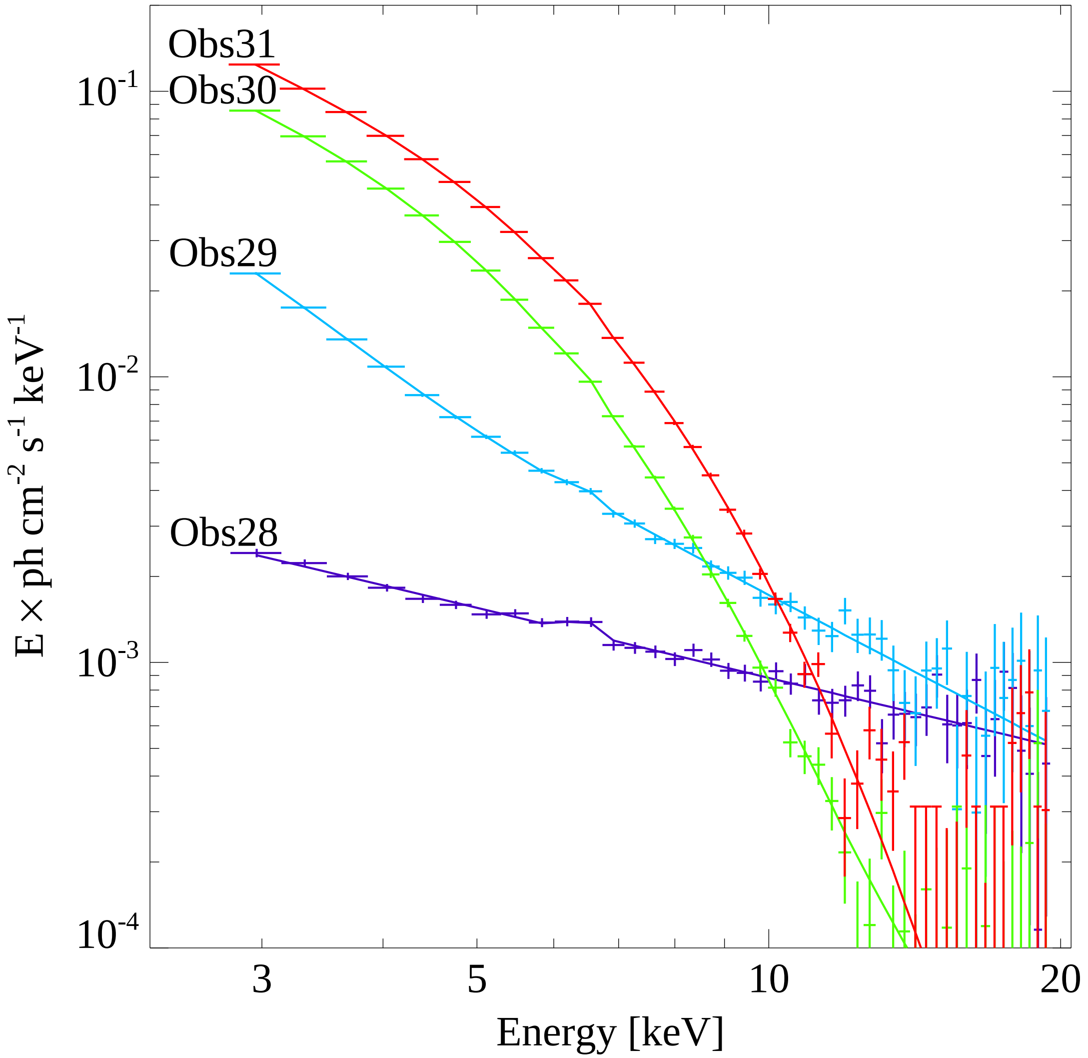
Possible origin: Magnetorotational instability (MRI): MHD instability amplifying B-fi eld inhomogeneities caused by small initial radial displacements in accretion disk
$\Longrightarrow$ angular momentum transport

(Balbus & Hawley 1991, going back to Velikhov 1959 and Chandrasekhar (1961).



(Hawley & Krolik, 2002)

Galactic Black Holes



LMC X-3, (Wilms et al., 2001)

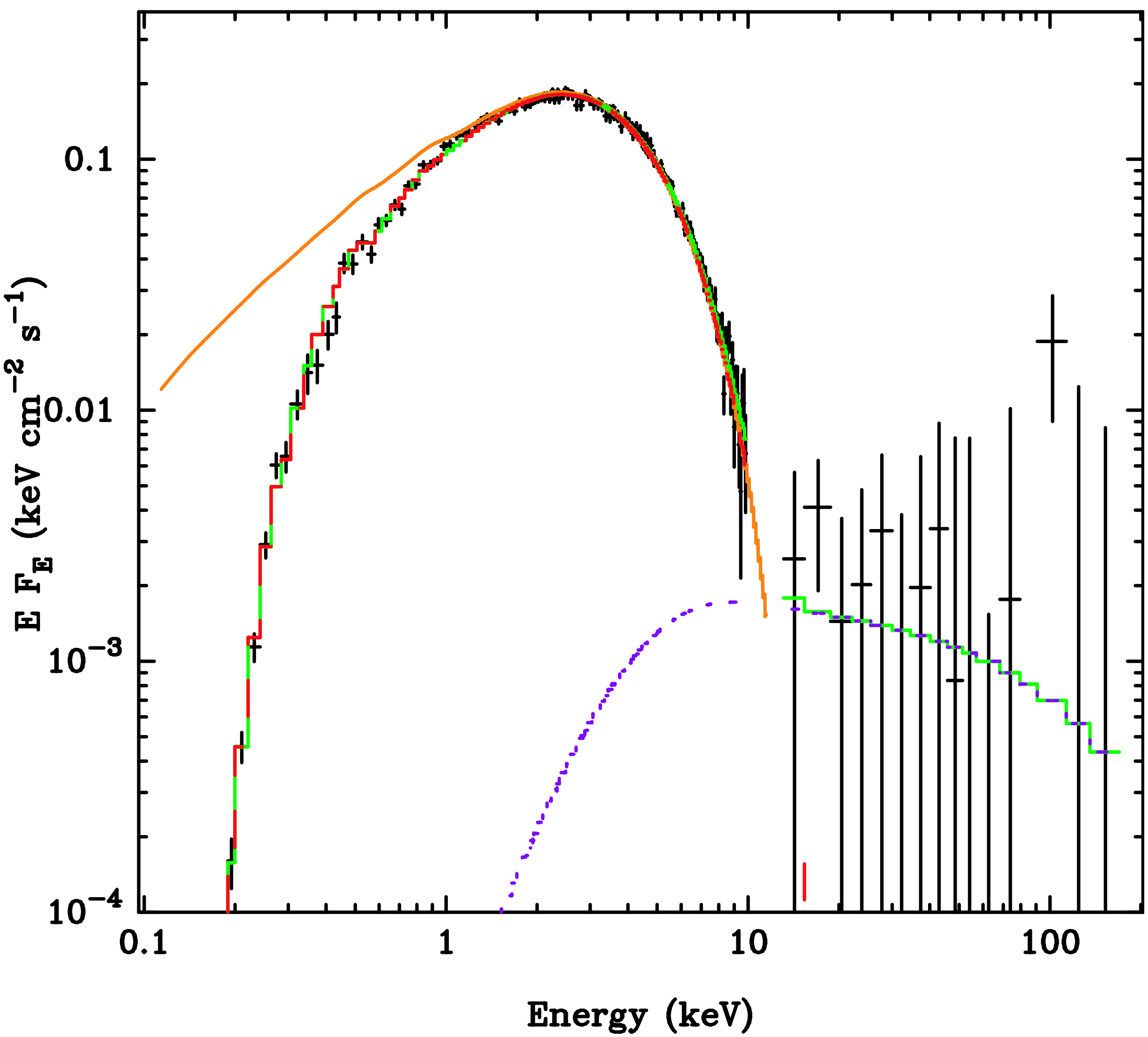
Problem with AGN: peak of disk in UV

$\Longrightarrow$ Galactic Black Holes: T is higher

Find ok agreement between accretion disk models and theory.

