Observational Cosmology

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Erlangen–Nürnberg

Introduction

Schedule

Introduction	01	16.10.	Introduction/History
	02	23.10.	Basic Facts
World Models	03	30.10.	World Models
Classical Cosmology	04	06.11.	Distances, H_0
	05	13.11.	Distances, H_0
The Early Universe	06	20.11.	Hot Big Bang Model
	07	27.11.	Nucleosynthesis
	08	04.12.	Inflation
		11.12.	**no lecture**
Large Scale Structures	09	18.12.	Ω and Λ
	10	08.01.	Dark Matter
	11	15.01.	Large Scale Structures
	12	22.01.	Structure Formation
	13	29.01.	Structure Formation
Summary	14	05.02.	Wrap Up

Literature

1. Cosmology Textbooks

SCHNEIDER, P., 2005, *Einführung in die Extragalaktische Astronomie und Kosmologie*, Heidelberg: Springer, 59.95■(English edition also available)

Well written introduction to cosmology, approximately at the level of this lecture. Recommended.

PEACOCK, J.A., 1999, *Cosmological Physics*, Cambridge: Cambridge Univ. Press, 49.50■

Very exhaustive, but difficult to read since the entropy per page is *very* high... still: a "must buy".

LONGAIR, M.S., 1998, *Galaxy Formation*, Berlin: Springer, 53.45■

Clear and pedagogical treatment of structure formation, recommended.

Literature

BERGSTRÖM, L. & GOOBAR, A., 1999, *Cosmology and Particle Astrophysics*, New York: Wiley, 47.90■

Nice description of the physics relevant to cosmology and high energy astrophysics, focusing on concepts. Less detailed than Peacock, but easier to digest.

PADMANABHAN, T., 1996, *Cosmology and Astrophysics Through Problems*, Cambridge: Cambridge Univ. Press, $36.95

Large collection of standard astrophysical problems (with solutions) ranging from radiation processes and hydrodynamics to cosmology and general relativity

PADMANABHAN, T., 1993, *Structure Formation in the Universe*, Cambridge: Cambridge Univ. Press, 46.50■

Mathematical treatment of cosmology, focusing on the formation of structure . . . Less astrophysical than the book by Longair.

ISLAM, J.N., 2002, *An Introduction to Mathematical Cosmology*, Cambridge: Cambridge Univ. Press, 42.50■

Useful summary of the facts of classical theoretical cosmology, recently revised.

Literature

KOLB, E.W. & TURNER, M.S., 1990, *The Early Universe*, Reading: Addison-Wesley, 49.90■

Graduate-level text, the section on phase transitions and inflation in the early universe is especially recommended.

PEEBLES, P.J.E., 1993, *Principles of Physical Cosmology*, Princeton: Princeton Univ. Press (antiquarian only, do not pay more than $30!)

700p introduction to modern cosmology by one of its founders, in some parts quite readable, however, many forward references make the book very difficult to read for beginners.

Literature

2. Textbooks on General Relativity

WEINBERG, S., 1972, *Gravitation and Cosmology*, New York: Wiley, 129■

Classical textbook on GR, still one of the best introductions. Nice section on classical cosmology.

SCHUTZ, B.F., 1985, *A First Course in General Relativity*, Cambridge: Cambridge Univ. Press, 45.90■

Nice and modern introduction to GR. The cosmology section is very short, though.

MISNER, C.W., THORNE, K.S. & WHEELER, J.A., 1973, *Gravitation*, San Francisco: Freeman, 104.90■

Commonly called "MTW", this book is as heavy as the subject... Uses a weird notation. The cosmology section is outdated.

WALD, R.M., 1984, *General Relativity*, Chicago: Univ. Chicago Press (only antiquarian, $\sim$\$40)

Modern introduction to GR for the mathematically inclined.

History

Prehistory

Pre-Babylonian astronomy: no written records known

But: Observations of the sky must have been important!

“Adorant” from the Geißenklösterle cave near Blaubeuren (Lkr. Ulm; 3.8 cm × 1.4 cm); Back side shows marks which have been interpreted as a lunar calendar.

Babylon

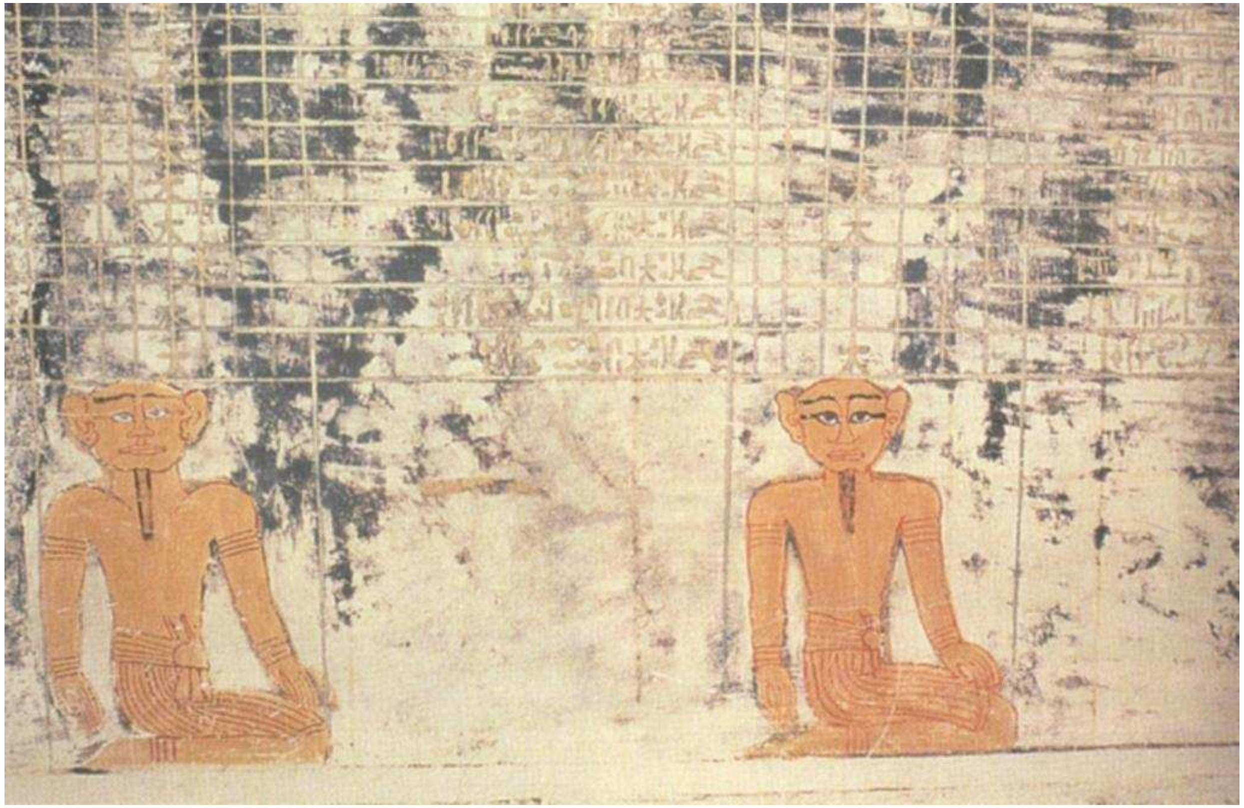
Babylonian astronomy: Earliest astronomy with influence on us: $\sim$360 d year
$\Longrightarrow$ sexagesimal system [360:60:60], 24h day, 12×30 d year,...

Enuma Elish myth ($\sim$1100BC): Universe is place of battle between Earth and Sky, born from world parents.

Note similar myth in the Genesis...

Image: Mul.Apin cuneiform tablet (British Museum, BM 86378, 8 cm high), describes rising and setting of constellations through the babylonian calendar. Summarizes astronomical knowledge as of before $\sim$690 BC.

Egypt



Egyptian coffin lid showing two assistant astronomers, 2000...1500 BC; hieroglyphs list stars ("decans") whose rise defines the start of each hour of the night.

(Aveni, 1993, p. 42)

$\sim$2000 BC: 365 d calendar (12×30 d plus 5 d extra), fixed to Nile flood (heliacal rising of Sirius), star clocks.

heliacal rising: first appearance of star in eastern sky at dawn, after it has been hidden by the Sun.

Greek/Roman, I

Atlas Farnese, 2c A.D., Museo Archeologico Nazionale, Napoli

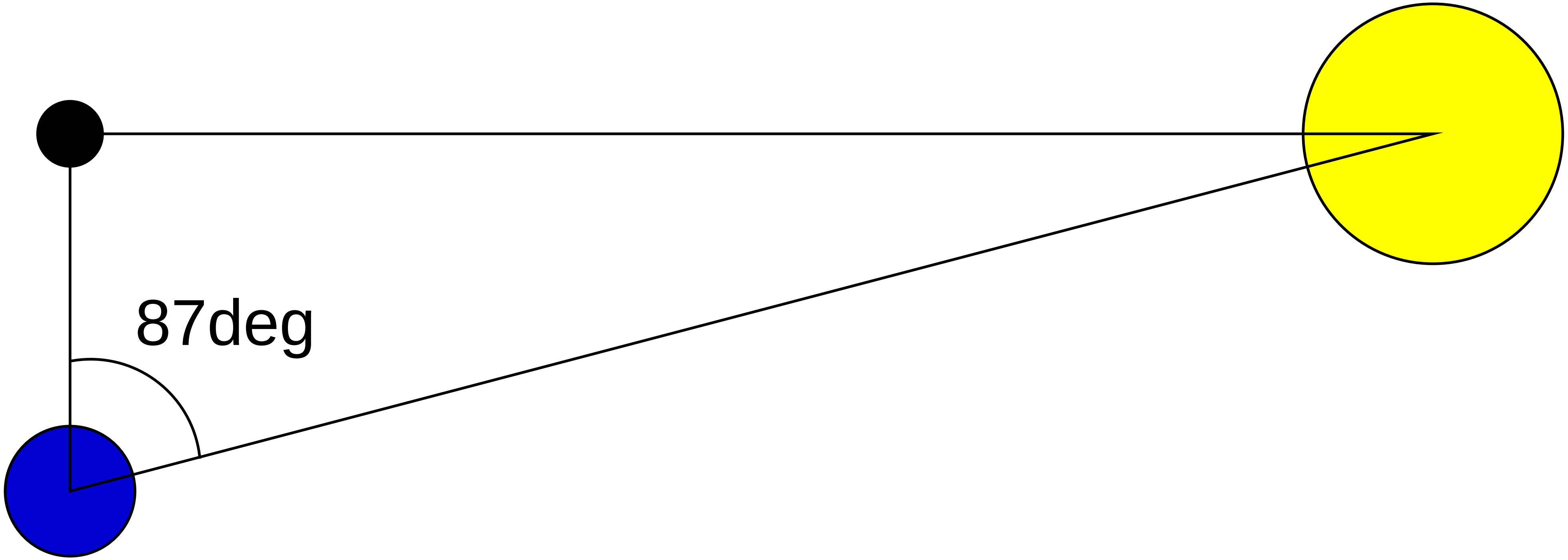
Early Greek astronomy: folk tale astronomy (Hesiod (730?–? BC), *Works and Days*). Constellations.

Thales (624–547 BC): Earth is flat, surrounded by water.

Anaxagoras (500–428 BC): Earth is flat, floats in nothingness, stars are far away, fixed on sphere rotating around us. Lunar eclipses: due to Earth's shadow, Sun is hot iron sphere

Eudoxus (408–355 BC): Geocentric, planets affixed to concentric crystalline spheres. First real model for planetary motions!

Greek/Roman, II

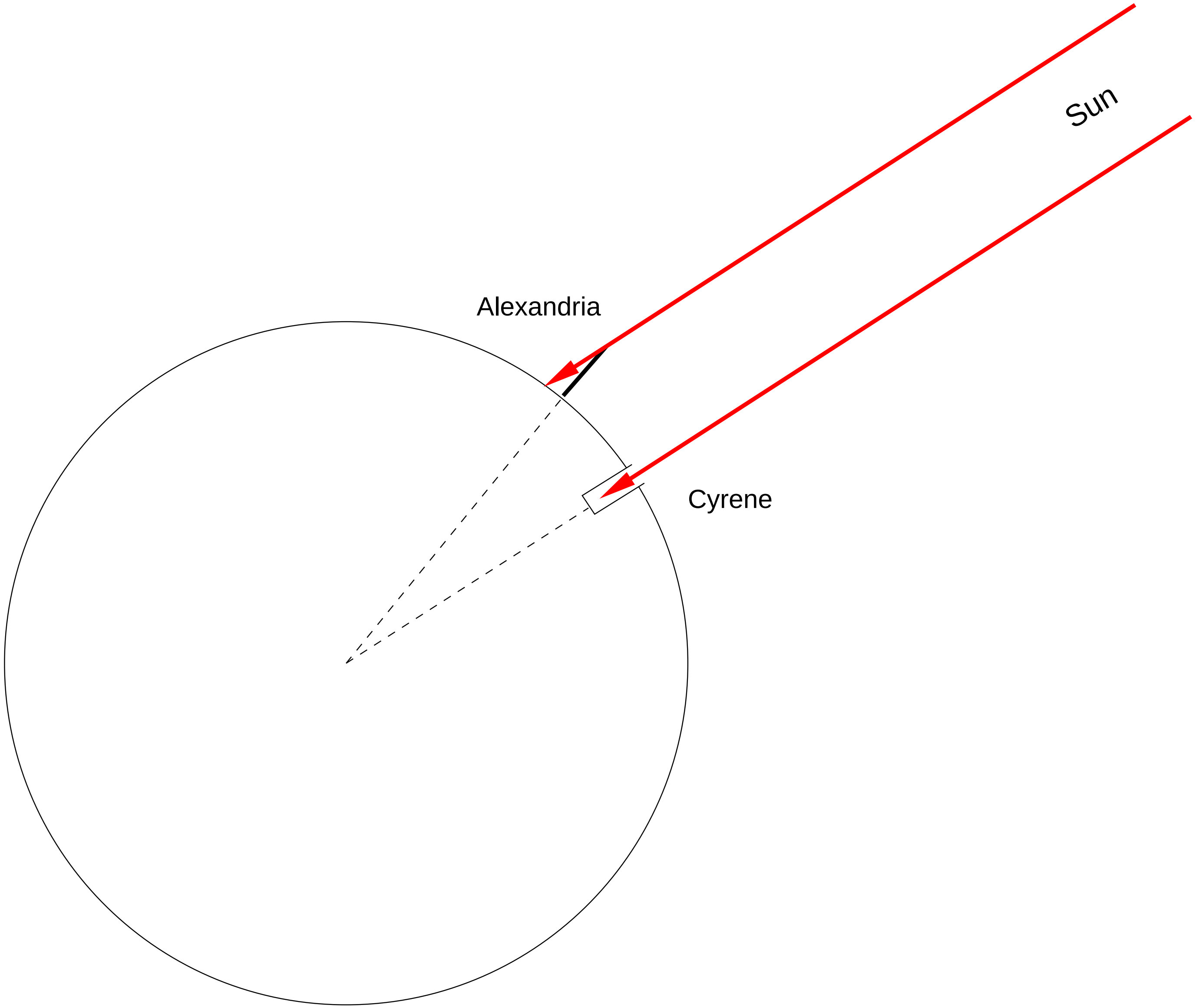


First attempts to measure scale of the universe:

Aristarch (310–230 BC): Determination of the relative distance between the Moon and the Sun: Sun is 20× farther away than the Moon

reality: 400×

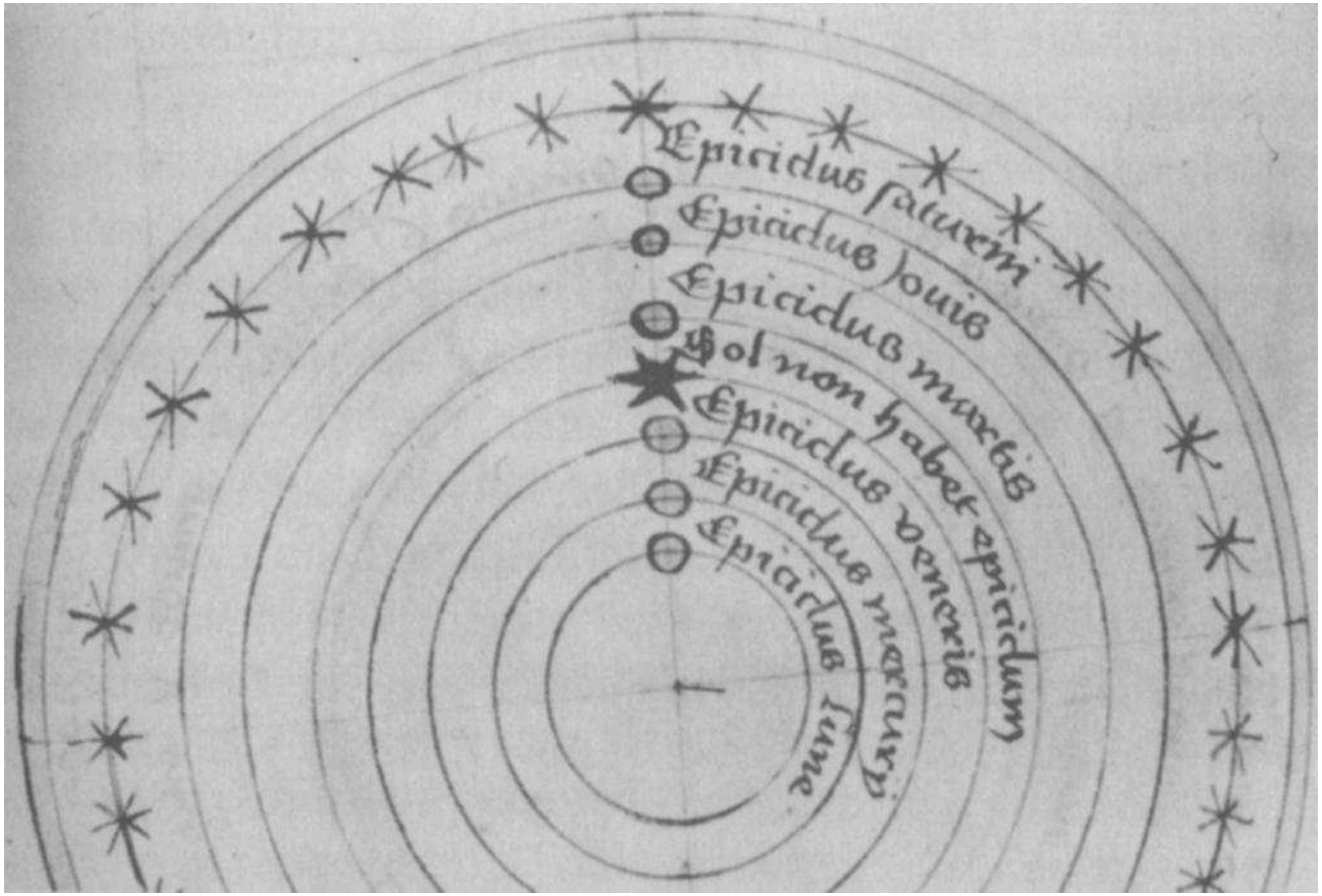
Greek/Roman, III



Eratosthenes von Cyrene (276–196 BC): Measurement of the radius of the Earth: Distance between Cyrene (Assuan) and Alexandria, diameter of Earth is 250000 stadia

The length of a stadium is unknown $\Longrightarrow$ we do not know how precise he was.

