Friedrich-Alexander-Universität
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Active Galactic Nuclei

Sommersemester 2010
Übungsaufgaben 1
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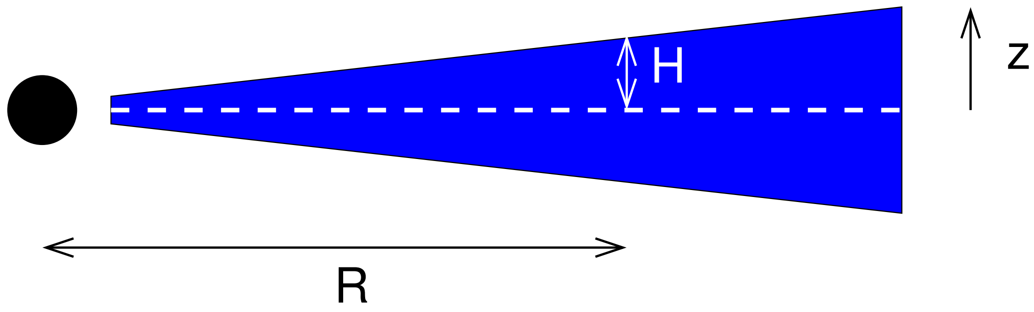
Question 1: *Accretion Disks*



In this exercise we will be looking in more detail at the structure of thin accretion disks, i.e., at disks for which the height of the disk, H, scales with distance R from the central compact object such that $H \ll R$.

a) **Accretion disks are strongly supersonic**.

In order to get an order of magnitude feeling for the condition within the accretion disk, we will first take a look at the vertical structure of a disk, i.e., at the z-direction. Since thin disks are gas pressure supported, we will ignore radiation pressure. For a stationary system, this means that the gravitational force in z-direction is balanced by the force due to the pressure gradient in that direction. Convince yourself that the force due to a pressure gradient scales as

$$\frac{1}{\rho}\frac{\partial P}{\partial z} \approx \frac{1}{\rho_c}\frac{P_c}{H} \tag{1.1}$$

where P_c is some characteristic pressure and ρ_c some characteristic density. Using these assumptions, show that accretion disks must be strongly supersonic

(*Hint:* the speed of sound is $c_s = \sqrt{P/\rho}$, the Kepler speed is $v_\phi = \sqrt{GM/R}$).

b) **The vertical density decays exponentially**

Convince yourself that the vertical density gradient in the accretion disk must scale as

$$n_R(z) \propto \exp\left(-\frac{z}{H}\right) \tag{1.1}$$

where $n_R(z)$ is the particle density at radius R and height z (you can assume the disk to be isothermal).

c) **Gas particles move on quasi-spherical orbits**

In the lectures, it was claimed that the gas motion in the accretion disk is primarily on circular orbits, or, in other words, that the radial velocity of gas in the disk, v_R, is significantly smaller than the Kepler speed, v_ϕ. Using the same approximations as above, show that the radial acceleration due to a radial gradient in gas pressure is negligable compared to the acceleration due to gravitation, and that therefore $v_R \ll v_\phi$.

d) **Mass Conservation**

In a stationary disk, the mass flow towards the black hole through the disk, $\dot{M}$, must be conserved. Assuming you know v_R and using the *surface density*

$$\Sigma(R) = \int n_R(z)\,\mathrm{d}z \tag{1.1}$$

write down an equation connecting $\dot{M}$ and $\Sigma(R)$ (this can be used to calculate the normalization constant in Eq. (1.1)).

e) **Angular Momentum Transport**

Since the disk is rotating differentially, we need to transport angular momentum away. This is somewhat tricky…

i) Calculate the specific angular momentum, i.e., the angular momentum per unit mass, for a ring of the accretion disk that rotates with the Kepler speed, and convince yourself that the ring material needs to loose angular momentum in order to move towards the black hole.

ii) To get rid of the angular momentum, viscous forces are invoked. The definition of the *coefficient of kinematic viscosity*, ν, is such that the force per unit length of between two media moving with relative speed Δv with respect to each other can be written

$$\mathcal{F} = \nu\Sigma \cdot \Delta v \tag{1.1}$$

Calculate the total torque on a ring at distance R, $G(R)$. To simplify your life it is useful to write Δv in terms of $d\Omega/dR$ where Ω is the angular velocity of the disk.

$$\Omega = \sqrt{\frac{GM}{R^3}} \tag{1.2}$$

iii) So far, you have calculated the torque between *two* rings. However, the disk consists of *many* rings, and therefore the total torque available for balancing the change in angular momentum is the net torque $\frac{dG}{dR}$ only. Use this *Ansatz* to show that

$$\nu\Sigma = \frac{\dot{M}}{3\pi} + \frac{\text{const.}}{R^{1/2}} \tag{1.1}$$

(*Note:* The change in angular momentum per unit time to be balanced by the torque is $\dot{M}d\mathcal{L}/dR$)

iv) The constant in Equation (1.1) depends on the so-called inner boundary condition. For a black hole, one often makes the assumption that no torque is acting at the inner edge of the accretion disk, at radius R_* (for black holes, $R_* = 6GM/c^2$, but this is not important here), i.e., that $dG/dR(R_*) = 0$. Show that this means that

$$\nu\Sigma = \frac{\dot{M}}{3\pi}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right] \tag{1.1}$$

f) **Temperature Profile**

The energy dissipated per unit area can be calculated from

$$D(R) = \frac{1}{2}\nu\Sigma\Delta v^2 \tag{1.1}$$

Assuming the disk is optically thick, use the Stefan-Boltzmann law and your previous results to determine the temperature profile of the accretion disk.

(*Note:* Do not forget that the disk has two sides…)

g) **Total disk luminosity**

Assuming the disk's outer radius is at infinity, calculate the total luminosity of the accretion disk. Compare your result with the total available energy.