In general: models with just $T \propto r^{-3/4}$ and no additional (atomic) physics seem to work best?!?

Galactic Black Holes



(Davis, Done & Blaes, 2006)

Comparison of self-consistent accretion disk model with LMC X-3 data $\Longrightarrow$ good agreement, although values of α smaller than expected (fi ts fi nd $0.01 < \alpha < 0.1$ instead of 0.1–0.8).

Top red line: inferred accretion disk spectrum without interstellar absorption.

Wind Accretion

Early type stars (O, B, mass $\gtrsim$ 10 $M_\odot$):

- strong winds, driven by radiation pressure in absorption lines
- mass loss rates 10^{-10} to $10^{-6}\,M_\odot\,\mathrm{yr}^{-1}$
- Wind velocity

$$v(r) \sim v_\infty \left(1 - \frac{R_\star}{r}\right)^\beta \tag{4.54}$$

with $v_\infty \sim 2000\,\mathrm{km\,s^{-1}}$ and $\beta \sim 0.5 \ldots 1.0$

A fraction of the wind can be accreted by a compact object

$\Longrightarrow$ $\sim$spherical accretion

$\Longrightarrow$ Bondi-Hoyle accretion

(Bondi & Hoyle, 1944)

The simplest case of wind accretion is spherically symmetric accretion.

For spherically symmetric accretion, we can derive the exact solution for the gas flow from the equations of gas dynamics:

Conservation of mass is described by the continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0 \tag{4.55}$$

while the conservation of momentum is described by the Euler equation

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} + \rho \boldsymbol{v} \cdot \nabla \boldsymbol{v} = -\nabla P + \boldsymbol{f} \tag{4.56}$$

where $\boldsymbol{f}$ is a force density (force per unit volume).

By definition, in the spherically symmetric case the flow has only a radial component. Furthermore, if the flow is steady, then all time derivatives vanish. This means that the equation of continuity now reads

$$\nabla \cdot (p\boldsymbol{v}) = \frac{1}{r^2}\frac{d}{dr}\left(r^2 \rho v\right) = 0 \tag{4.57}$$

and therefore

$$r^2 \rho v = \text{const.} = C \tag{4.58}$$

The constant is related to the mass accretion rate: Since the inward flux of mass is given by $\rho|v|$, the mass accretion rate is

$$\dot{M} = 4\pi r^2 \rho |v| \tag{4.59}$$

and therefore

$$C = \frac{\dot{M}}{4\pi} \tag{4.60}$$

To obtain the velocity profile, we use the Euler equation (Eq.4.56). Because of Newton's law of gravitation

$$\boldsymbol{F} = \frac{GMm}{r^2}\frac{\boldsymbol{r}}{r} \tag{4.61}$$

the force density has a radial component only and is given by

$$f = -\frac{GM\rho}{r^2} \tag{4.62}$$

Inserting this into Euler's equation then results in

$$\rho v \frac{dv}{dr} = -\frac{dP}{dr} - \frac{GM\rho}{r^2} \tag{4.63}$$

which simplifies to

$$v\frac{dv}{dr} = -\frac{GM}{r^2} - \frac{1}{\rho}\frac{dP}{dr} \tag{4.64}$$

This differential equation can be solved under the boundary condition of some velocity at infinity. Furthermore, we need to know the equation of state, i.e., how the pressure relates to other quantities in the system. Here, we will be using the polytropic equation of state

$$P = K\rho^\gamma \tag{4.65}$$

where K is some constant. As shown in lectures on thermodynamics, if the gas is isothermal, then $\gamma = 1$, if the flow is adiabatic instead, then $\gamma = 5/3$ (γ is the ratio of specific heats).

With this equation of state, the speed of sound is

$$c_\mathrm{s}^2 = \frac{\partial P}{\partial \rho} = K\gamma\rho^{\gamma-1} \tag{4.66}$$

We now insert the equation of state into Eq. (4.64):

$$v\frac{dv}{dr} = -\frac{GM}{r^2} - \frac{1}{\rho}\gamma K\rho^{-\gamma-1}\frac{d\rho}{dr} = -\frac{GM}{r^2} - c_\mathrm{s}^2\frac{1}{\rho}\frac{d\rho}{dr} \tag{4.67}$$

But because of Eq. (4.57)

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\rho v\right) = 0 \tag{4.57}$$

we have

$$\frac{1}{r^2}\left(\frac{d\rho}{dr}\left(r^2 v\right) + \rho\frac{d}{dr}\left(r^2 v\right)\right) = 0 \quad \Longleftrightarrow \quad \frac{1}{\rho}\frac{d\rho}{dr} = -\frac{1}{vr^2}\frac{d}{dr}\left(r^2 v\right) \tag{4.68}$$

Inserting this into Eq. (4.67) gives

$$v\frac{dv}{dr} = -\frac{GM}{r^2} + c_\mathrm{s}^2\frac{1}{vr^2}\frac{d}{dr}\left(r^2 v\right) = -\frac{GM}{r^2} + c_\mathrm{s}^2\left(\frac{2}{r} + \frac{1}{v}\frac{dv}{dr}\right) \tag{4.69}$$

Multiplying by v then results in

$$v^2\frac{dv}{dr} = -\frac{GMv}{r^2} + \frac{2v}{r}c_\mathrm{s}^2 + c_\mathrm{s}^2\frac{dv}{dr} \tag{4.70}$$

and therefore

$$\left(v^2 - c_\mathrm{s}^2\right)\frac{dv}{dr} = v\left(\frac{2c_\mathrm{s}^2}{r} - \frac{GM}{r^2}\right) \tag{4.71}$$

Bondi-Hoyle Accretion, I

Spherical symmetric accretion:

$$\left(v^2 - c_{\mathrm{s}}^2\right)\frac{dv}{dr} = v\left(\frac{2c_{\mathrm{s}}^2}{r} - \frac{GM}{r^2}\right) \tag{4.71}$$

For r large: right hand side is positive.

Since $dv/dr < 0$ for accretion, this means that for large r: $v < c_{\mathrm{s}}$.

Similarly, for small r: $v > c_{\mathrm{s}}$

$\Longrightarrow$ sonic point for $v = c_{\mathrm{s}}$ at

$$r_{\mathrm{sonic}} = \frac{GM}{2c_{\mathrm{s}}^2} \tag{4.72}$$

$\Longrightarrow$ *If* the flow goes supersonic, it does so at $r = r_{\mathrm{sonic}}$

Note that c_{s} depends on r, several other solutions are possible, but the above one is the most common one for the objects we're looking at. See Holzer & Axford (1970) for details.

Bondi-Hoyle Accretion, II

To finish the discussion of Bondi-Hoyle accretion, we now explicitly integrate Euler's equation

$$v\frac{dv}{dr} + \frac{GM}{r^2} + \frac{1}{\rho}\frac{dP}{dr} = 0 \quad (4.64)$$

over r:

$$\int v\frac{dv}{dr}\,dr + \int \frac{GM}{r^2}\,dr + \int \frac{dP}{\rho} = 0 \quad (4.73)$$

inserting $dP = K\gamma\rho^{\gamma-1}d\rho$ and integrating then gives the Bernoulli integral

$$\frac{1}{2}v^2 + \frac{\gamma}{\gamma-1}K\rho^{\gamma-1} - \frac{GM}{r} = \text{const.} \quad (4.74)$$

which obviously is related to energy conservation and can be written as

$$\frac{1}{2}v^2 + \frac{c_\text{s}^2}{\gamma-1} - \frac{GM}{r} = \text{const.} = \frac{c_{\text{s},\infty}}{\gamma-1} \quad (4.75)$$

where $c_{\text{s},\infty}$ is the speed of sound at $r = \infty$.