Greek/Roman, IV



Aristotle (384–322 BC, *de caelo*): Refinement of Eudoxus model: add spheres to ensure smooth motion

$\Longrightarrow$ Universe filled with crystalline spheres (*nature abhors vacuum*).

Ether in celestial spheres, not on Earth (everything falls, except for planets and stars); Stars are very distant since they do not show parallaxes.

$\Longrightarrow$ Central philosophy until $\sim$1450AD!

Hipparcus

Hipparchus (?? – ∼127 BC): Refi nement of geocentric Aristotelian model into tool to make predictions.

- Catalogue of 850 stars
- magnitudes
- lunar parallax
- Table of “chords” (=early trigonometry)
- Discovery of precession

  Difference between the durations of the siderial and the tropical year [$365.25 - 1/300$ d vs. $365.25 + 1/400$ d], through comparison with babylonian measurements

- different duration of seasons
- conversion of geocentric model of Aristotele into a tool to make predictions.

Ptolemy

(Aveni, 1993, p. 58)

Ptolemy (∼140AD): *Syntaxis* (aka Almagest): Refinement of Aristotelian theory into model useable for computations

Foundation of astronomy until Copernicus

⟹ Ptolemaic System.

After Hipparcus and Ptolemy: end of the golden age of early astronomy. Greek works are continued by arabs and further refi ned.
Aristotele’s philosophy remains foundation of science of medieval ages and is not questioned (in Europe).

Copernicus, I

Nicolaus Copernicus (1473–1543): Earth centred Ptolemaic system is too complicated, a Sun-centred system is more elegant.

Copernicus, II

(Gingerich, 1993, p. 165)

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De revolutionibus orbium coelestium: "In no other way do we perceive the clear harmonious linkage between the motions of the planets and the sizes of their orbs."

Copernicus, III

(Gingerich, 1993, p. 165)

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De revolutionibus orbium coelestium: “In no other way do we perceive the clear harmonious linkage between the motions of the planets and the sizes of their orbs.”

Copernican principle: The Earth is not at the center of the universe.

net, in quo terram cum orbe lunari tanquam epicyclo contineri diximus. Quinto loco Venus nono mense reducitur. Sextum denique locum Mercurius tenet, octuaginta dierum spacio circu currens. In medio uero omnium residet Sol. Quis enim in hoc

pulcherrimo templo lampadem hanc in alio uel meliori loco poneret, quàm unde totum simul possit illuminare? Siquidem non inepte quidam lucernam mundi, alij mentem, alij rectorem uocant. Trimegistus uisibilem Deum, Sophoclis Electra intuentē omnia. Ita profecto tanquam in solio regali Sol residens circum agentem gubernat Astrorum familiam. Tellus quoque minime fraudatur lunari ministerio, sed ut Aristoteles de animalibus ait, maximam Luna cum terra cognationē habet. Cōcipit interea à Sole terra, & impregnatur annuo partu. Inuenimus igitur sub hac

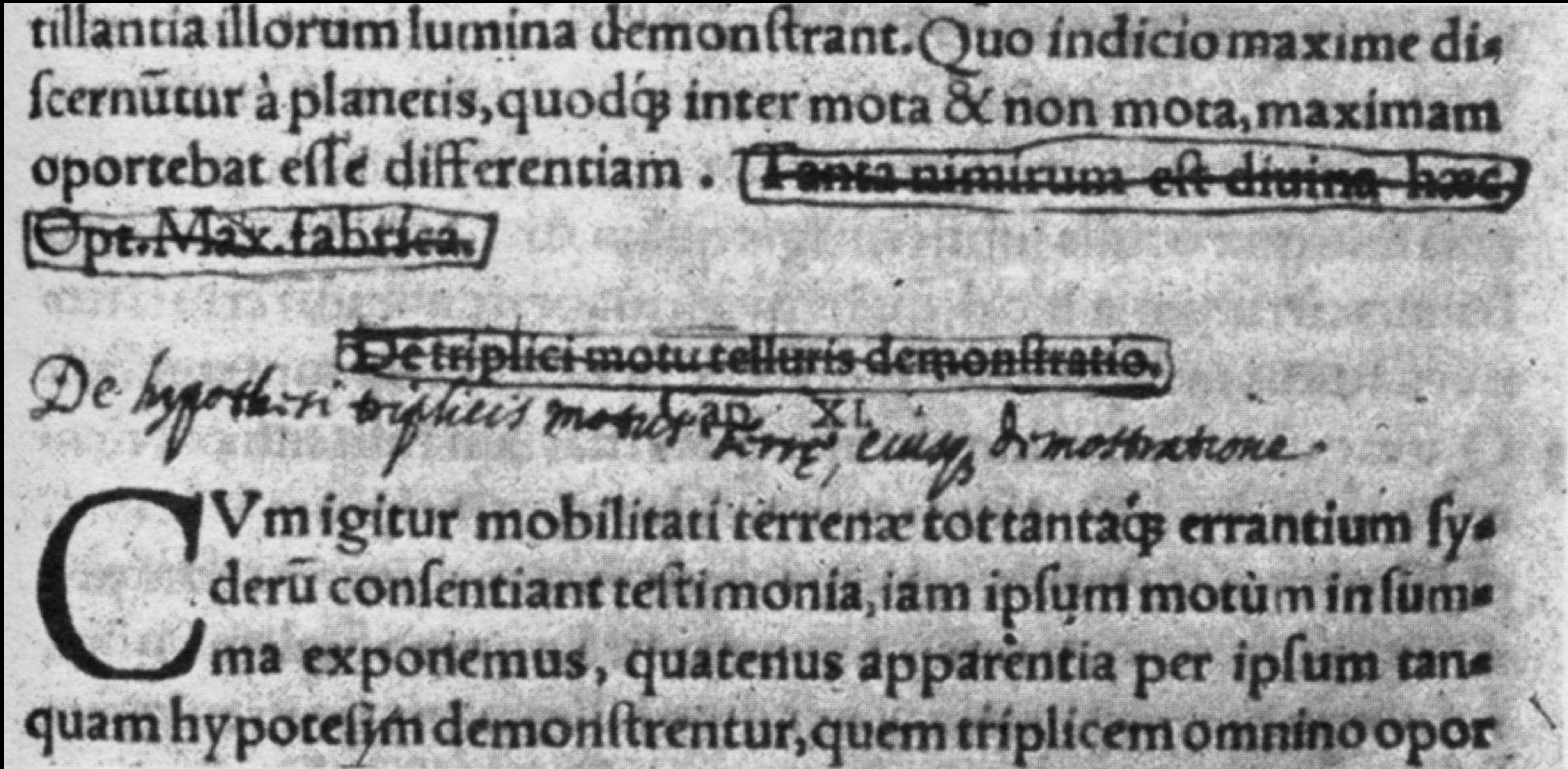
hac ordinatione admirandam mundi symmetriam, ac certū harmoniæ nexum motus & magnitudinis orbium: qualis alio modo reperiri non potest. Hic enim licet animaduertere, non segniter contemplanti, cur maior in Ioue progressus & regressus appareat, quàm in Saturno, & minor quàm in Marte: ac rursus maior in Venere quàm in Mercurio. Quod'que frequentior appareat in Saturno talis reciprocatio, quàm in Ioue: rarior adhuc in Marte, & in Venere, quàm in Mercurio. Præterea quòd Saturnus, Iupiter, & Mars acronycti propinquiores sint terræ, quàm circa eorū occultationem & apparitionem. Maxime uero Mars pernox factus magnitudine Iouem æquare uidetur, colore duntaxat rutilo discretus: illic autem uix inter secundæ magnitudinis stellas inuenitur, sedula obseruatione sectantibus cognitus. Quæ omnia ex eadem causa procedūt, quæ in telluris est motu. Quòd autem nihil eorum appareat in fixis, immensam illorum arguit celsitudinem, quæ faciat etiam annui motus orbem siue eius imaginem ab oculis euanescere. Quoniam omne uisibile longitudinem distantiæ habet aliquam, ultra quam non amplius spectatur, ut demonstratur in Opticis. Quòd enim à supremo errantium Saturno ad fixarum sphæram adhuc plurimum intersit, scintillantia illorum lumina demonstrant. Quo indicio maxime discernūtur à planetis, quodque inter mota & non mota, maximam oportebat esse differentiam. ~~Tanta nimirum est diuina hæc Opt. Max. fabrica.~~

~~De triplici motu telluris demonstratio.~~

De hypothesi triplicis motus terræ, eiusque demonstratione.

CVm igitur mobilitati terrenæ tot tantaque errantium syderū consentiant testimonia, iam ipsum motum in summa exponemus, quatenus apparentia per ipsum tanquam hypotesim demonstrentur, quem triplicem omnino oportet admittere. Primum quem diximus νυχθημερινὸν à Græcis uocari, diei noctisque circuitum proprium, circa axem telluris, ab occasu in ortum uergentem, prout in diuersum mundus ferri putatur, æquinoctialem circulum describendo, quem nonnulli æquidialem dicunt, imitantes significationem Græcorum, apud

 quos


tillantia illorum lumina demonſtrant. Quo indicio maxime di-
ſcernũtur à planetis, quodq; inter mota & non mota, maximam
oportebat eſſe differentiam. ~~Tanta nimirum eſt diuina hæc
Opt. Max. fabrica.~~

~~De triplici motu telluris demonſtratio,~~

De hypothesi triplicis motus terrae, eiusque demonstratione.

Cap. XI.

CVm igitur mobilitati terrenæ tot tantaq; errantium ſy-
derũ conſentiant teſtimonia, iam ipſum motum in ſum-
ma exponemus, quatenus apparentia per ipſum tan-
quam hypoteſim demonſtrentur, quem triplicem omnino opor

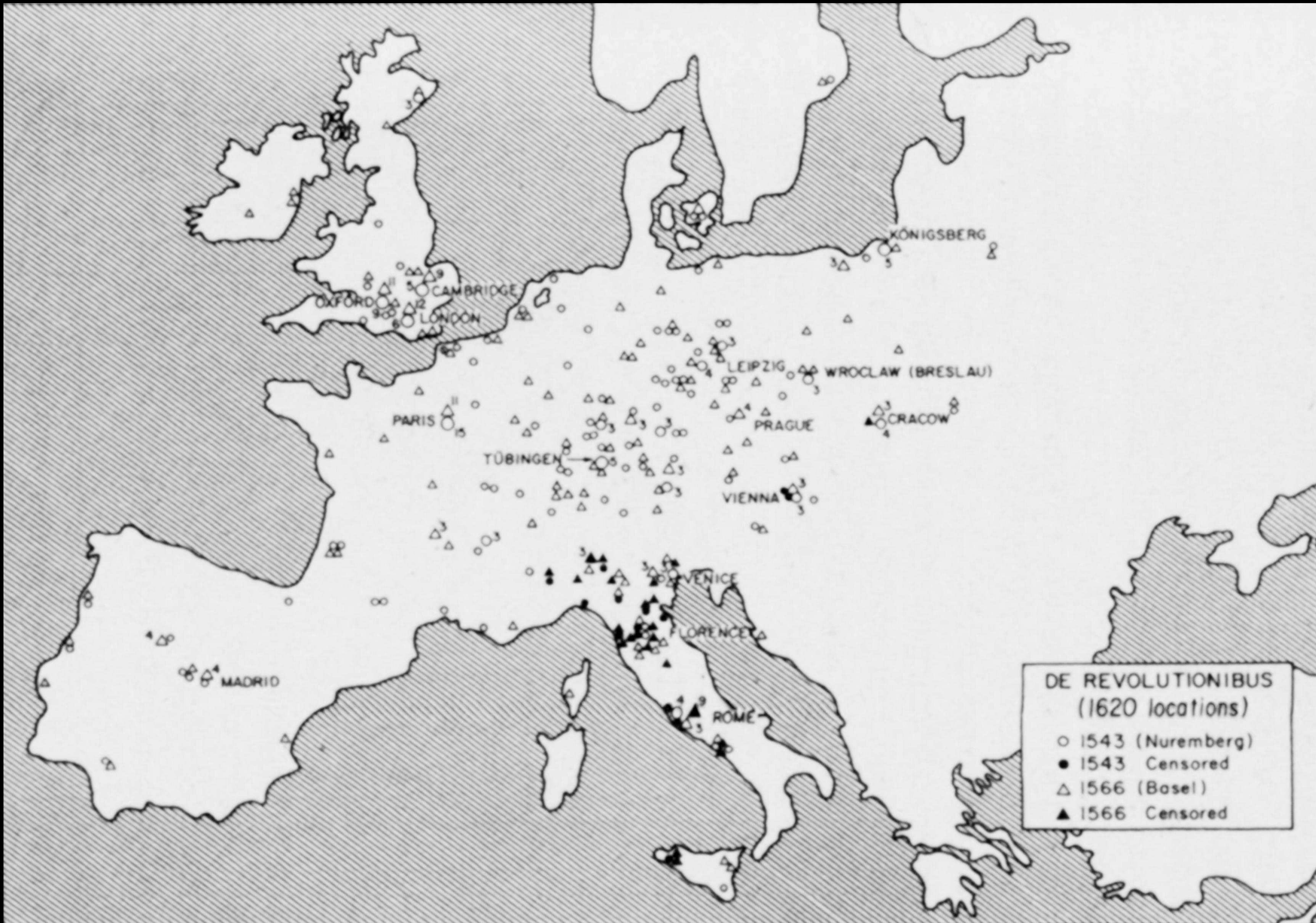
(Gingerich, 2005)

The “censored” copy of Galileo’s “de revolutionibus”

Deleted: “Indeed, large is the work of . . . God”

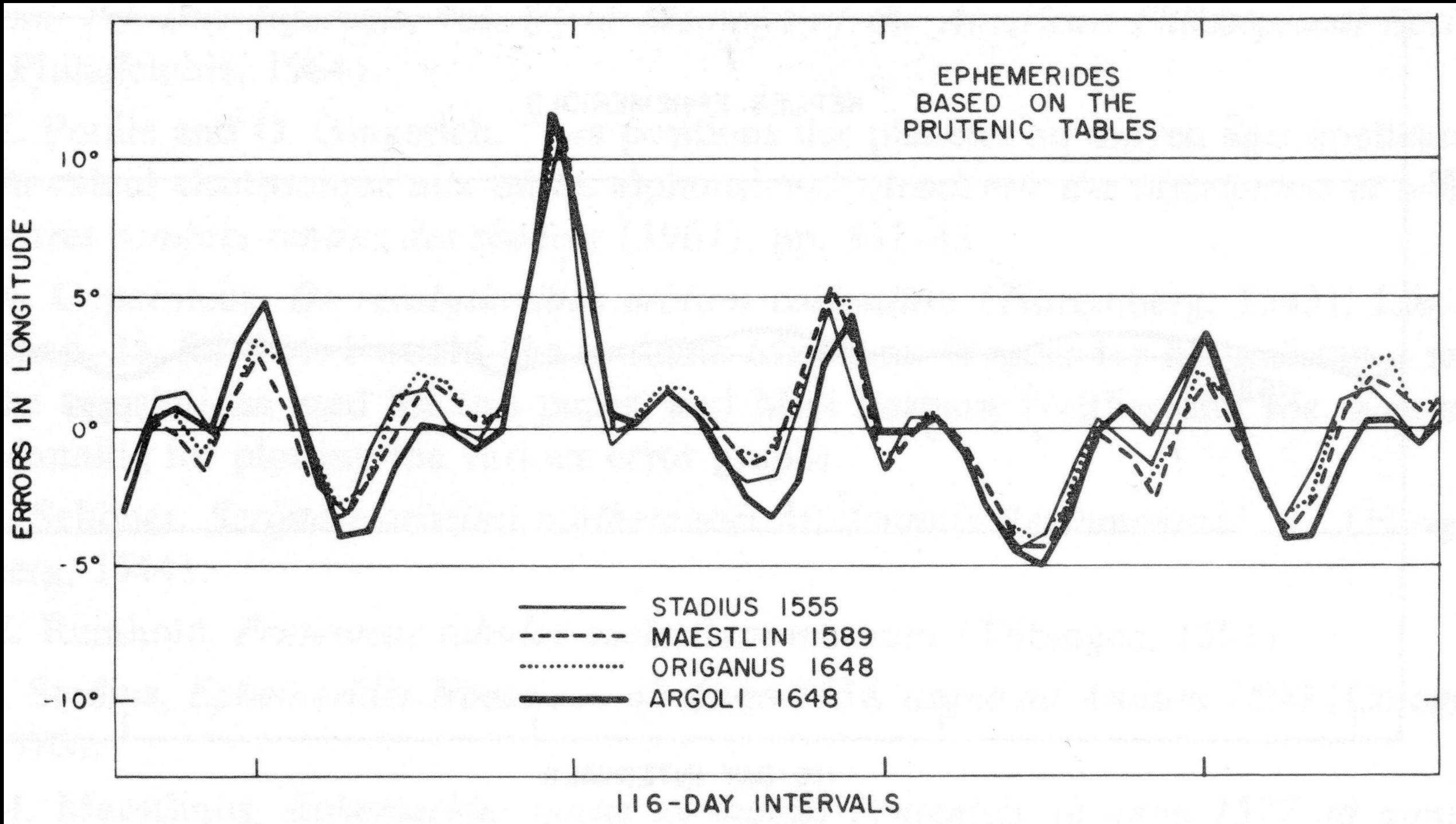
Changed: “On the explanation of the triple motion of the Earth”

$\Longrightarrow$ “On the hypothesis of the triple motion of the Earth”



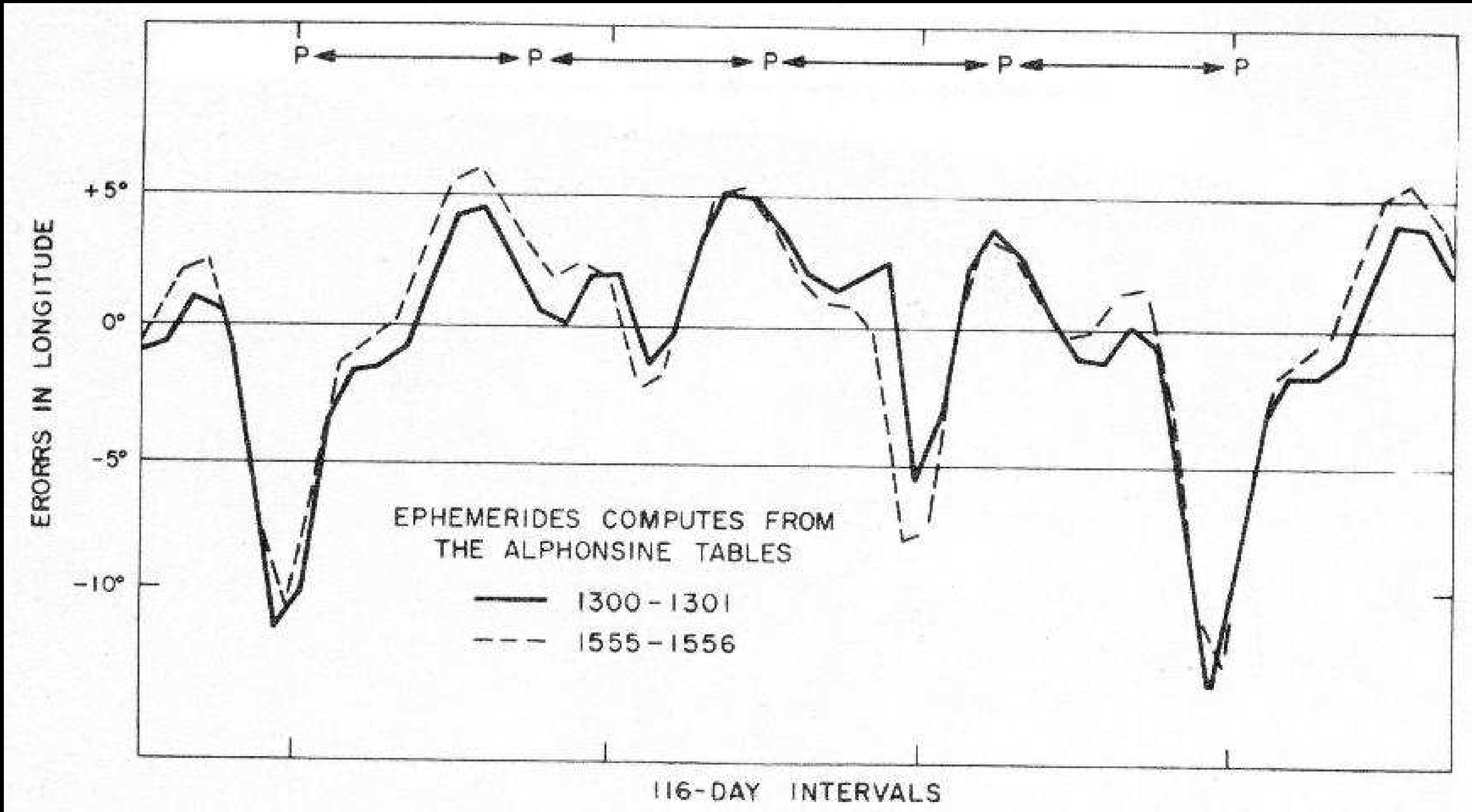
(Gingerich, 2005)

Distribution of the censored copies of “De revolutionibus”



(Gingerich, 1993)

The error in the Copernican position of Mercury. . .



. . . is not smaller than the error in the ptolemaic Alfonsinian Tables

Brahe

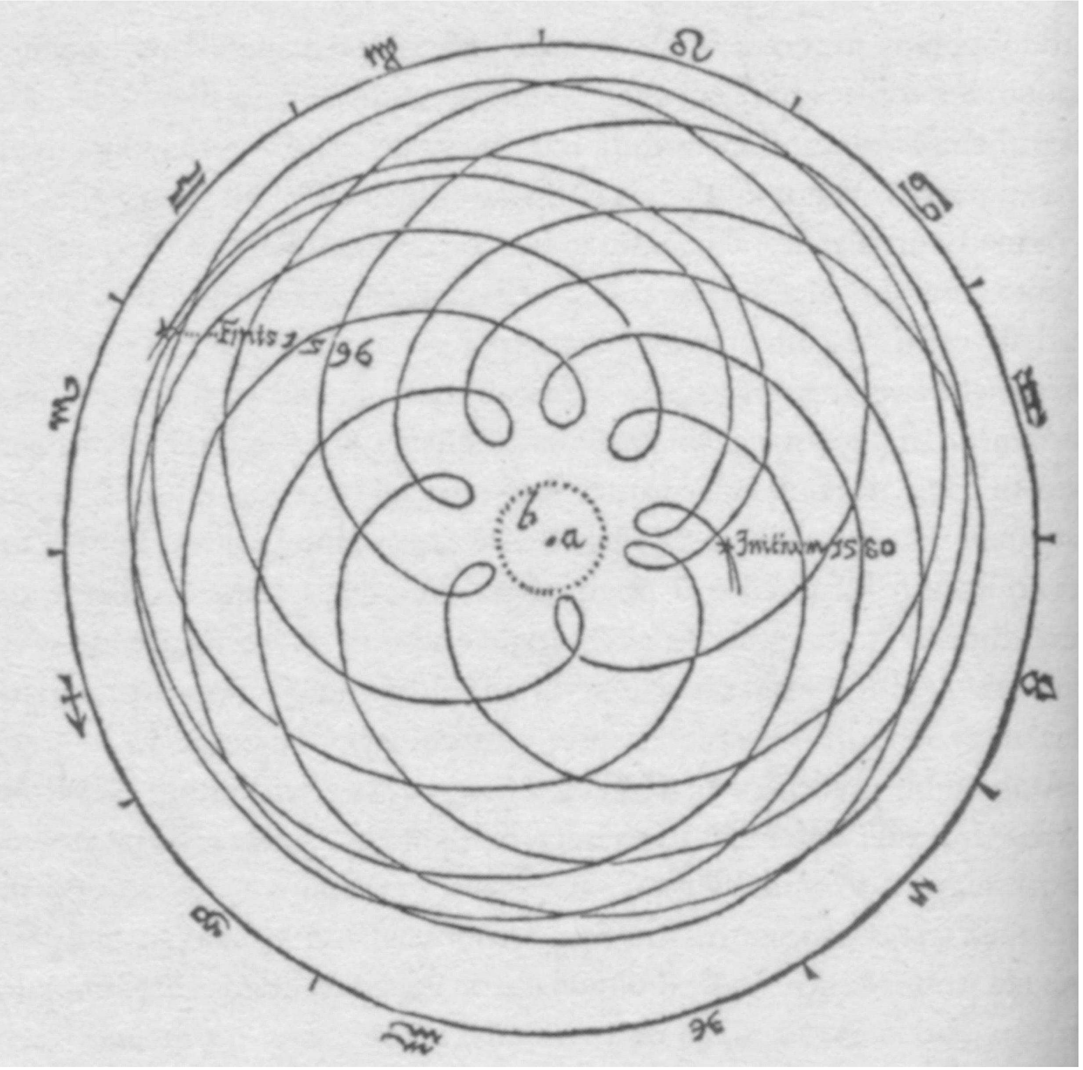
Tycho Brahe (1546–1601): Visual planetary positions of highest precision reveal flaws in Ptolemaic positions.

Kepler, I

Johannes Kepler (1571–1630):

- 27.12.1571, Weil der Stadt
- Studies in Tübingen with Maestlin
- 1594–1600: Graz
- 1596: Mysterium Cosmographicum
- 1600–1612: Prag, with Brahe, court astrologer, theory of planets, discovery of the supernova of 1604,. . .
- 1609: Astronomia Nova

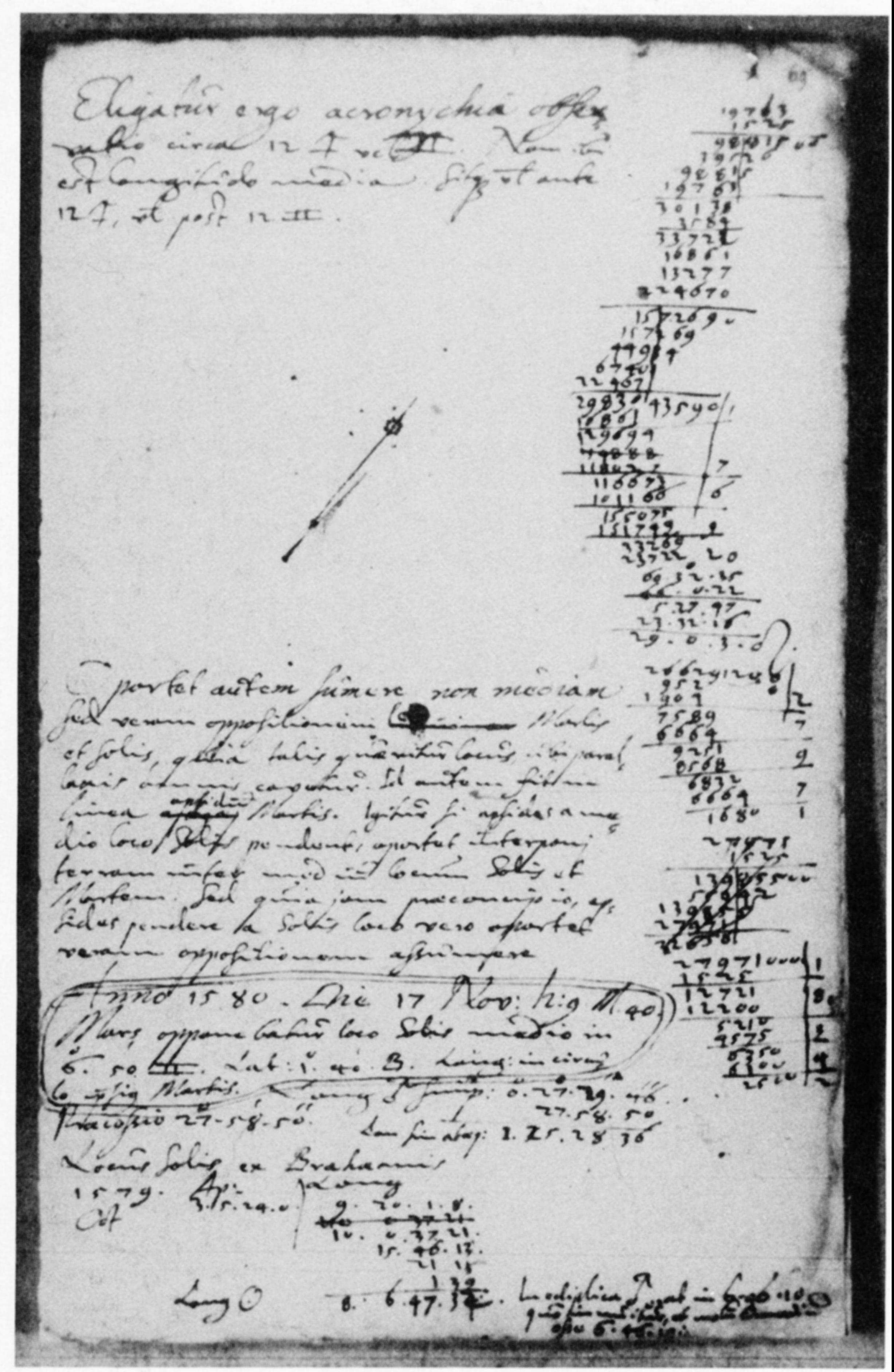
Kepler, II



Kepler's theory of planetary motion: Astronomia nova (Prag, 1609)

Critique of epicycles: “panis quadragesimalis” (Osterbrezel) $\Longrightarrow$ inelegant!

Astronomia Nova, chapter 1: Motion of Mars in the theory of epicycles



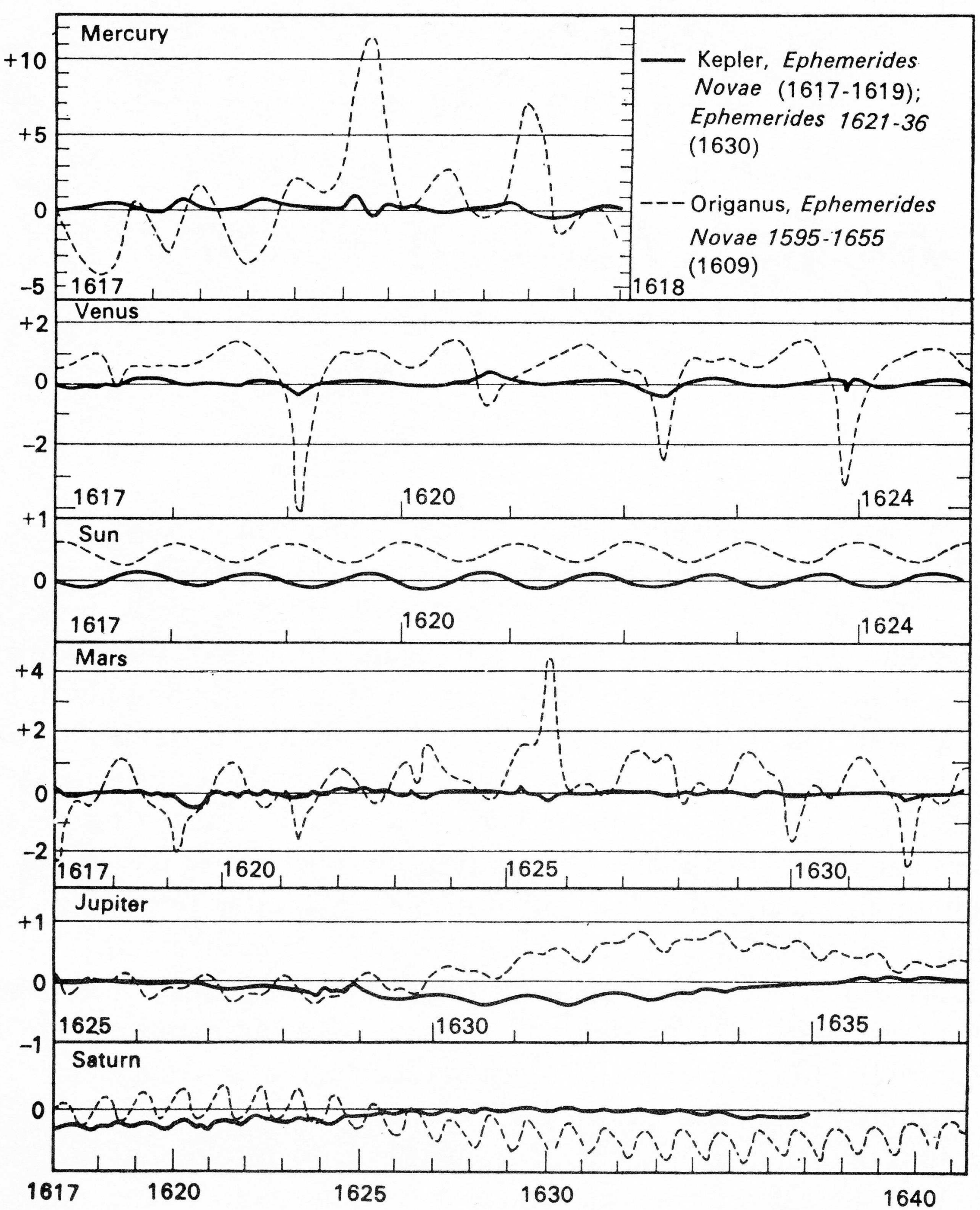
Kepler’s laboratory book
Drawing of Mars in opposition highlighted: one of the few positions of Mars done by Brahe which Kepler was allowed to use

(Gingerich, 1993)

Kepler, IV

Tabulae Rudolphinae, 1627
Best planetary positions
(error only $\sim 5'$!)

(Gingerich, 2005)



Comparison of positions, Kepler vs. copernican theory

$\Longrightarrow$ extreme improvement!

(Gingerich, 1993)

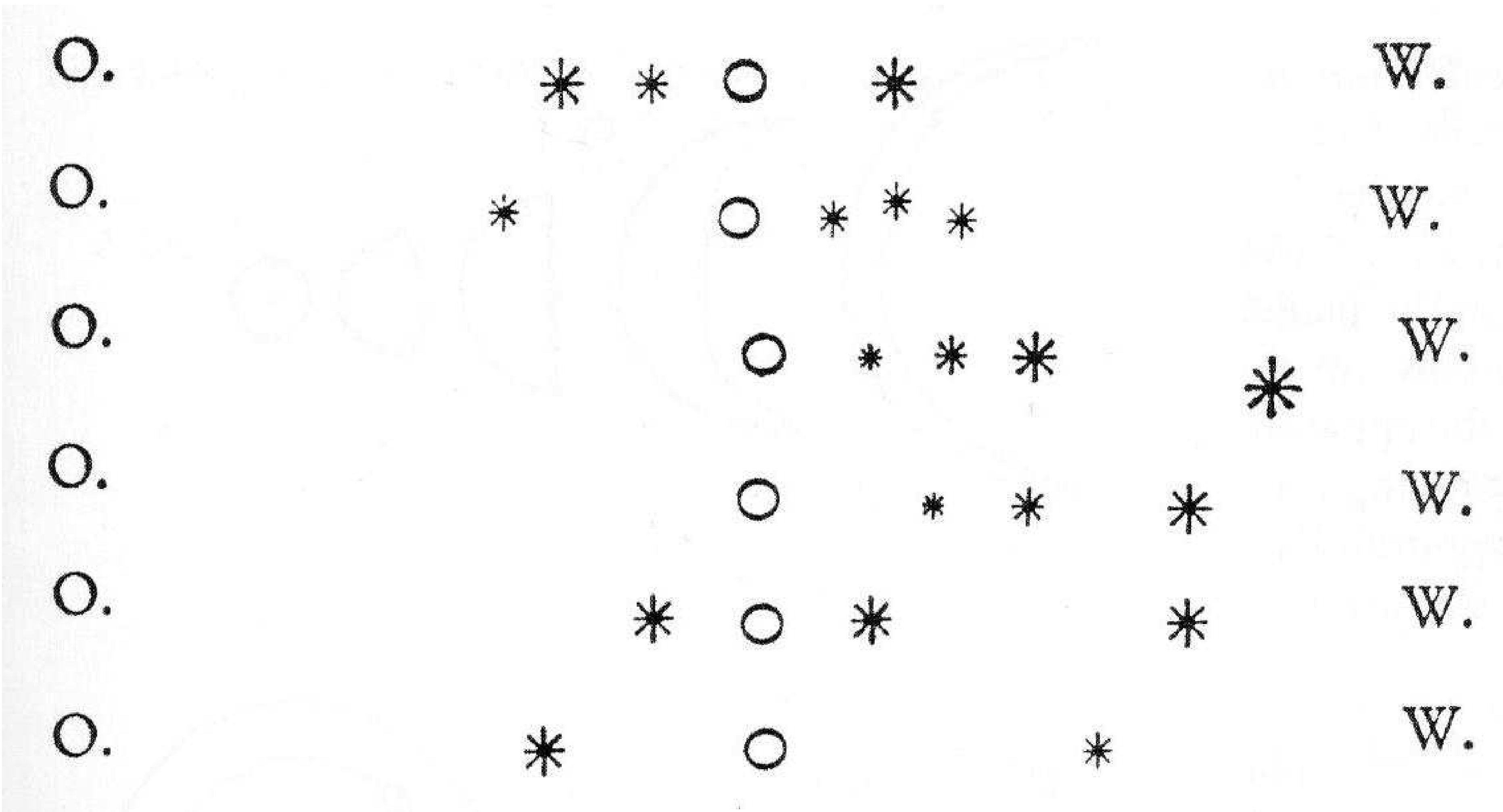
Galilei

Galileo Galilei (1564–1642): Telescope

$\Longrightarrow$ Observations!