This follows since $v(r \to \infty) = 0$.

Bondi-Hoyle Accretion, III

From Eq. (4.75) we can now determine the speed of sound at the sonic point

$$c_s^2(r_{\text{sonic}}) = c_{s,\infty} \left(\frac{2}{5-3\gamma} \right)^{1/2} \tag{4.76}$$

and the mass accretion rate is

$$\dot{M} = 4\pi r^2 \rho |v| = 4\pi r_{\text{sonic}}^2 \rho(r_{\text{sonic}}) c_s(r_{\text{sonic}}) \tag{4.77}$$

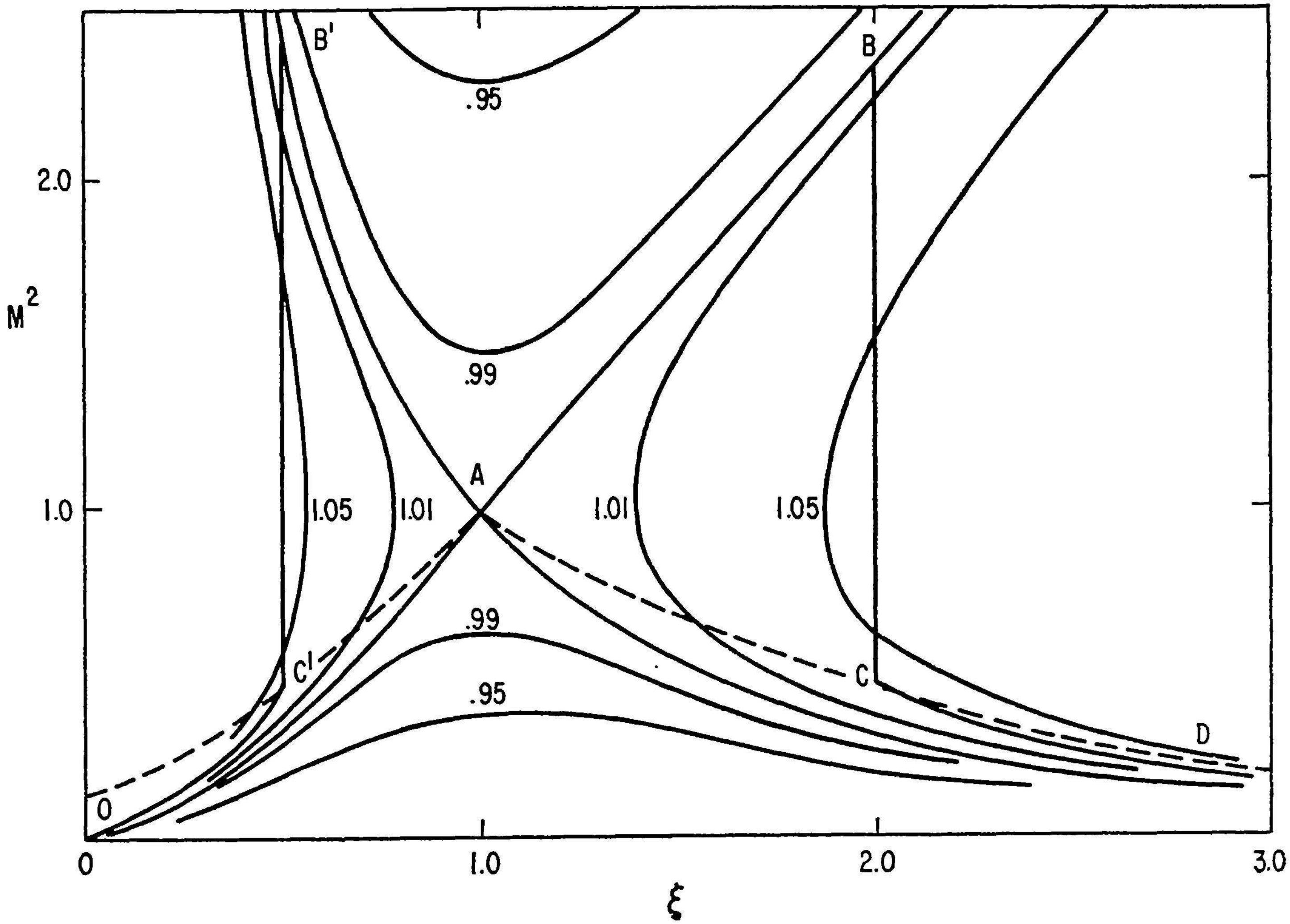
Since $c_s^2 \propto \rho^{\gamma-1}$,

$$\rho(r_{\text{sonic}}) = \rho_\infty \left(\frac{c_s(r_{\text{sonic}})}{c_{s,\infty}} \right)^{2/(\gamma-1)} \tag{4.78}$$

Therefore

$$\dot{M} = \pi G^2 M^2 \frac{\rho_\infty}{c_{s,\infty}^3} \left(\frac{2}{5-3\gamma} \right)^{(5-3\gamma)/2(\gamma-1)} \tag{4.79}$$

Bondi-Hoyle Accretion, IV



Mach number ($M = v(r)/c_{\mathrm{s}}(r)$) as a function of radial distance, $\xi = r/r_{\mathrm{sonic}}$, for all possible solutions of the spherical accretion problem.

(Holzer & Axford, 1970, Fig. 1)

Bondi-Hoyle Accretion, V

Taking $\gamma = 5/3$, Eq. (4.79) becomes

$$\begin{aligned}\dot{M} &= \pi G^2 M^2 \frac{\rho_\infty}{c_{\mathrm{s},\infty}^3} \\ &= \pi \left(\frac{GM}{c_{\mathrm{s},\infty}^2}\right)^2 \rho_\infty c_{\mathrm{s},\infty} \\ &= \pi r_{\mathrm{acc}}^2 \rho_\infty c_{\mathrm{s},\infty}\end{aligned} \tag{4.80}$$

where the accretion radius is defined as

$$r_{\mathrm{acc}} = \frac{GM}{c_{\mathrm{s},\infty}^2} \tag{4.81}$$

Often, r_{acc} is defined as $r_{\mathrm{acc}} = 2GM/c_{\mathrm{s}}$, see next slide for the reason why.

r_{acc} defines the approximate radius of influence of an accreting body.

Wind accretion, I

If the ambient medium is not at rest: wind accretion. In principle, we can do a similar calculation as for Bondi-Hoyle accretion, however, this would take too long, so let's do an approximate treatment here.

Let the wind's velocity be v_∞. The material in the wind is captured once

$$\frac{1}{2}v_\infty^2 = \frac{GM}{r_{\mathrm{acc}}} \tag{4.82}$$

such that the accretion radius for wind accretion is

$$r_{\mathrm{acc}} = \frac{2GM}{v_\infty^2} \tag{4.83}$$

...explaining why many people like to have a factor 2 also in the defi nition of r_{acc} for Bondi-Hoyle accretion.