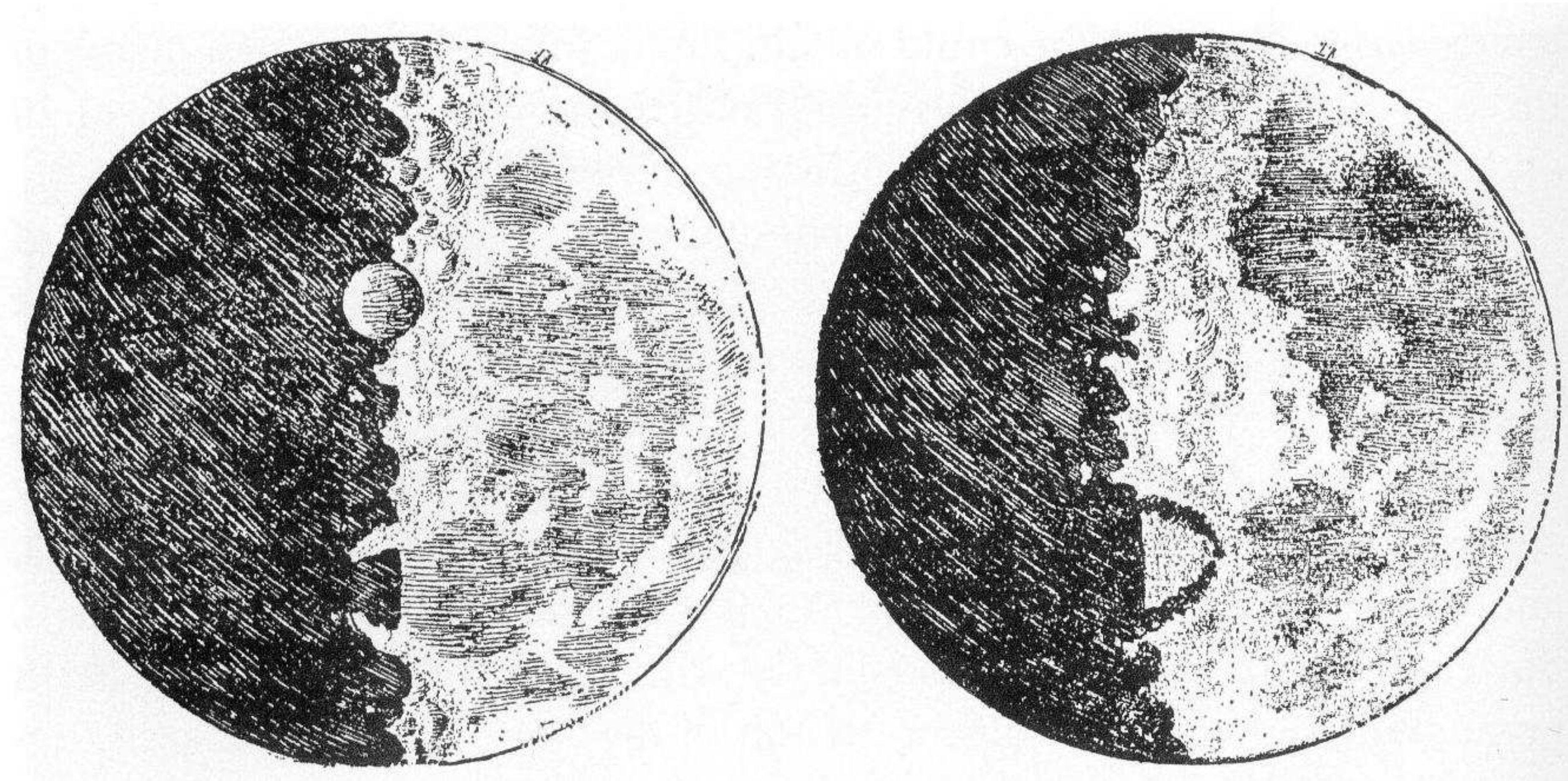
$\Longrightarrow$ Siderius Nuncius (1610)

Galilei



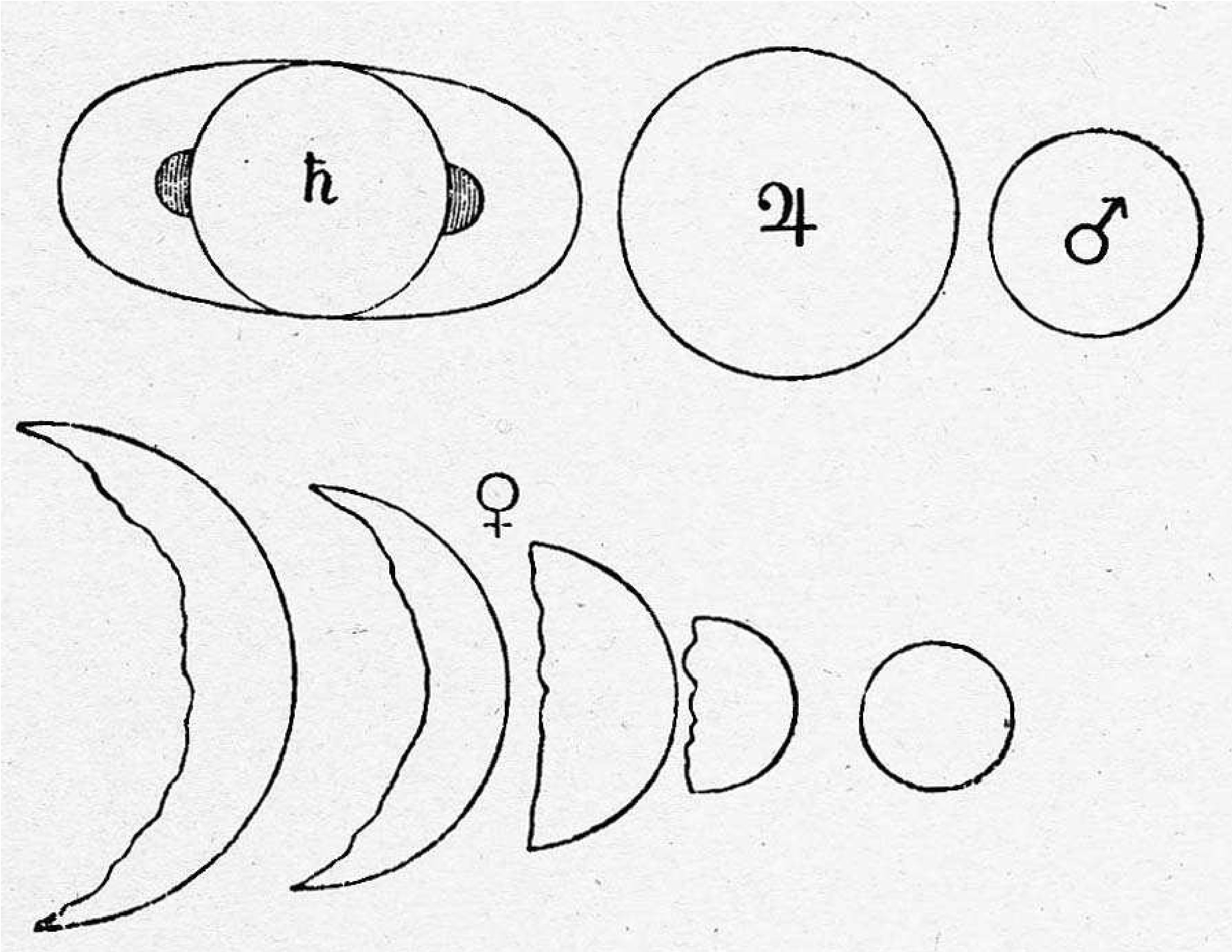
The moons of Jupiter move around Jupiter
($\Longrightarrow$ similar to the heliocentric model!). . .

Galilei



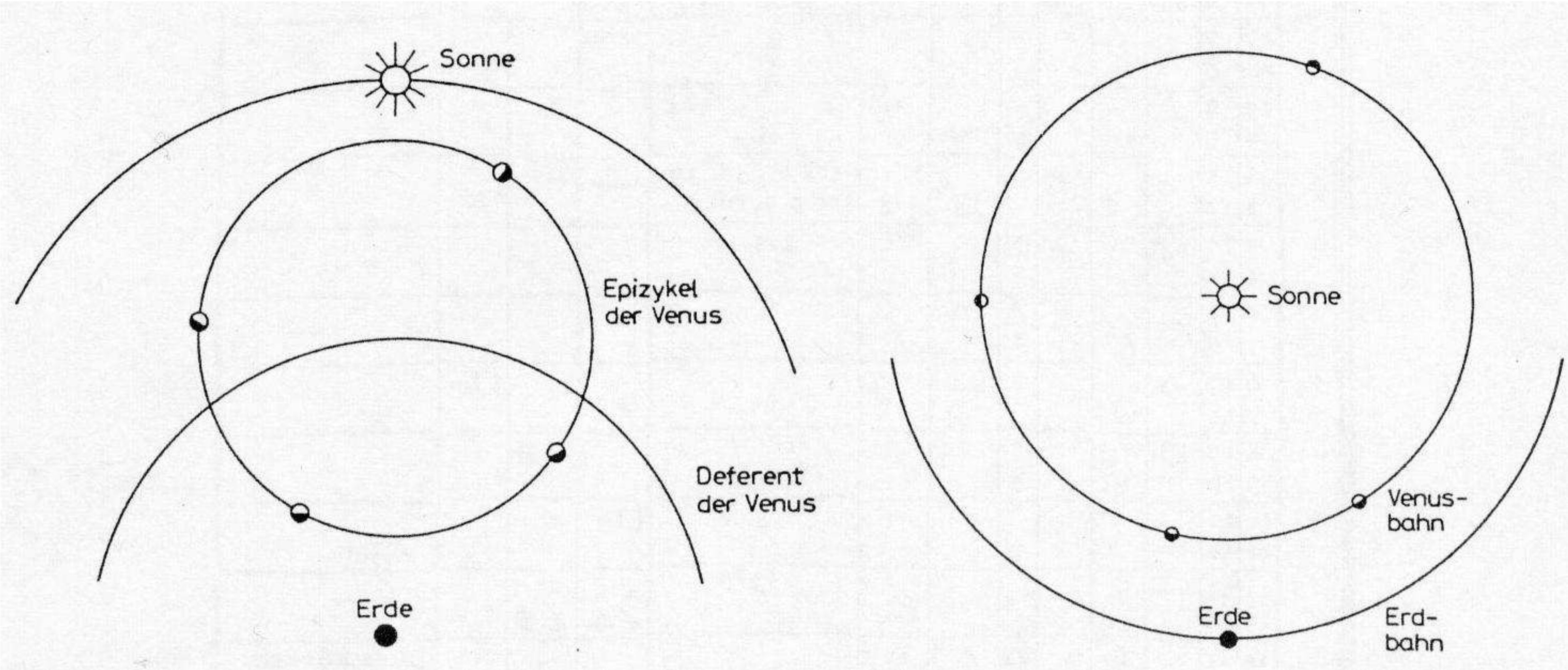
Moon has surface features, shadows, and “wiggles” (libration!).

Galilei



Discovery of the phases of Venus (Il Saggiatore, 1623)

Galilei



The observed sequence of the phases of Venus *cannot* be explained by the geocentric theory, only by a heliocentric theory.

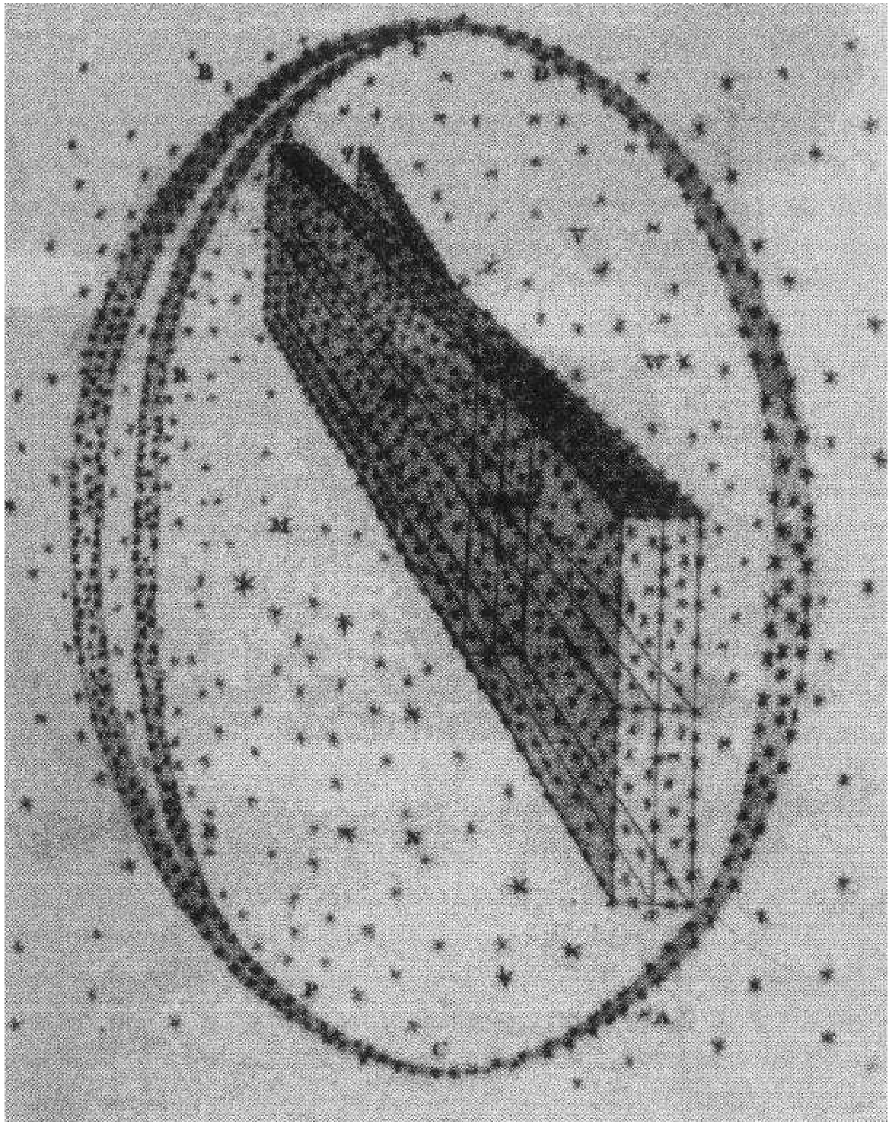
Newton

(Newton, 1730)

Isaac Newton (1642–1727): Newton's laws, physical cause for shape of orbits is gravitation (*De Philosophiae Naturalis Principia Mathematica*, 1687).

$\Longrightarrow$ Begin of modern physics based astronomy.

18th and 19th century



Galileo: Milky Way consists of stars.

Newton: Stars are distant suns

William Herschel (1738–1822): Milky Way is a flattened disk of stars, Sun is at center (see figure).

Immanuel Kant (1724–1804): “Nebulae are galaxies” (disputed until the 1910s).

Friedrich Bessel (1784–1846): Distance to 61 Cyg (1838), positions of 50000 stars

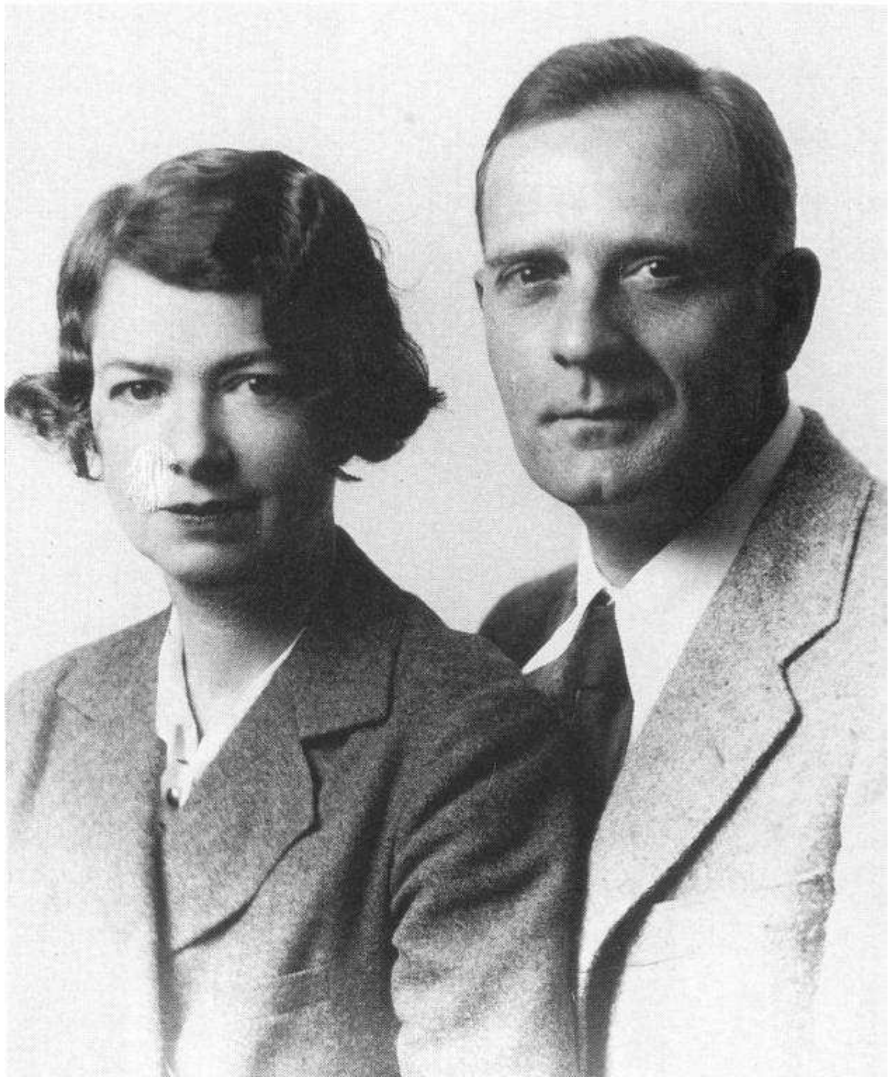
John Herschel (1792–1871): General Catalogue of Galaxies (1864, 5079 Objects)

John Dreyer (1852–1926): NGC+IC (15000 Objects)

Albert Einstein

Albert Einstein (1879–1955): Theory of gravitation, applicability of theory to evolution of the universe as a whole.

Edwin Hubble



Christianson, 1995, p. 165

Edwin Hubble (1889–1953):

- Realization of galaxies as being outside of the Milky Way
- Discovery that universe is expanding

Founder of modern extragalactic astronomy

Aveni, A. F., 1993, Ancient Astronomers, (Washington, D.C.: Smithsonian Books)

Gingerich, O., 1993, The Eye of Heaven – Ptolemy, Copernicus, Kepler, (New York: American Institute of Physics)

Gingerich, O., 2005, The book nobody read, (London: arrow books)

Newton, I., 1730, Opticks, Vol. 4th, (London: William Innys), reprint: Dover Publications, 1952

Basic Facts

Basic Facts

Cosmology deals with answering the questions about the universe as a whole.

The main question is:

How did the universe evolve into what it is now?

For this, *four major facts* need to be taken into account:

The universe is:
- expanding,
- isotropic,
- and homogeneous.

The isotropy and homogeneity of the universe is called the *cosmological principle*.