Therefore, analoguously to Eq. (4.80),

$$\dot{M} = \pi r_{\mathrm{acc}}^2 \rho_\infty v_\infty = \frac{4\pi G^2 M^2 \rho_\infty}{v_\infty^3} \tag{4.84}$$

Wind accretion, II

To estimate the typical parameters of a wind accretor, we need to estimate v_∞ for a compact object at a distance a from the donor star

The typical velocity consists of two contributions:

1. The stellar wind velocity profile

$$v_{\text{wind}}(a) \sim v_{\text{wind},\infty} \left(1 - \frac{R_\star}{a}\right)^\beta \tag{4.54}$$

2. The orbital velocity of the compact object

$$v_{\text{compact}}(a) = \sqrt{\frac{GM}{a}} \tag{4.85}$$

Therefore

$$v_\infty^2 \sim v_{\text{wind}}^2 + v_{\text{compact}}^2 = \frac{GM}{a} + v_{\text{wind},\infty}^2 \left(1 - \frac{R_\star}{a}\right)^{2\beta} \sim \frac{GM}{a} + v_{\text{wind},\infty}^2 \tag{4.86}$$

the last is true assuming that the compact object is outside of the wind acceleration zone

Wind accretion, III

Finally, making use of the fact that the wind density is

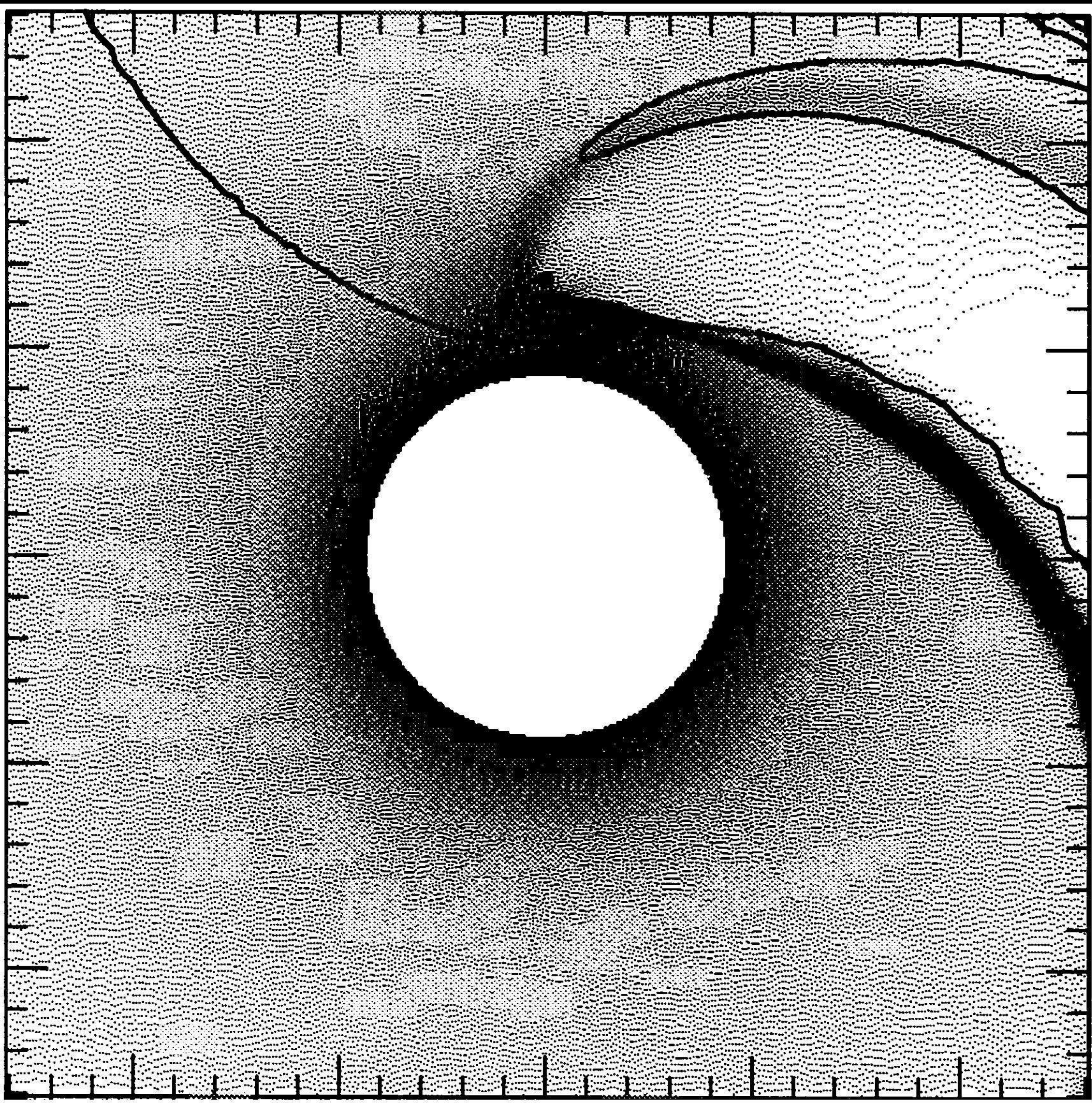
$$\rho_\infty = \frac{\dot{M}_\mathrm{W}}{4\pi a^2 v_{\mathrm{wind},\infty}} \tag{4.87}$$

where $\dot{M}_\mathrm{W}$ is the wind loss rate of the donor.

Therefore, the accretion rate of the compact object is

$$\begin{aligned}\dot{M} &= \frac{G^2M^2}{a^2 v_{\mathrm{wind},\infty}\left(\frac{GM}{a}+v^2_{\mathrm{wind},\infty}\right)^{3/2}}\dot{M}_\mathrm{W} \\ &= \begin{cases}\left(\frac{GM}{av^2_{\mathrm{wind},\infty}}\right)^{1/2}\dot{M}_\mathrm{W} & \text{for } v_\mathrm{orbit} \gg v_{\mathrm{wind},\infty}\\ \frac{G^2M^2}{a^2v^4_{\mathrm{wind},\infty}}\dot{M}_\mathrm{W} & \text{for } v_\mathrm{orbit} \ll v_{\mathrm{wind},\infty}\end{cases}\end{aligned} \tag{4.88}$$