Perhaps (for us) the most important fact is:

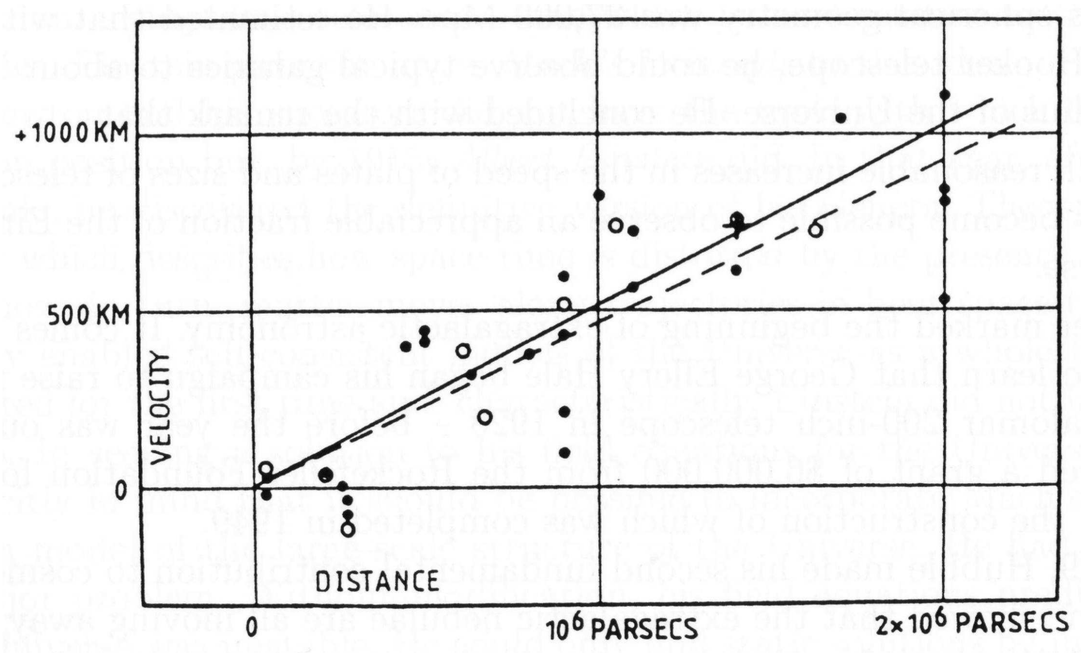
- The universe is habitable to humans.

i.e., the *anthropic principle*.

The one question cosmology does not attempt to answer is: How came the universe into being?

$\Longrightarrow$ Realm of theology!

Expansion, I



(Hubble, 1929, Fig. 1)

Hubble (1929): Velocity v (defined as $v/c := z = \Delta\lambda/\lambda$) for galaxy at distance r is

$$v(r) = H_0 r + v_X \cos\alpha\cos\delta + v_Y \sin\alpha\cos\delta + v_Z \sin\delta \quad (3.1)$$

(v_X, v_Y, v_Z) velocity due to motion of solar system ($\sim 350\,\mathrm{km\,s^{-1}}$ towards $l = 264^\circ$, $b = 48^\circ$, Bennet et al., 1996)

H_0: "Hubble parameter"; *intrinsic* component of velocity due to *expansion* of the universe.

Old usage: "Hubble constant", but $H_0 \neq$ const. (cf. Eq. (4.36)).

Dome of the 5 m Hale Telescope on Mt. Palomar (©I. Kreykenbohm)

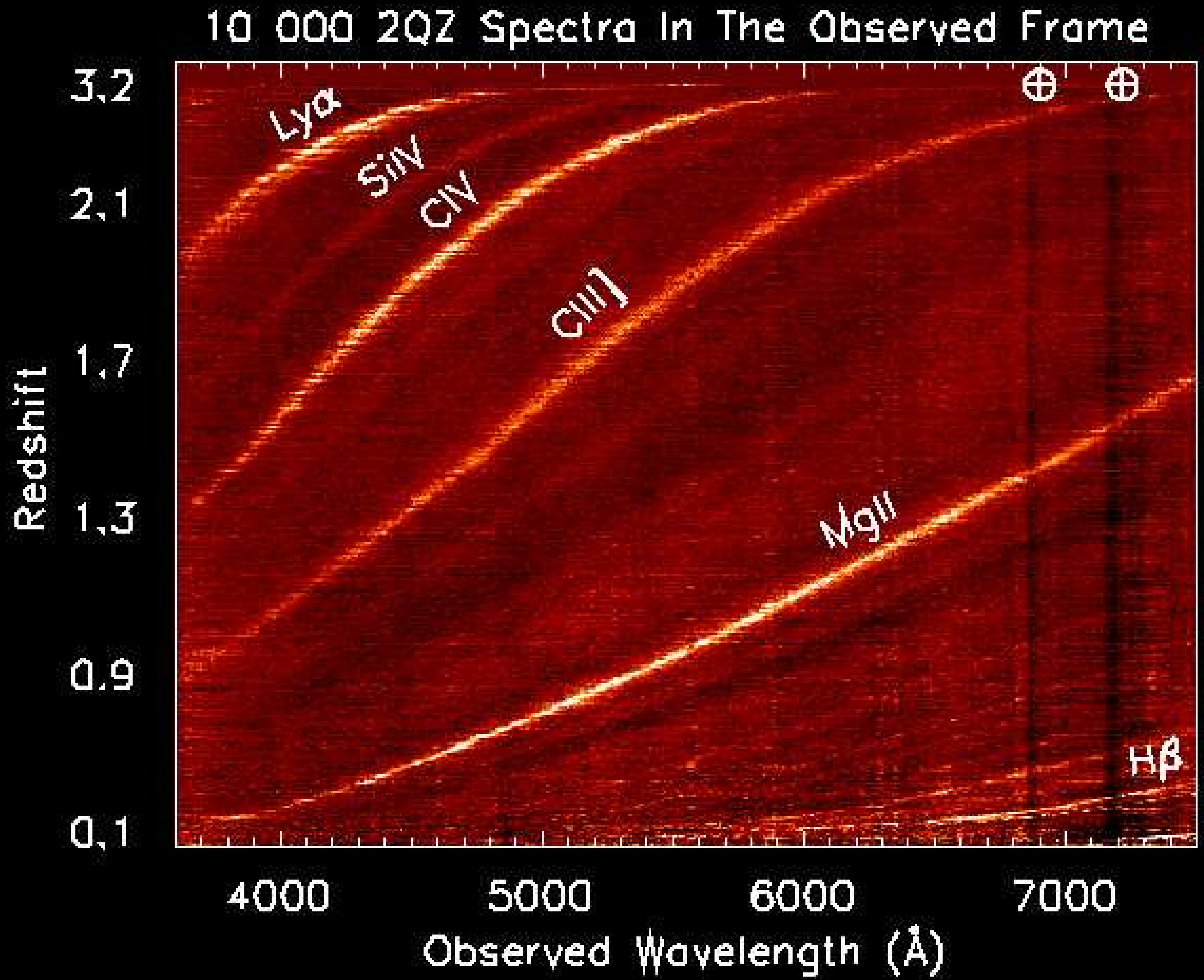
Dome of the 5 m Hale Telescope on Mt. Palomar (© J. Wilms)

The 5 m Hale Telescope (© I. Kreykenbohm)

The 5 m Hale Telescope (© I. Kreykenbohm)

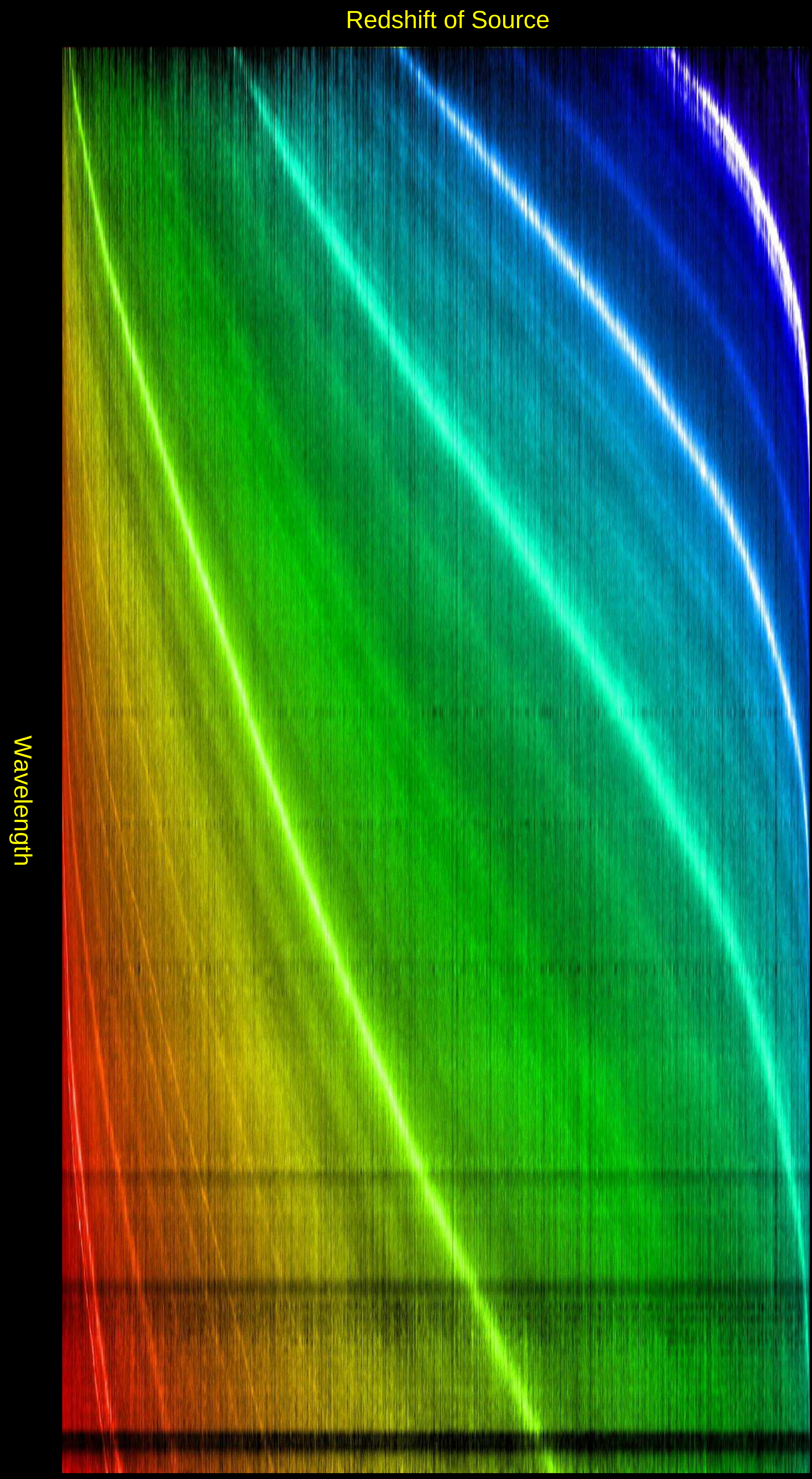
Mount of the 5 m Hale Telescope (© I. Kreykenbohm)

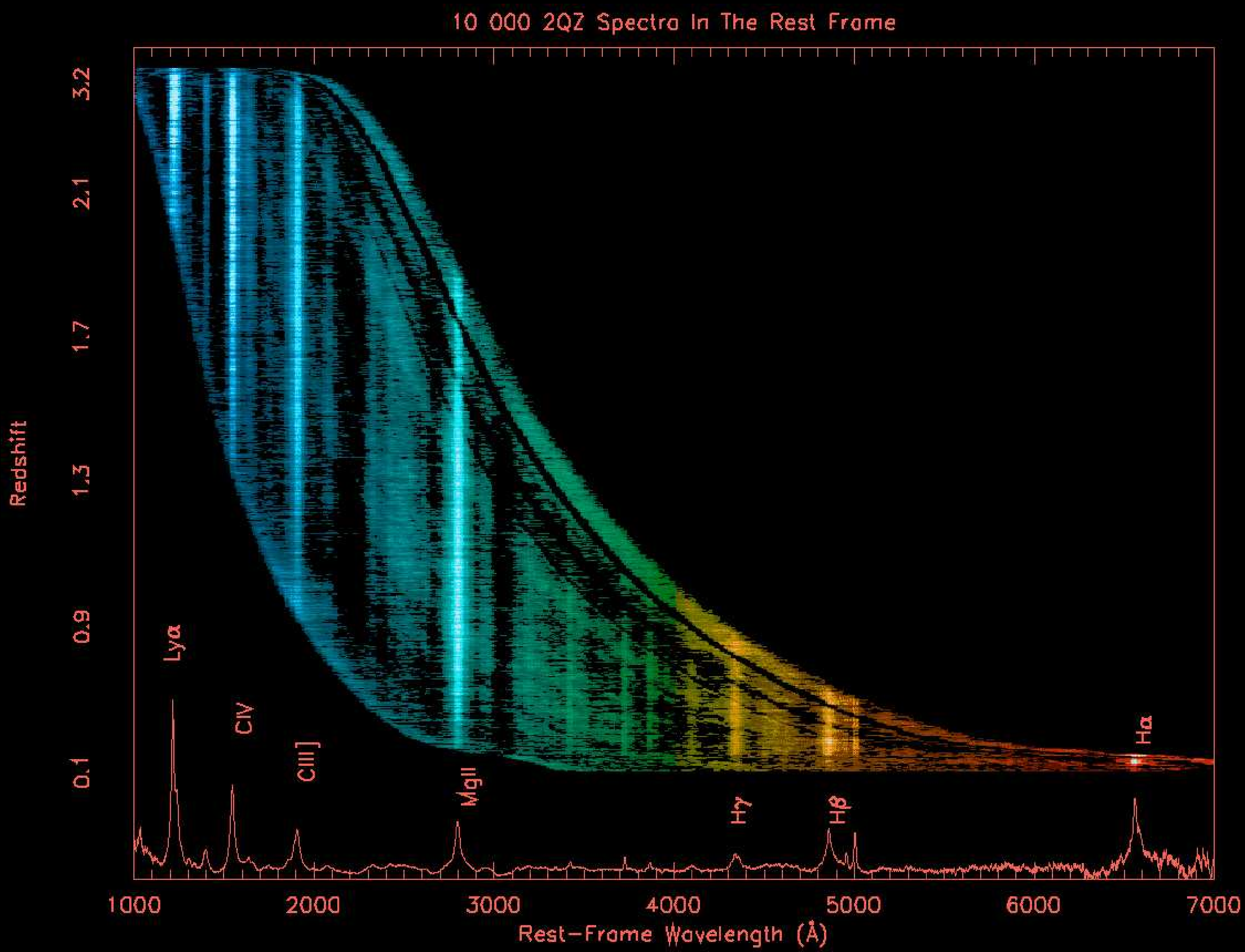
No comment (© I. Kreykenbohm)



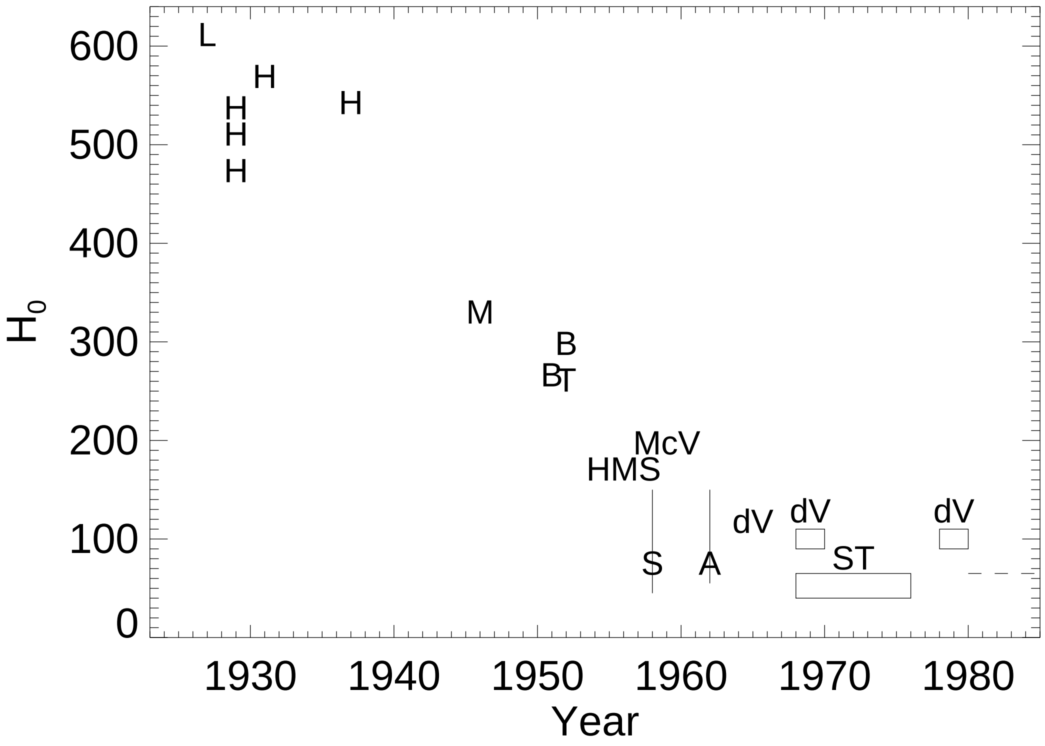
courtesy 2dF QSO Redshift survey

As a consequence of the cosmological redshift, for different z different parts of the spectrum of a distant source are visible.


Redshift of Source
Wavelength


10 000 2QZ Spectra In The Rest Frame
Redshift
3.2
2.1
1.7
1.3
0.9
0.1
Lyα
CIV
CIII]
MgII
Hγ
Hβ
Hα
1000
2000
3000
4000
5000
6000
7000
Rest-Frame Wavelength (Å)

Expansion, XI



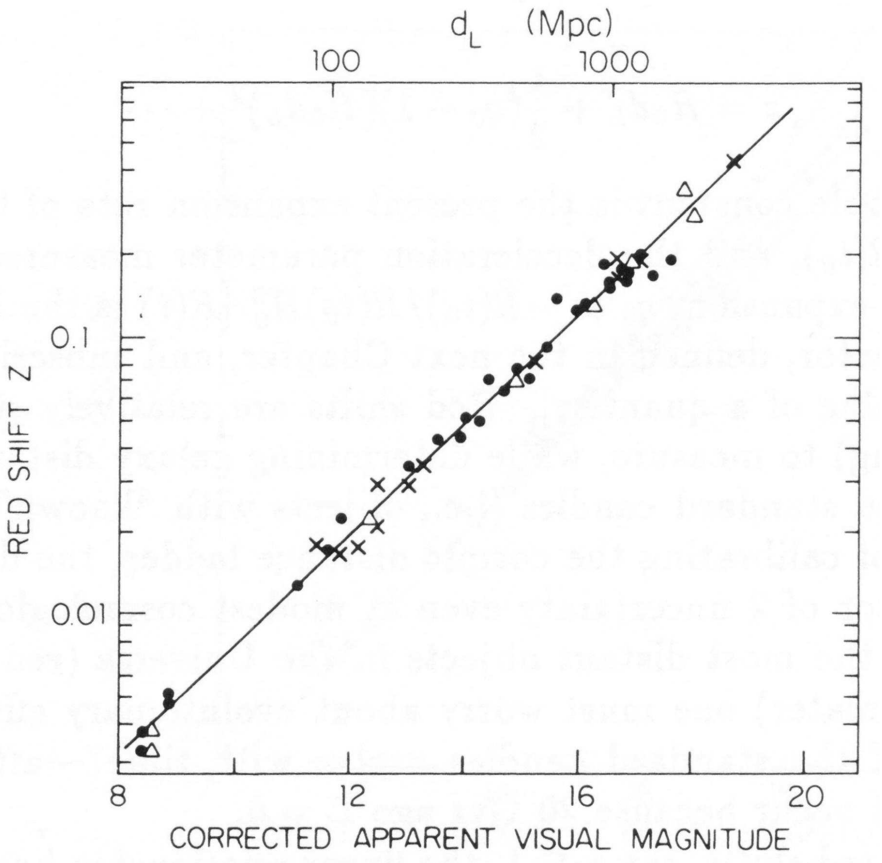
(after Trimble, 1997)

Currently accepted value:
$H_0 \sim 75\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$.
The systematic uncertainty of H_0 is $\sim 10\,\text{km}^{-1}\,\text{s}^{-1}\,\text{Mpc}^{-1}$.
Parameterize uncertainty in formulae by defi ning

$$\begin{aligned} H_0 &= 100\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1} \cdot h \\ H_0 &= 75\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1} \cdot h_{75} \end{aligned} \tag{3.2}$$

Note: H_0^{-1} has units of time: $H_0^{-1} = 9.78\,\text{Gyr}/h$: Hubble-Time;
for $h = 0.75$, the Hubble-Time is 13 Gyr.