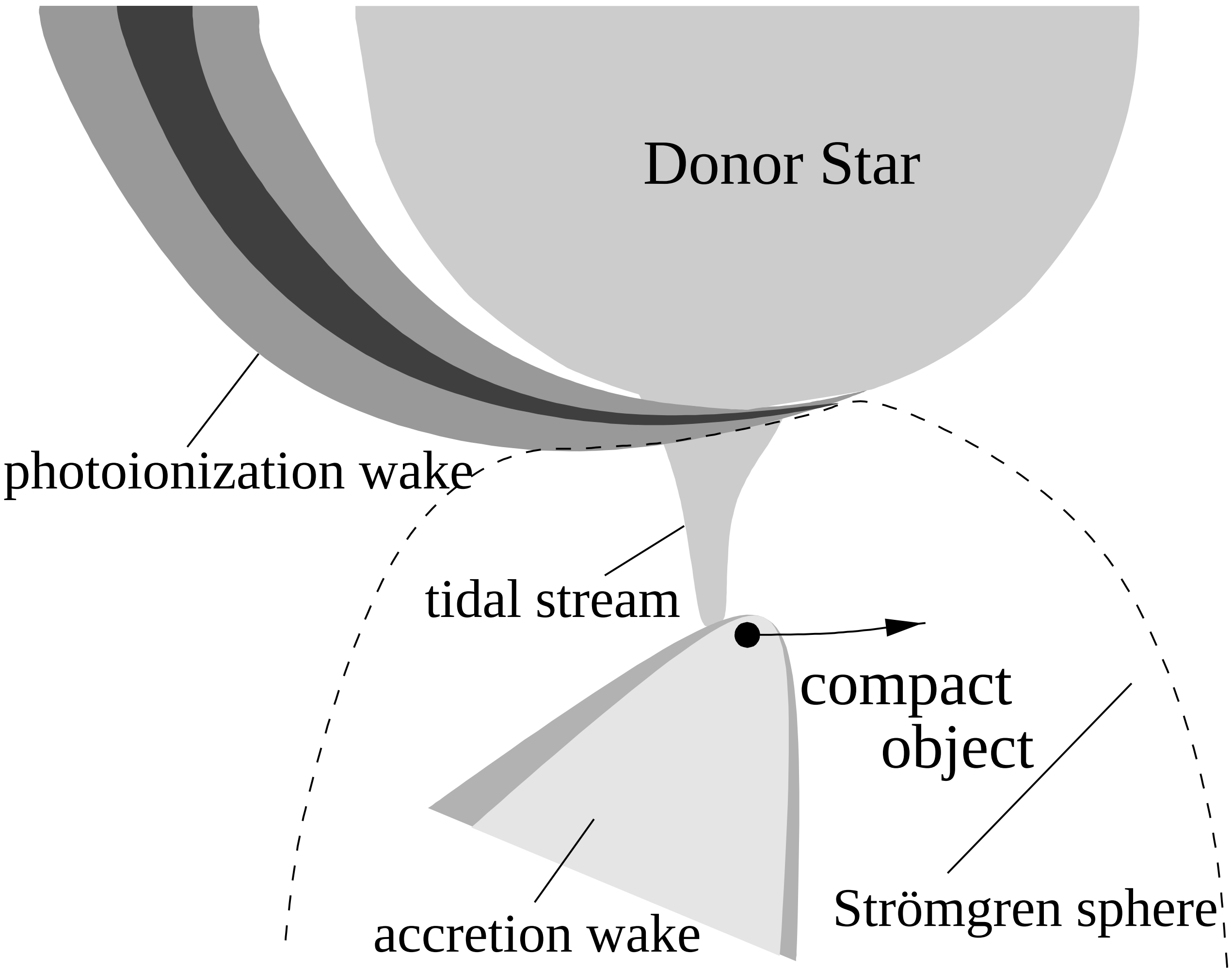
So, for $M = 1.44\,M_\odot$, $v_{\mathrm{wind},\infty} = 500\,\mathrm{km\,s^{-1}}$, $a = 10^7\,\mathrm{km}$, $\dot{M} = 6\times10^{-3}\dot{M}_\mathrm{W}$, i.e., the Eddington rate ($\dot{M}_\mathrm{Edd} = 2.9\times10^{-8}\,M_\odot\,\mathrm{yr}^{-1}$ for $1.44\,M_\odot$) is reached for $\dot{M}_\mathrm{W} = 4.8\times10^{-6}\,M_\odot\,\mathrm{yr}^{-1}$, which is very realistic.



(Blondin 1994, Fig. 4)

Realistic hydrodynamical computations are diffi cult (asymmetry of accretion process, ionization of wind, large range of length-scales involved,. . .).

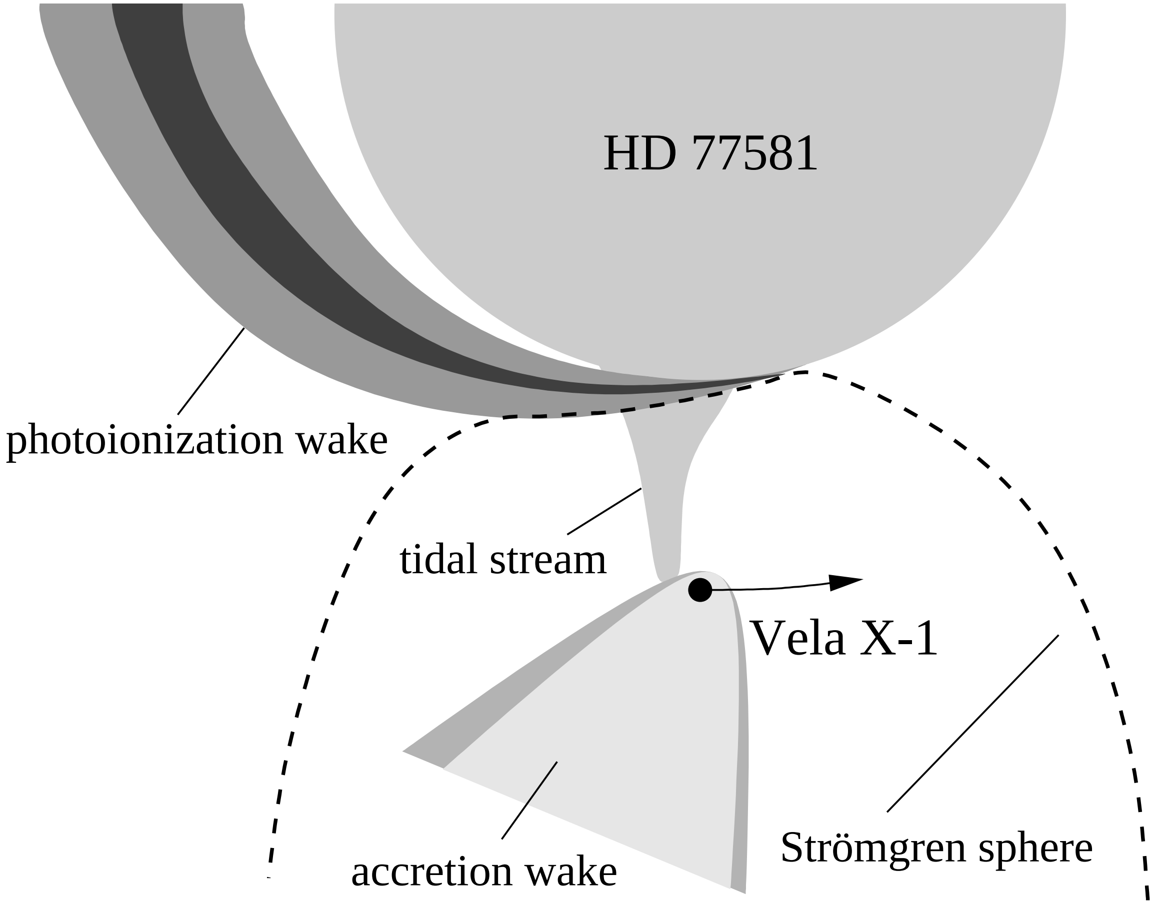
Accretion in HMXB



Principal components for wind-accretion:

- Ionized Strömgren region (wind ionized by X-rays from compact object).
- Accretion shock around compact object (since $v_{\mathrm{orb}} > c_{\mathrm{s}}$).
- Ionization wake where material is overdense.