Expansion, XII



For standard candles, i.e., objects where the absolute luminosity L is known, the Hubble law can be written using observed quantities only:
Euclidean space $\Longrightarrow$ observed flux

$$f = \frac{L}{4\pi d_{\mathrm{L}}^2} \quad \Longleftrightarrow \quad d_{\mathrm{L}} = \left(\frac{L}{4\pi f}\right)^{1/2} \tag{3.3}$$

where d_{L} is the luminosity distance.
Using the Hubble law eq. (3.1)

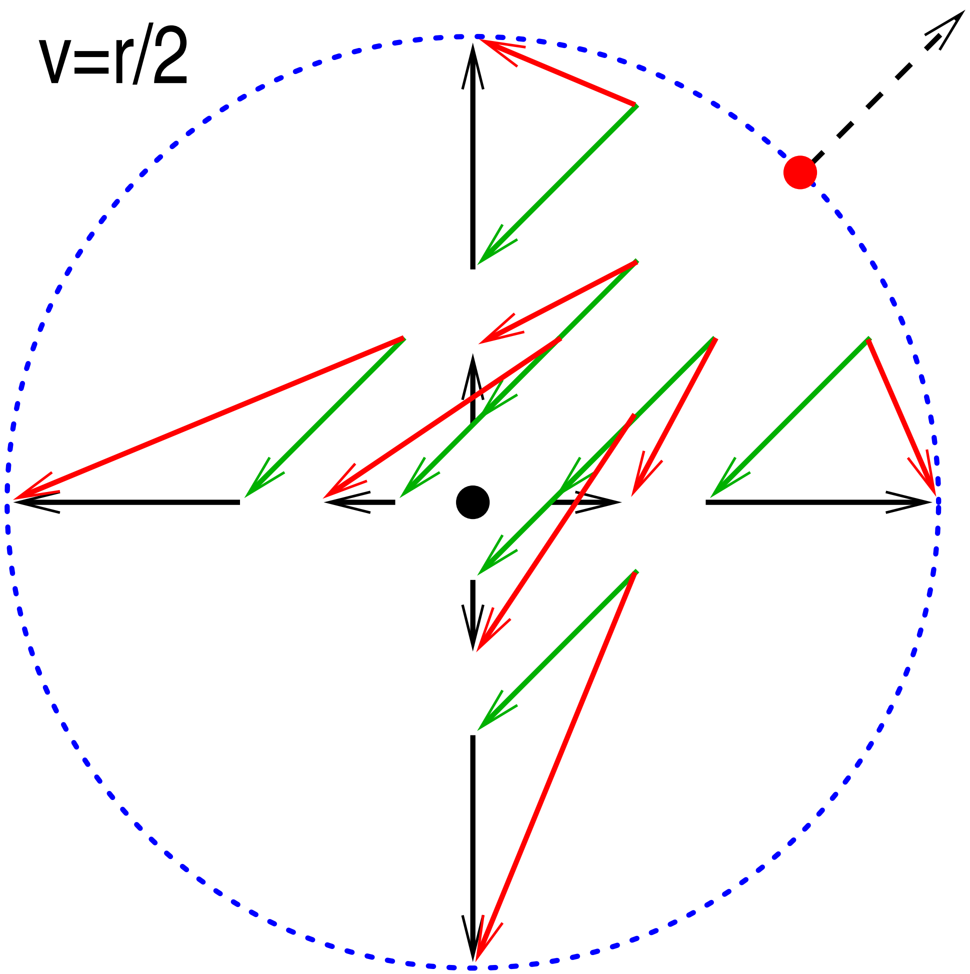
$$H_0 d_{\mathrm{L}} = cz \quad \Longrightarrow \quad z \propto H_0 \left(\frac{L}{4\pi f}\right)^{1/2} \tag{3.4}$$

Since *magnitudes* are defined via $m \propto -2.5 \log f$:

$$\log z \propto \log H_0 + \frac{1}{2}(\log L - \log f) \quad \Longrightarrow \quad \log z = a + b(m - M) \tag{3.5}$$

where $m - M$: distance modulus.

Expansion, XIII



Expansion law $\boldsymbol{v} = H_0\boldsymbol{r}$ is unchanged under rotation and translation: isomorphism.

Proof:

Rotation: Trivial.

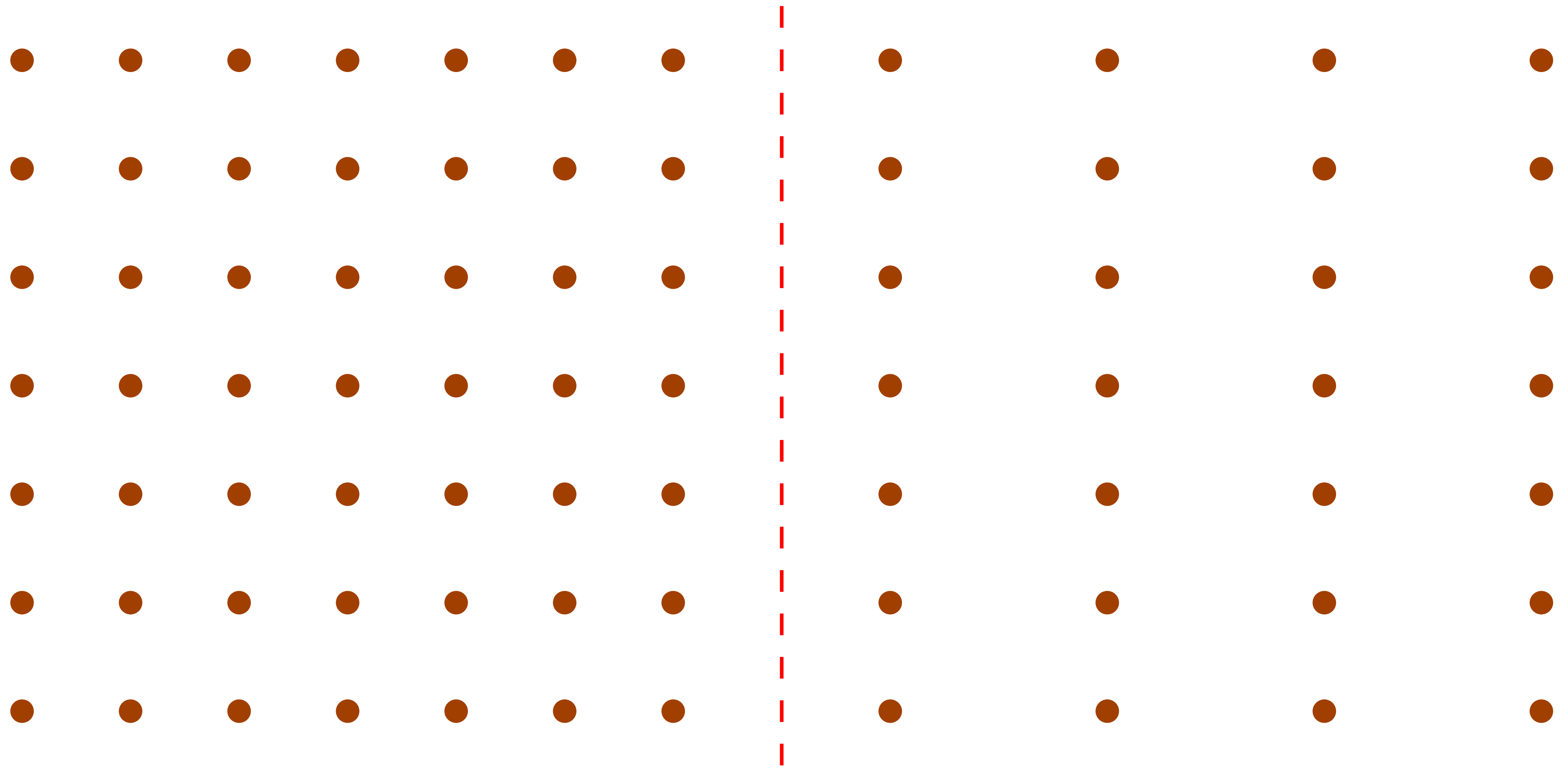
Translation: Observations from place with position $\boldsymbol{r}'$ and velocity $\boldsymbol{v}'$: Observed distance is $\boldsymbol{r}_\mathrm{o} = \boldsymbol{r} - \boldsymbol{r}'$, observed velocity is $\boldsymbol{v}_\mathrm{o} = \boldsymbol{v} - \boldsymbol{v}'$. Because of the Hubble law,

$$\boldsymbol{v}_\mathrm{o} = H_0\boldsymbol{r} - H_0\boldsymbol{r}' = H_0\left(\boldsymbol{r} - \boldsymbol{r}'\right) = H_0\boldsymbol{r}_\mathrm{o}$$

This isomorphism is a direct consequence of the homogeneity of the universe.

Despite everything receding from us, we are not at the center of the universe $\Longrightarrow$ Copernicus principle still holds.

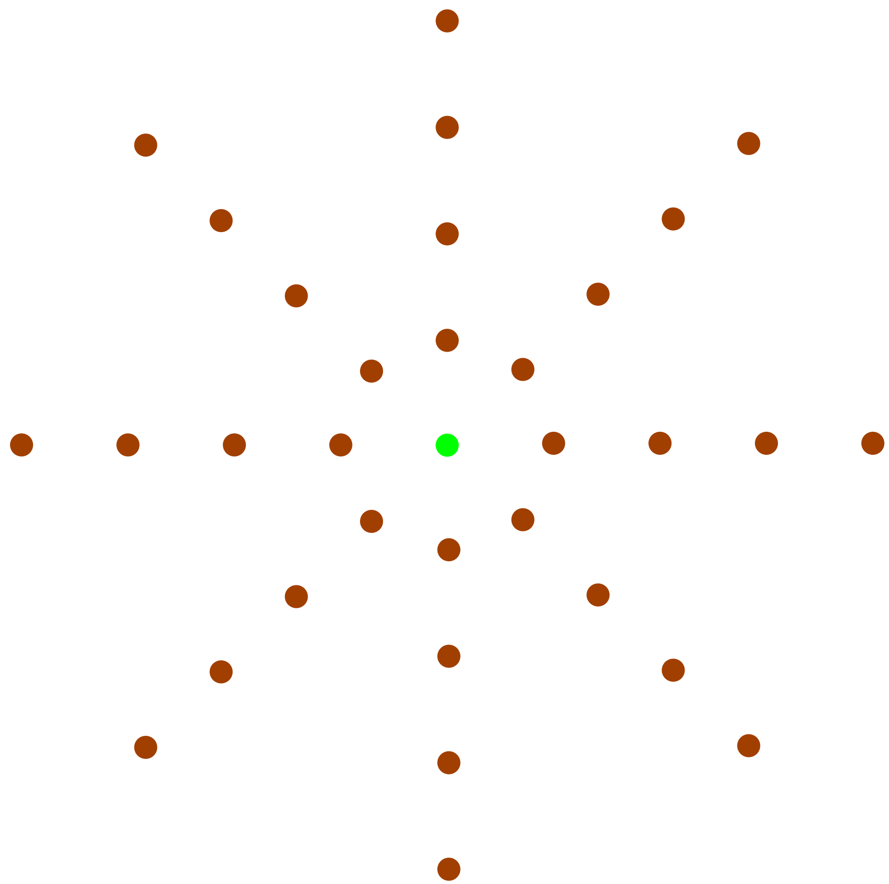
Homogeneity and Isotropy, I



after Silk (1997, p. 8).

Note that homogeneity does not imply isotropy!

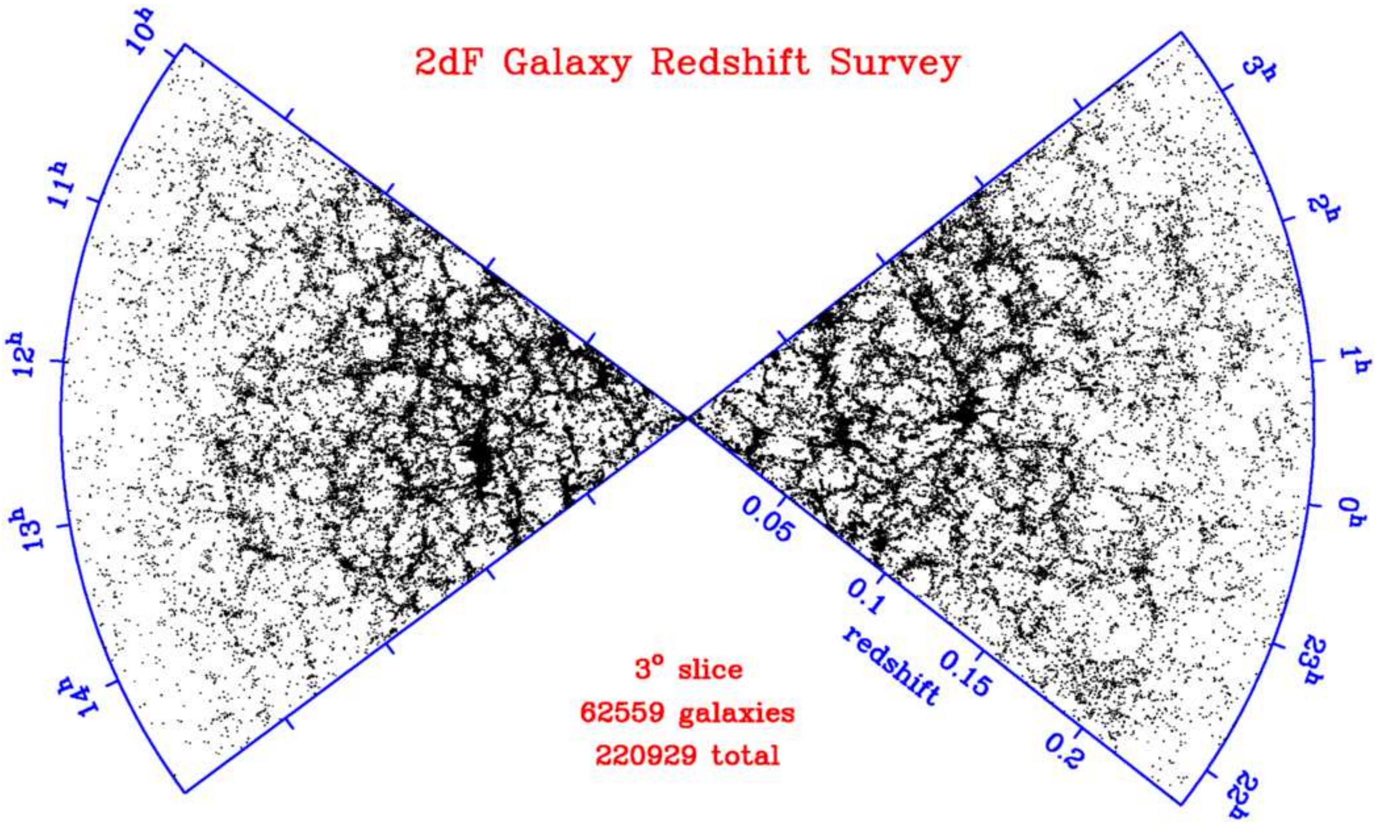
Homogeneity and Isotropy, II



Neither does isotropy *around one point* imply homogeneity!

$\Longrightarrow$ *Both* assumptions need to be tested.

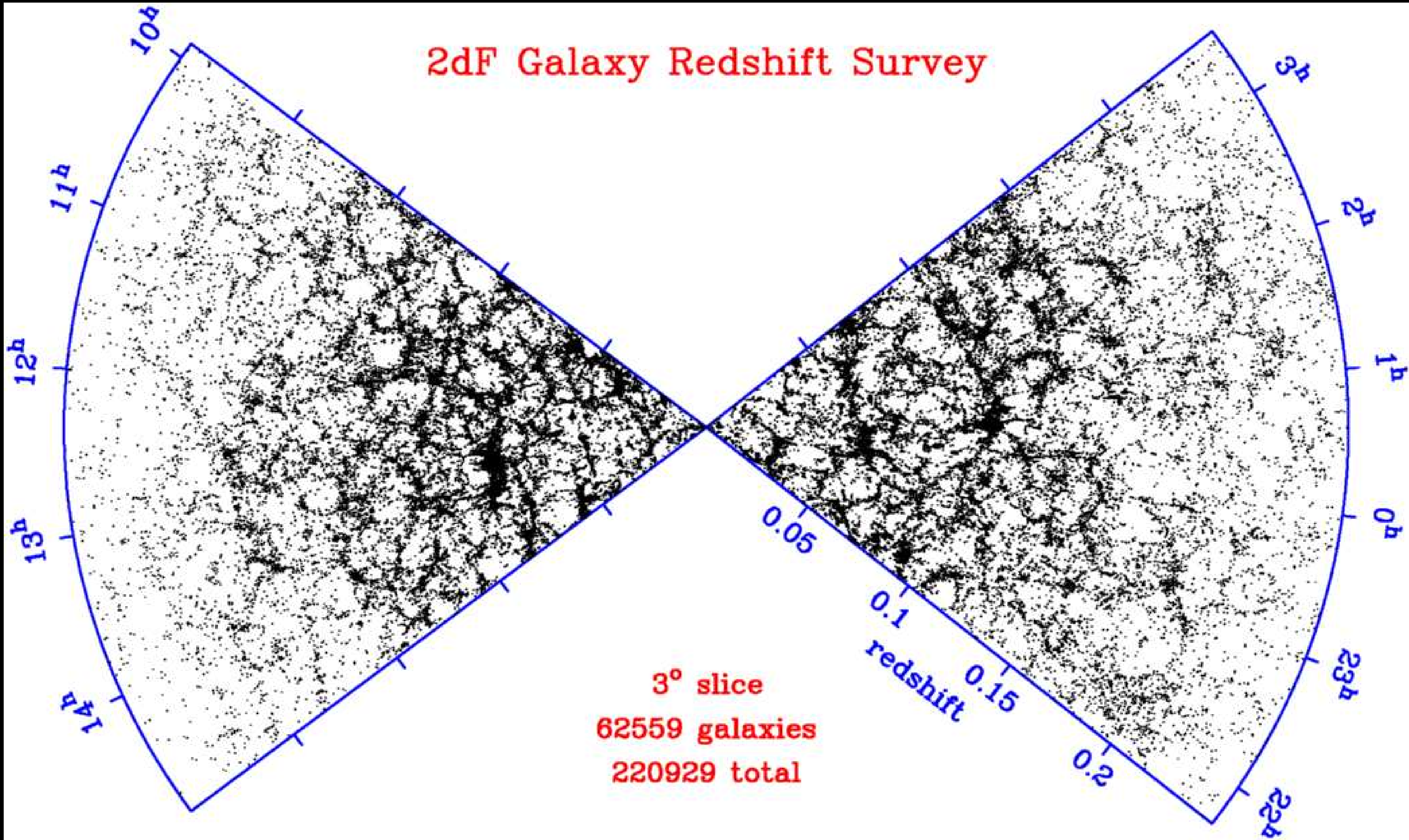
Homogeneity, I



2dF Survey, $\sim$220000 galaxies total

The universe is homogeneous $\Longleftrightarrow$ The universe looks the same everywhere in space

Testable by observing spatial distribution of galaxies.