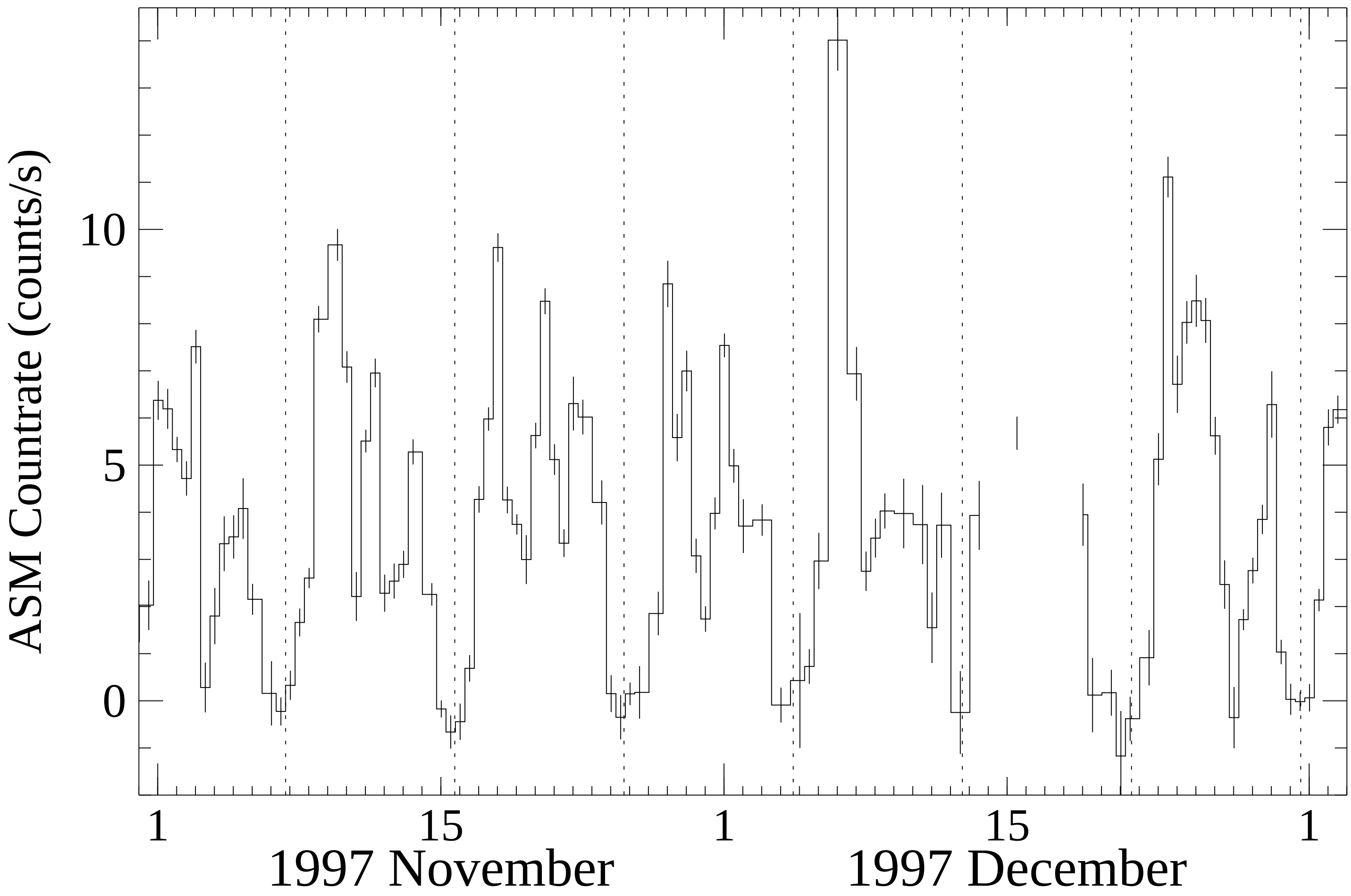
Accretion in HMXB



In realistic HMXB, because the accreted material still has some angular momentum, a small accretion disk still forms.

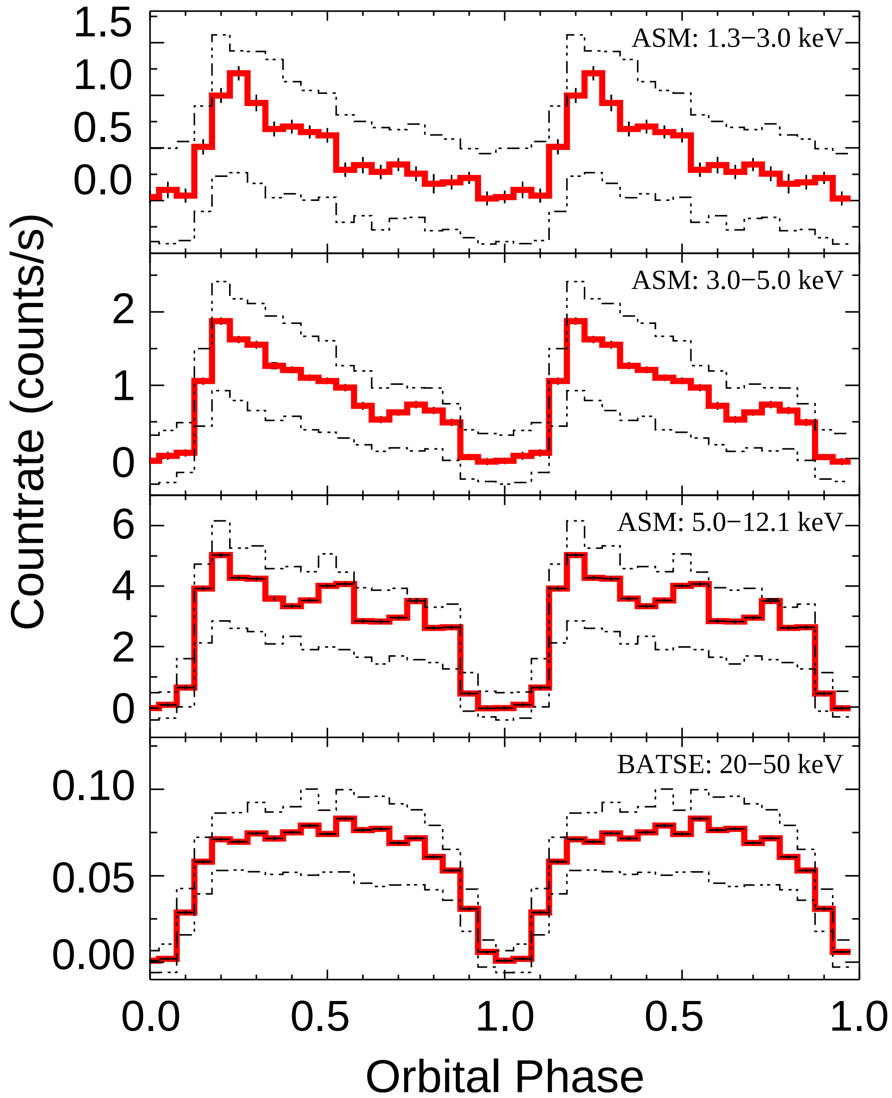
J. Blondin: "The disk is being BASHED by the stellar wind, BATTERED by the tidal stream, and BLASTED by X-rays"

Accretion in HMXB

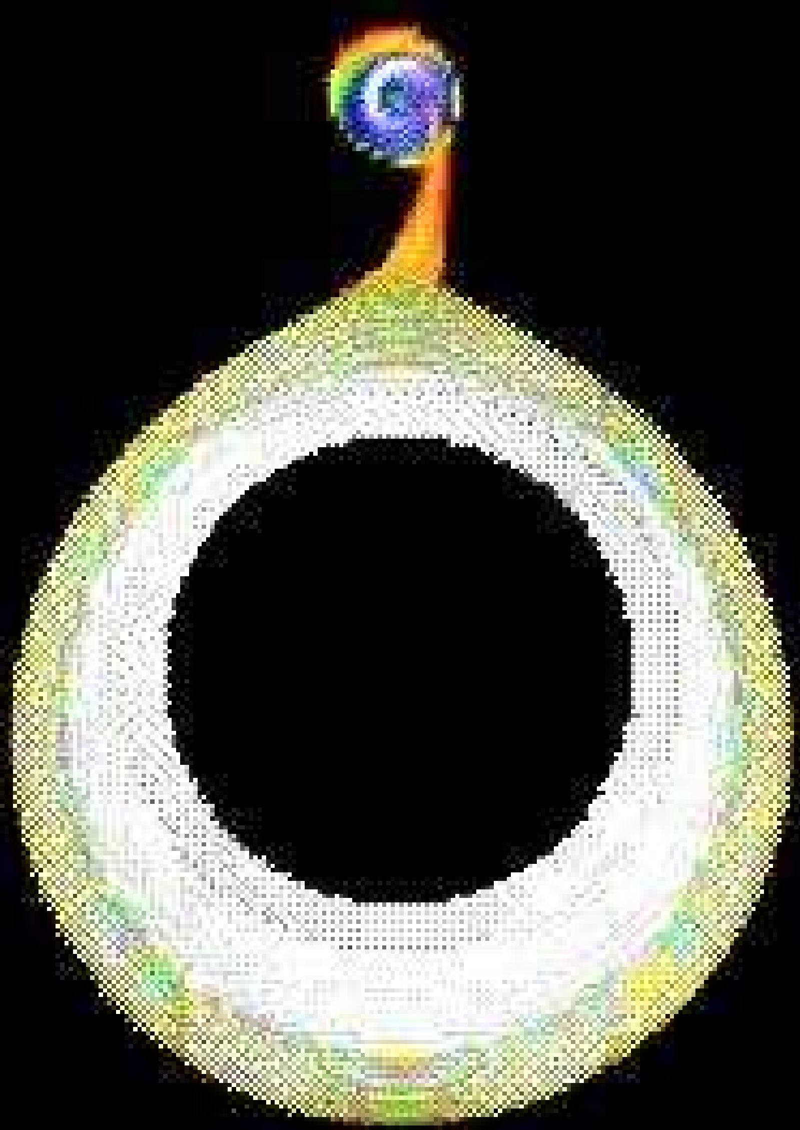


RXTE-ASM 2–10 keV lightcurve of the HMXB Vela X-1

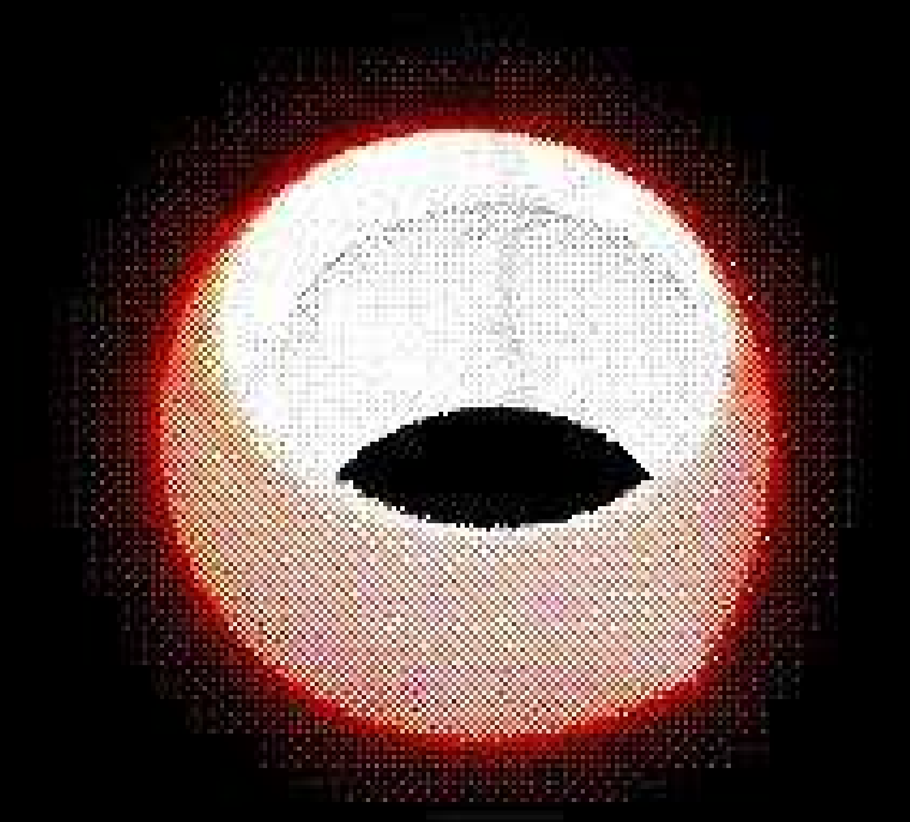
Accretion in HMXB



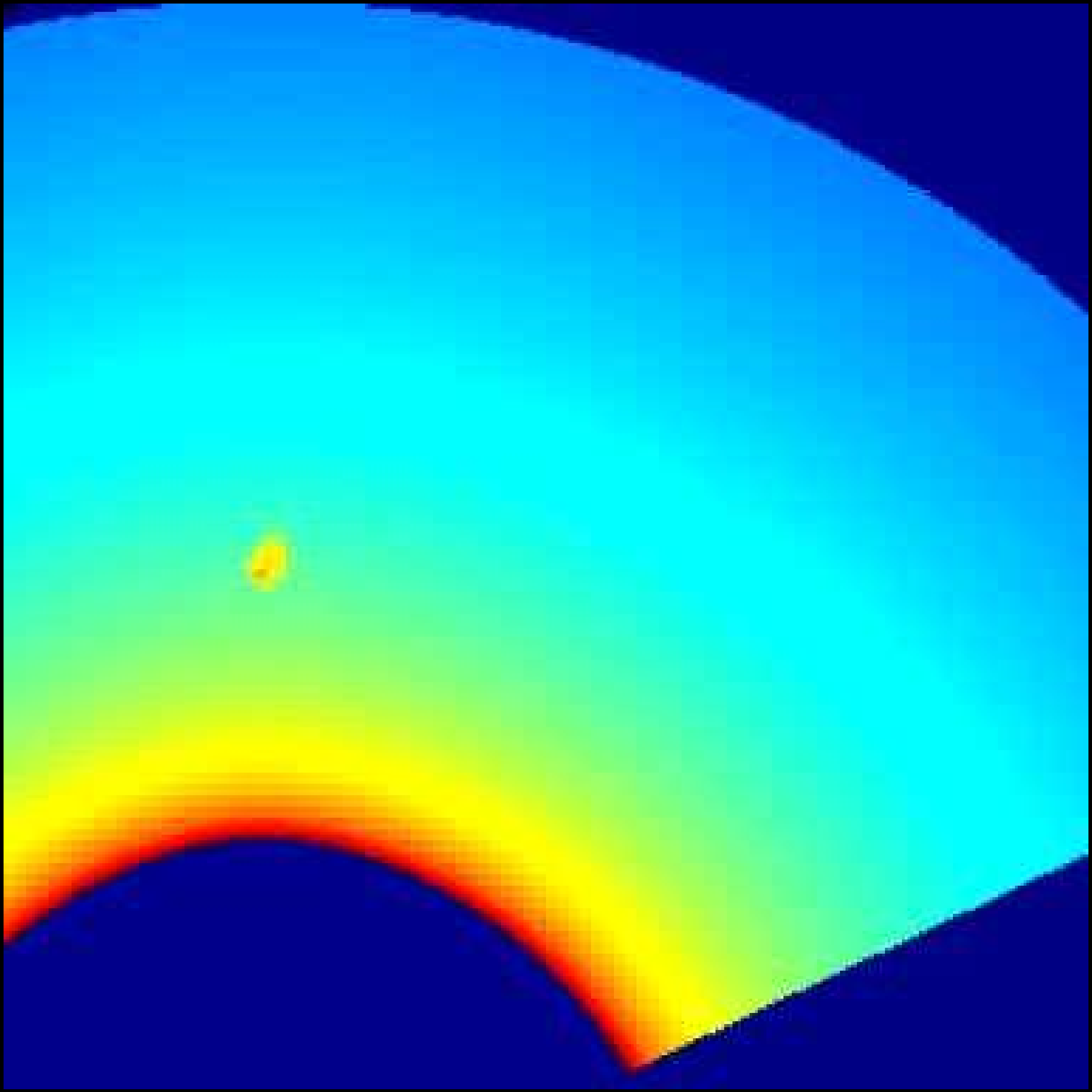
Orbit averaged, energy resolved lightcurves of Vela X-1