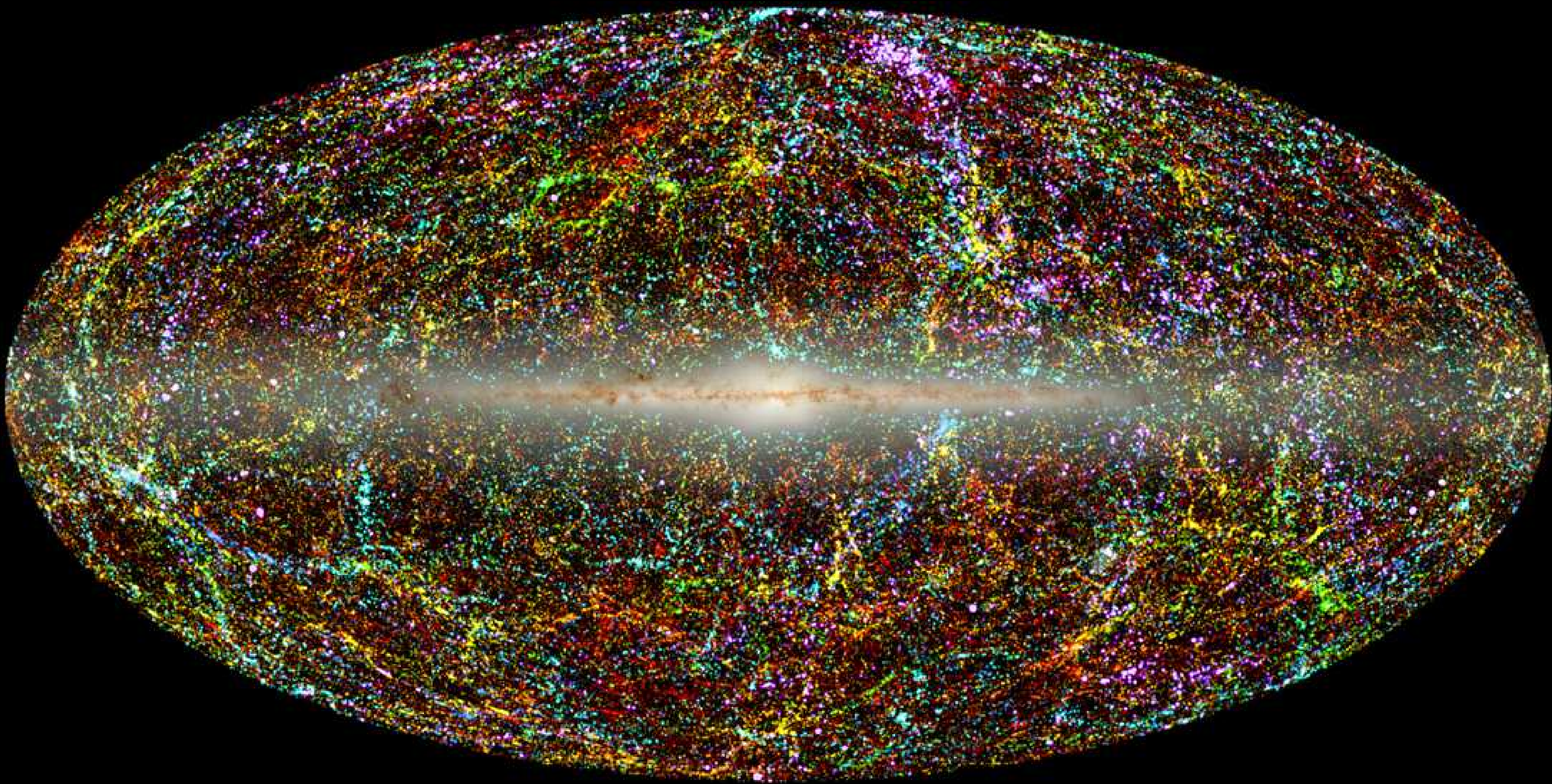


2dF Survey, ∼220000 galaxies total

On scales ≫100 Mpc the universe looks indeed the same.

Below that: structure.

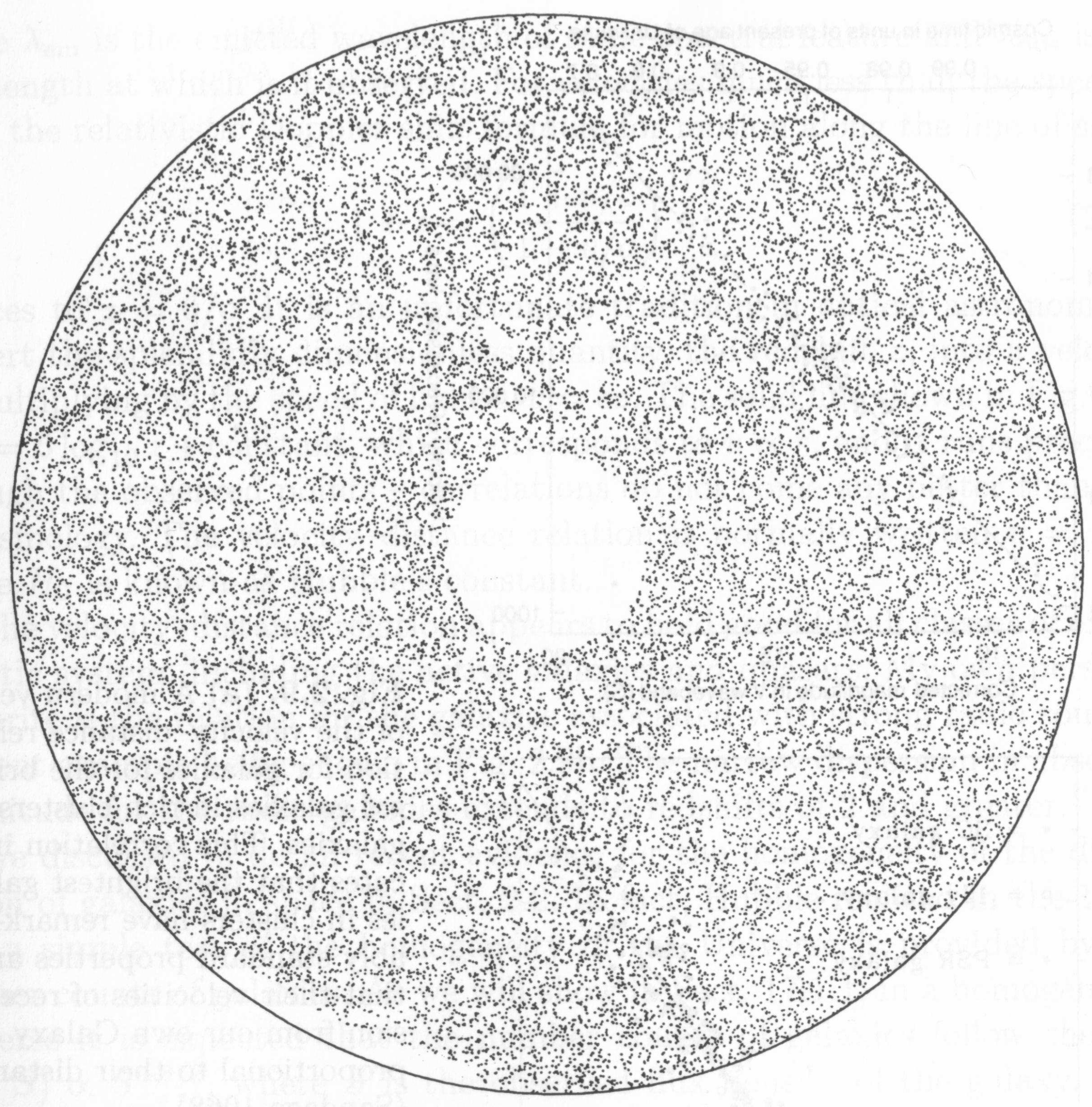
Structures seen are galaxy clusters (gravitationally bound) and superclusters (larger structures, not [yet] gravitationally bound).



(Jarrett, 2004, Fig. 1)

Distribution of Galaxy redshifts in the 2MASS galaxy catalogue

Isotropy



The universe is isotropic

$\Longleftrightarrow$ The universe looks the same in all directions

Radio galaxies are mainly quasars

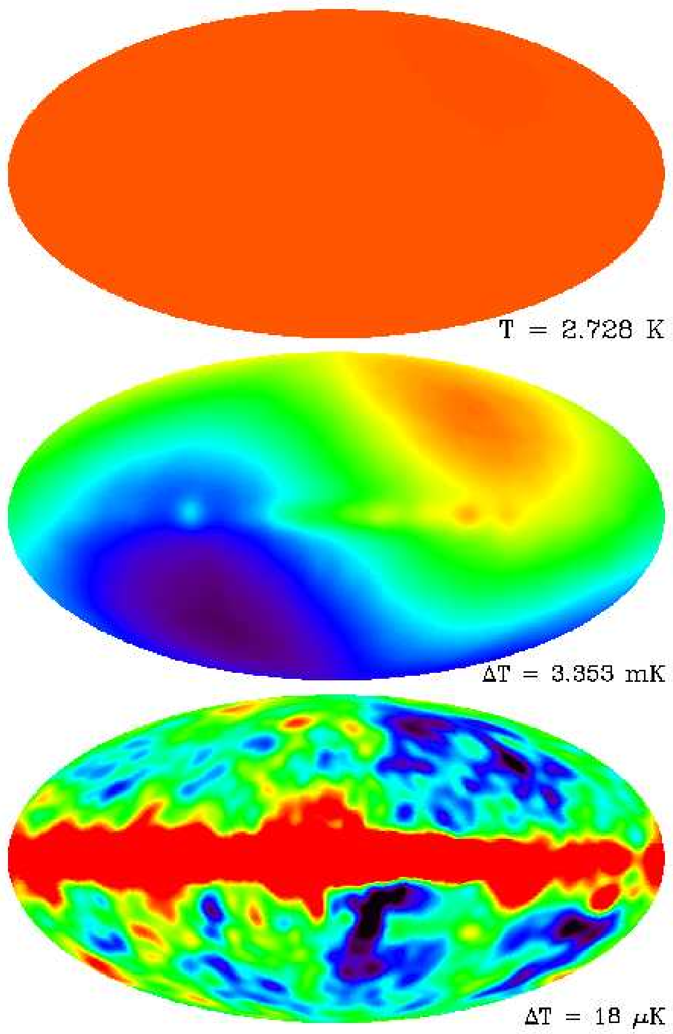
$\Longrightarrow$ Sample large space volume ($z \gtrsim 1$)

$\Longrightarrow$ Clear isotropy.

Peebles (1993): Distribution of 31000 objects at $\lambda = 6$ cm from the Greenbank Catalogue.

Anisotropy in the image: galactic plane, exclusion region around Cyg A, Cas A, and the north celestial pole.

Isotropy



Best evidence for isotropy: Intensity of 3 K Cosmic Microwave Background (CMB) radiation.

First: dipole anisotropy due to motion of Sun (see slide 3–3), after subtraction: $\Delta T/T \lesssim 10^{-4}$ on scales from $10''$ to 180°.

At level of 10^{-5}: structure in CMB due to structure of surface of last scattering of the CMB photons, i.e., structure at the time when Hydrogen recombined.

Bennet, C. L., et al., 1996, ApJ, 464, L1

Hubble, E. P., 1929, Proc. Natl. Acad. Sci. USA, 15, 168

Jarrett, T., 2004, Proc. Astron. Soc. Aust., 21, 396

Peebles, P. J. E., 1993, Principles of Physical Cosmology, (Princeton: Princeton Univ. Press)

Silk, J., 1997, A Short History of the Universe, Scientifi c American Library 53, (New York: W. H. Freeman)

Trimble, V., 1997, Space Sci. Rev., 79, 793

World Models

Structure

Observations: cosmological principle holds: The universe is homogeneous and isotropic.

$\Longrightarrow$ Need theoretical framework obeying the cosmological principle.

Use combination of

- General Relativity
- Thermodynamics
- Quantum Mechanics

$\Longrightarrow$ Complicated!

For 99% of the work, the above points can be dealt with separately:

1. Define metric obeying cosmological principle.
2. Obtain equation for evolution of universe using Einstein field equations.
3. Use thermo/QM to obtain equation of state.
4. Solve equations.

GRT vs. Newton

Before we can start to think about universe: Brief introduction to assumptions of general relativity.

$\Longrightarrow$ See theory lectures for the gory details, or check with the literature (Weinberg or MTW).

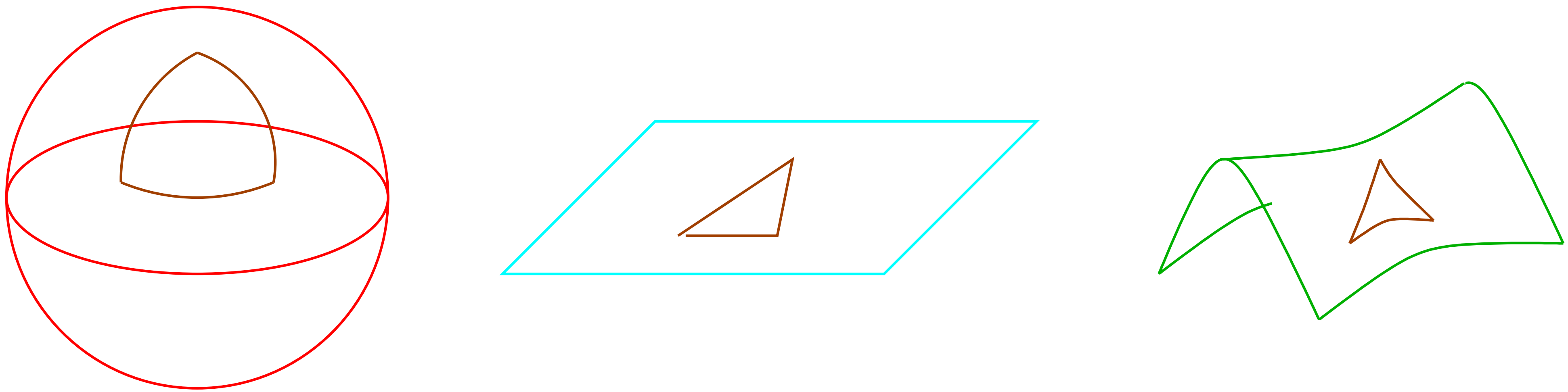
Assumptions of GRT:

- Space is 4-dimensional, might be curved
- Matter (=Energy) modifi es space (Einstein fi eld equation).
- Covariance: physical laws must be formulated in a coordinate-system independent way.
- Strong equivalence principle: There is no experiment by which one can distinguish between free falling coordinate systems and inertial systems.
- At each point, space is locally Minkowski (i.e., locally, SRT holds).

$\Longrightarrow$ Understanding of geometry of space necessary to understand physics.

2D Metrics

Before describing the 4D geometry of the universe: fi rst look at 2D spaces (easier to visualize).



After Silk (1997, p. 107)

There are three classes of isotropic and homogeneous two-dimensional spaces:

- 2-sphere ($\mathscr{S}^2$) positively curved
- x-y-plane ($\mathbb{R}^2$) zero curvature
- hyperbolic plane ($\mathscr{H}^2$) negatively curved

(curvature ≈ $\sum$angles in triangle $>$, $=$, or $<$ 180°)

We will now calculate what the metric for these spaces looks like.

2D Metrics

The metric describes the local geometry of a space.

Differential distance, $\mathrm{d}s$, in Euclidean space, $\mathbb{R}^2$:

$$\mathrm{d}s^2 = \mathrm{d}x_1^2 + \mathrm{d}x_2^2 \tag{4.1}$$

The metric tensor, $g_{\mu\nu}$, is defined through

$$\mathrm{d}s^2 = \sum_\mu \sum_\nu g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu =: g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu \tag{4.2}$$

(Einstein's summation convention)

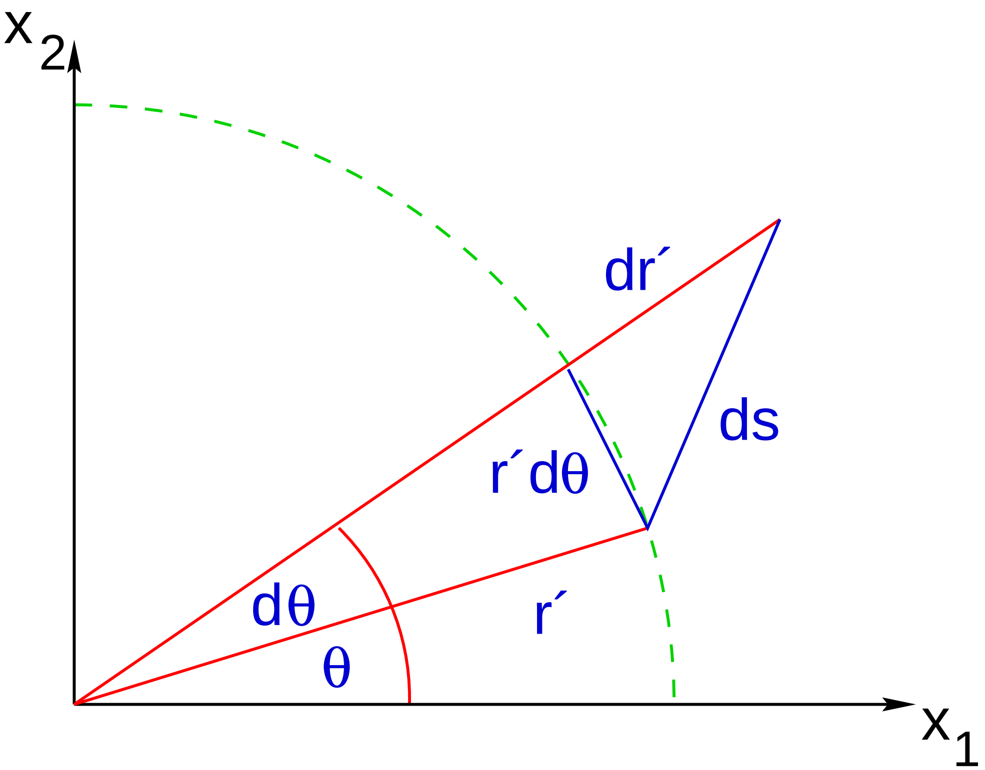
Thus, for the $\mathbb{R}^2$,

$$\begin{aligned} g_{11} &= 1 \qquad & g_{12} &= 0 \\ g_{21} &= 0 \qquad & g_{22} &= 1 \end{aligned} \tag{4.3}$$

2D Metrics

But: Other coordinate-systems are also possible in the plane!