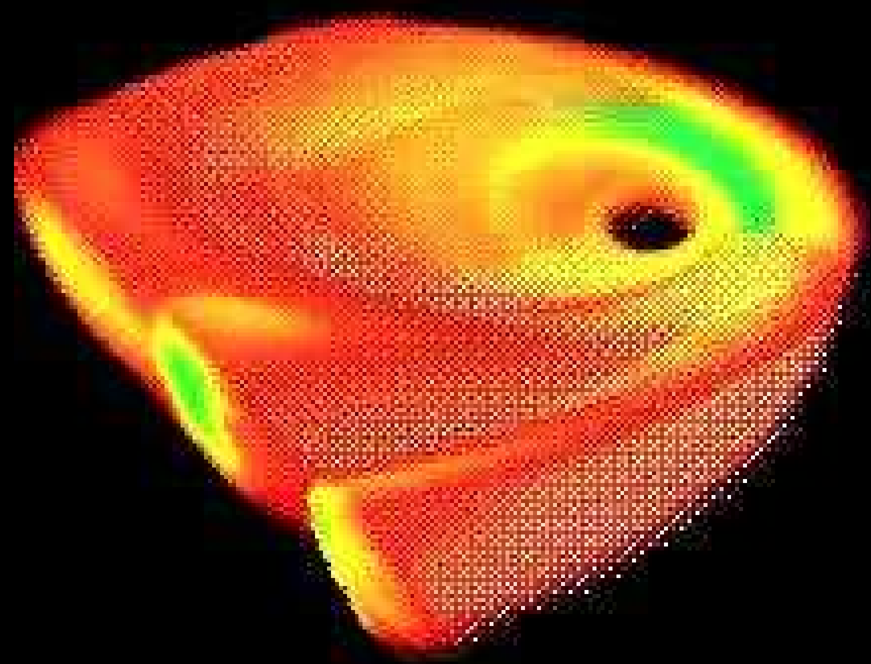
courtesy J. Blondin



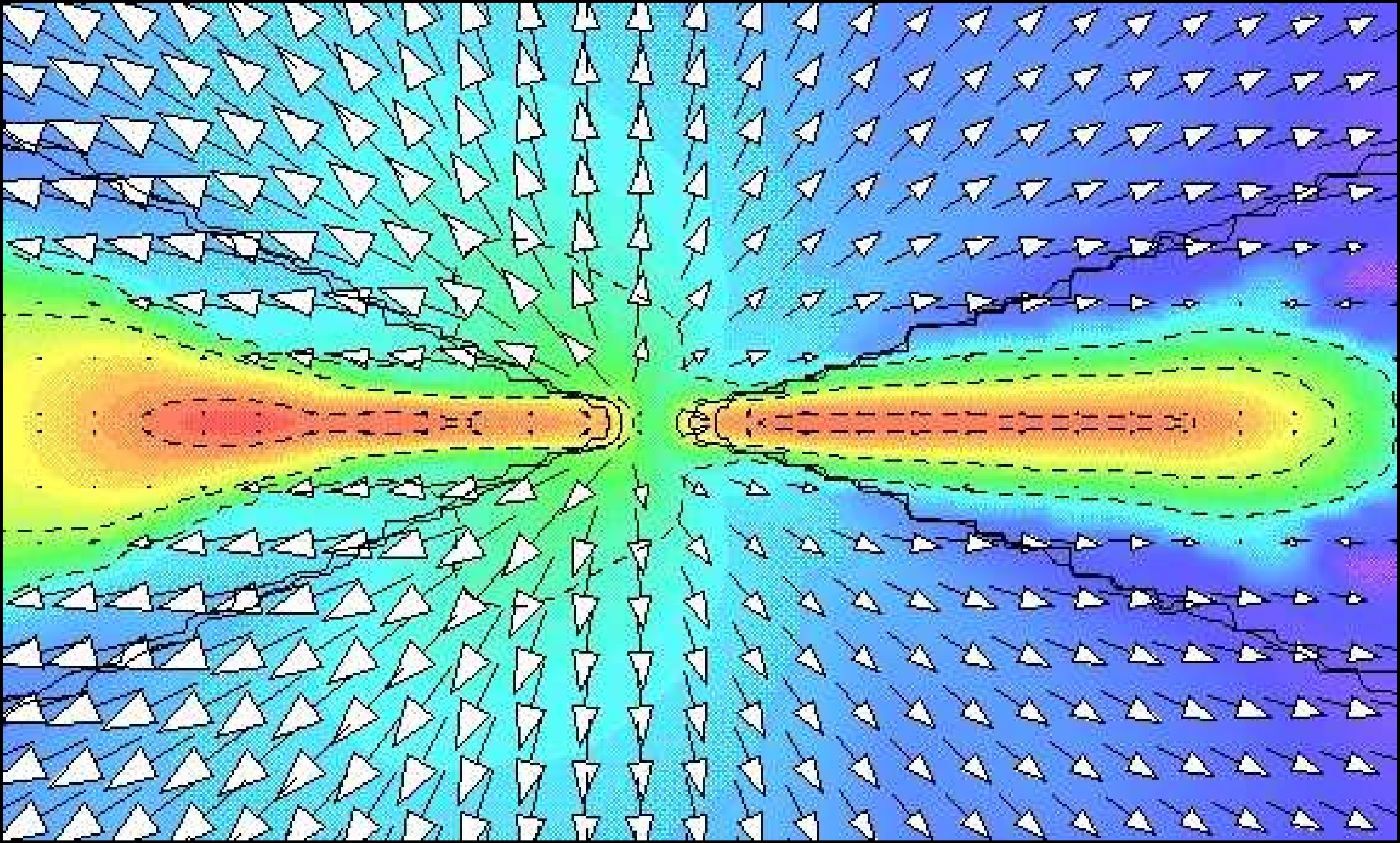
courtesy J. Blondin



courtesy J. Blondin



courtesy J. Blondin



courtesy J. Blondin

X-rays from central source heat disk surface, drive a strong wind.

Balbus, S. A., & Hawley, J. F., 1991, ApJ, 376, 214

Blondin, J. M., 1994, ApJ, 435, 756

Bondi, H., & Hoyle, F., 1944, MNRAS, 104, 273

Chandrasekhar, S., 1961, Hydrodynamic and Hydromagnetic Stability, (Oxford: Oxford Univ. Press), (reprinted 1981 by Dover, New York)

Davis, S. W., Blaes, O. M., Hubeny, I., & Turner, N. J., 2005, ApJ, 621, 372

Davis, S. W., Done, C., & Blaes, O. M., 2006, ApJ, 647, 525

Hawley, J. F., & Krolik, J. H., 2002, ApJ, 566, 164

Holzer, T. E., & Axford, W. I., 1970, Ann. Rev. Astron. Astrophys., 8, 31

Lubow, S. H., & Shu, F. H., 1975, ApJ, 198, 383

Velikhov, E. P., 1959, Sov. Phys. – JETP, 9, 995

Wilms, J., Nowak, M. A., Pottschmidt, K., Heindl, W. A., Dove, J. B., & Begelman, M. C., 2001, MNRAS, 320, 327

Accretion onto Magnetized Neutron Stars