Changing to polar coordinates r', θ, defined by



$$x_1 =: r' \cos\theta$$
$$x_2 =: r' \sin\theta \qquad (4.4)$$

it is easy to see that

$$\mathrm{d}s^2 = \mathrm{d}r'^2 + r'^2\,\mathrm{d}\theta^2 \qquad (4.5)$$

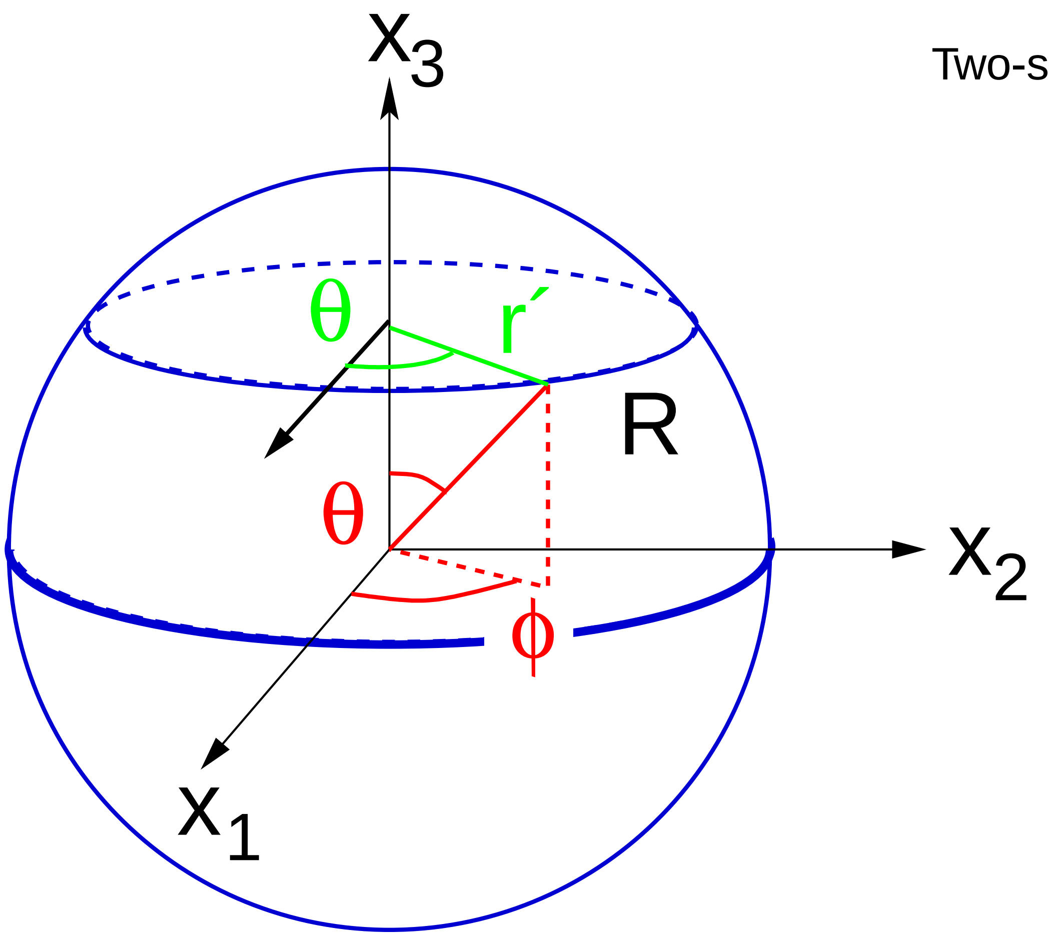
Performing a change of scale by substituting $r' = Rr$, then gives

$$\mathrm{d}s^2 = R\{\mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2\} \qquad (4.6)$$

2D Metrics

A more complicated case occurs if space is curved.

Easiest case: surface of three-dimensional sphere (a two-sphere).



After Kolb & Turner (1990, Fig. 2.1)

Two-sphere with radius R in $\mathbb{R}^3$:

$$x_1^2 + x_2^2 + x_3^2 = R^2 \tag{4.7}$$

Length element of $\mathbb{R}^3$:

$$\mathrm{d}s^2 = \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2$$

Eq. (4.7) gives

$$x_3 = \sqrt{R^2 - x_1^2 - x_2^2}$$

such that

$$\mathrm{d}x_3 = \frac{\partial x_3}{\partial x_1}\,\mathrm{d}x_1 + \frac{\partial x_3}{\partial x_2}\,\mathrm{d}x_2 = -\frac{x_1\,\mathrm{d}x_1 + x_2\,\mathrm{d}x_2}{\sqrt{R^2 - x_1^2 - x_2^2}} \tag{4.8}$$

2D Metrics

Introduce again polar coordinates r', θ in x_3-plane:

$$x_1 =: r' \cos\theta x_2 \ =: r' \sin\theta \tag{4.4}$$

(note: r', θ are only unique in upper or lower half-sphere)

The differentials are given by

$$\mathrm{d}x_1 = \cos\theta \,\mathrm{d}r' - r' \sin\theta \,\mathrm{d}\theta \quad \text{and} \quad \mathrm{d}x_2 = \sin\theta \,\mathrm{d}r' + r' \cos\theta \,\mathrm{d}\theta \tag{4.9}$$

In cartesian coordinates, the length element on $\mathscr{S}^2$ is

$$\mathrm{d}s^2 = \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \frac{(x_1 \,\mathrm{d}x_1 + x_2 \,\mathrm{d}x_2)^2}{R^2 - x_1^2 - x_2^2} \tag{4.10}$$

inserting eq. (4.9) gives after some algebra

$$= r'^2 \,\mathrm{d}\theta^2 + \frac{R^2}{R^2 - r'^2} \,\mathrm{d}r'^2 \tag{4.11}$$

finally, defining $r = r'/R$ (i.e., $0 \le r \le 1$) results in

$$\mathrm{d}s^2 = R^2 \left\{ \frac{\mathrm{d}r^2}{1 - r^2} + r^2 \,\mathrm{d}\theta^2 \right\} \tag{4.12}$$

2D Metrics

Alternatively, we can work in spherical coordinates on $\mathscr{S}^2$

$$\begin{aligned} x_1 &= R\sin\theta\cos\phi \\ x_2 &= R\sin\theta\sin\phi \\ x_3 &= R\cos\theta \end{aligned} \tag{4.13}$$

($\theta \in [0, \pi]$, $\phi \in [0, 2\pi]$).

Going through the same steps as before, we obtain after some tedious algebra

$$\mathrm{d}s^2 = R^2 \left\{\mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\phi^2\right\} \tag{4.14}$$

2D Metrics

(Important) remarks:

1. The 2-sphere has no edges, has no boundaries, but has still a finite volume, $V = 4\pi R^2$.
2. Expansion or contraction of sphere caused by variation of $R \Longrightarrow R$ determines the *scale* of volumes and distances on $\mathscr{S}^2$.

   R is called the *scale factor*

3. Positions on $\mathscr{S}^2$ are defined, e.g., by r and θ, *independent* on the value of R

   r and θ are called *comoving coordinates*

4. Although the metrics Eq. (4.10), (4.12), and (4.14) look *very* different, they still describe the *same* space $\Longrightarrow$ that's why physics should be covariant, i.e., independent of the coordinate system!

2D Metrics

The hyperbolic plane, $\mathscr{H}^2$, is defi ned by

$$x_1^2 + x_2^2 - x_3^2 = -R^2 \tag{4.15}$$

If we work in Minkowski space, where

$$\mathrm{d}s^2 = \mathrm{d}x_1^2 + \mathrm{d}x_2^2 - \mathrm{d}x_3^2 \tag{4.16}$$

then

$$= \mathrm{d}x_1^2 + \mathrm{d}x_2^2 - \frac{(x_1\,\mathrm{d}x_1 + x_2\,\mathrm{d}x_2)^2}{R^2 + x_1^2 + x_2^2} \tag{4.17}$$

$\Longrightarrow$ substitute $R \to iR$ (where $i = \sqrt{-1}$) to obtain same form as for sphere (eq. 4.11)!

Therefore,

$$\mathrm{d}s^2 = R^2 \left\{ \frac{\mathrm{d}r^2}{1 + r^2} + r^2\,\mathrm{d}\theta^2 \right\} \tag{4.18}$$

2D Metrics

The analogy to spherical coordinates on the hyperbolic plane are given by

$$\begin{aligned} x_1 &= R\sinh\theta\cos\phi \\ x_2 &= R\sinh\theta\sin\phi \\ x_3 &= R\cosh\theta \end{aligned} \tag{4.19}$$

($\theta \in [-\infty, +\infty]$, $\phi \in [0, 2\pi]$).

A session with Maple (see handout) will convince you that these coordinates give

$$\mathrm{d}s^2 = R^2 \left\{ \mathrm{d}\theta^2 + \sinh^2\theta \, \mathrm{d}\phi^2 \right\} \tag{4.20}$$

Remark:

$\mathscr{H}^2$ is unbound and has an infi nite volume.

Transcript of Maple session to obtain Eq. (4.20):

```
>  x1:=r*sinh(theta)*cos(phi);
```

$$x1 := r\sinh(\theta)\cos(\phi)$$

```
>  x2:=r*sinh(theta)*sin(phi);
```

$$x2 := r\sinh(\theta)\sin(\phi)$$

```
>  x3:=r*cosh(theta);
```

$$x3 := r\cosh(\theta)$$

```
>  dx1:=diff(x1,theta)*dtheta+diff(x1,phi)*dphi;
```

$$dx1 := r\cosh(\theta)\cos(\phi)\,dtheta - r\sinh(\theta)\sin(\phi)\,dphi$$

```
>  dx2:=diff(x2,theta)*dtheta+diff(x2,phi)*dphi;
```

$$dx2 := r\cosh(\theta)\sin(\phi)\,dtheta + r\sinh(\theta)\cos(\phi)\,dphi$$

```
>  ds2:=dx1*dx1+dx2*dx2-(x1*dx1+x2*dx2)^2/(r^2+x 1^2+x2^2);
```

$$\begin{aligned} ds2 := {} & (r\cosh(\theta)\cos(\phi)\,dtheta - r\sinh(\theta)\sin(\phi)\,dphi)^2 \\ & + (r\cosh(\theta)\sin(\phi)\,dtheta + r\sinh(\theta)\cos(\phi)\,dphi)^2 - (\\ & r\sinh(\theta)\cos(\phi)\,(r\cosh(\theta)\cos(\phi)\,dtheta - r\sinh(\theta)\sin(\phi)\,dphi) \\ & + r\sinh(\theta)\sin(\phi)\,(r\cosh(\theta)\sin(\phi)\,dtheta + r\sinh(\theta)\cos(\phi)\,dphi))^2 \Big/ (\\ & r^2 + r^2\sinh(\theta)^2\cos(\phi)^2 + r^2\sinh(\theta)^2\sin(\phi)^2) \end{aligned}$$

```
>  expand(ds2);
```

$$\begin{aligned} & r^2\cosh(\theta)^2\cos(\phi)^2\,dtheta^2 + r^2\sinh(\theta)^2\sin(\phi)^2\,dphi^2 + r^2\cosh(\theta)^2\sin(\phi)^2\,dtheta^2 \\ & + r^2\sinh(\theta)^2\cos(\phi)^2\,dphi^2 - \frac{r^4\sinh(\theta)^2\cos(\phi)^4\cosh(\theta)^2\,dtheta^2}{\%1} \\ & - 2\,\frac{r^4\sinh(\theta)^2\cos(\phi)^2\cosh(\theta)^2\,dtheta^2\sin(\phi)^2}{\%1} - \frac{r^4\sinh(\theta)^2\sin(\phi)^4\cosh(\theta)^2\,dtheta^2}{\%1} \\ & \%1 := r^2 + r^2\sinh(\theta)^2\cos(\phi)^2 + r^2\sinh(\theta)^2\sin(\phi)^2 \end{aligned}$$

```
>  simplify(",{cosh(theta)^2-sinh(theta)^2=1}, [cosh(theta)]);
```

$$r^2\,dtheta^2 + r^2\sinh(\theta)^2\,dphi^2$$

2D Metrics

To summarize:

Sphere: $$ds^2 = R^2 \left\{ \frac{dr^2}{1 - r^2} + r^2\, d\theta^2 \right\} \tag{4.12}$$

Plane: $$ds^2 = R^2 \left\{ dr^2 + r^2\, d\theta^2 \right\} \tag{4.6}$$

Hyperbolic Plane: $$ds^2 = R^2 \left\{ \frac{dr^2}{1 + r^2} + r^2\, d\theta^2 \right\} \tag{4.18}$$

$\Longrightarrow$ All three metrics can be written as

$$ds^2 = R^2 \left\{ \frac{dr^2}{1 - k\, r^2} + r^2\, d\theta^2 \right\} \tag{4.21}$$

where k defines the geometry:

$$k = \begin{cases} +1 & \text{spherical} \\ 0 & \text{planar} \\ -1 & \text{hyperbolic} \end{cases} \tag{4.22}$$

2D Metrics

For “spherical coordinates” we found:

Sphere: $$\mathrm{d}s^2 = R^2 \left\{ \mathrm{d}\theta^2 + \sin^2\theta \, \mathrm{d}\phi^2 \right\} \tag{4.14}$$

Plane: $$\mathrm{d}s^2 = R^2 \left\{ \mathrm{d}\theta^2 + \theta^2 \mathrm{d}\phi^2 \right\} \tag{4.6}$$

Hyperbolic Plane: $$\mathrm{d}s^2 = R^2 \left\{ \mathrm{d}\theta^2 + \sinh^2\theta \, \mathrm{d}\phi^2 \right\} \tag{4.20}$$

$\Longrightarrow$ All three metrics can be written as

$$\mathrm{d}s^2 = R^2 \left\{ \mathrm{d}\theta^2 + S_k^2(\theta) \, \mathrm{d}\phi^2 \right\} \tag{4.23}$$

where

$$S_k(\theta) = \begin{cases} \sin\theta & \text{for } k = +1 \\ \theta & \text{for } k = 0 \\ \sinh\theta & \text{for } k = -1 \end{cases} \quad \text{and} \quad C_k(\theta) = \sqrt{1 - kS_k^2(\theta)} = \begin{cases} \cos\theta & \text{for } k = +1 \\ 1 & \text{for } k = 0 \\ \cosh\theta & \text{for } k = -1 \end{cases} \tag{4.24}$$

The cos-like analogue of S_k, C_k, will be needed later

Note that, compared to the earlier formulae, some coordinates have been renamed. This is confusing, but legal. . .

RW Metric

- Cosmological principle $+$ expansion $\Longrightarrow$ $\exists$ freely expanding cosmical coordinate system.
  - Observers $=:$ fundamental observers
  - Time $=:$ cosmic time

  This is the coordinate system in which the 3K radiation is isotropic, clocks can be synchronized, e.g., by adjusting time to the local density of the universe.

  $\Longrightarrow$ Metric has temporal and spatial part.

  This also follows directly from the equivalence principle.

- *Homogeneity and isotropy* $\Longrightarrow$ spatial part is spherically symmetric:

$$d\psi^2 := d\theta^2 + \sin^2\theta\, d\phi^2 \tag{4.25}$$

- *Expansion:* $\exists$ scale factor, $R(t)$ $\Longrightarrow$ measure distances using comoving coordinates.

$\Longrightarrow$ metric looks like

$$ds^2 = c^2\, dt^2 - R^2(t)\left[f^2(r)\, dr^2 + g^2(r)\, d\psi^2\right] \tag{4.26}$$

where $f(r)$ and $g(r)$ are arbitrary.

RW Metric

Metrics of the form of eq. (4.26) are called Robertson-Walker (RW) metrics (introduced in 1935).

Previously studied by Friedmann and Lemaître. . .

One common choice is

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - R^2(t)\left[\mathrm{d}r^2 + S_k^2(r)\,\mathrm{d}\psi^2\right] \tag{4.27}$$

where

$R(t)$: scale factor, containing the physics

t: cosmic time

r, θ, ϕ: comoving coordinates (remember Eq. (4.25) ($\mathrm{d}\psi^2 := \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2$)!)

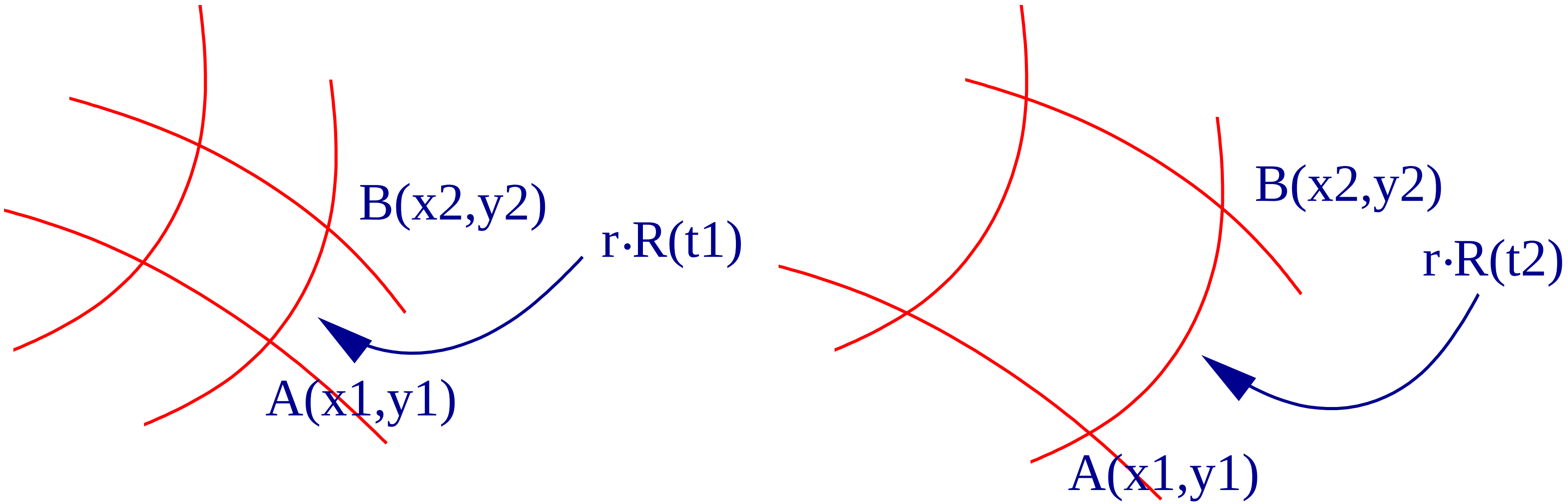
k: defi nes curvature, integer

$S_k(r)$ was defined in Eq. (4.24).

Remark: θ and ϕ describe directions on sky, as seen from the arbitrary center of the coordinate system (=us), r can be interpreted as a radial coordinate.

RW Metric

The RW metric defi nes an universal coordinate system tied to expansion of space:



Scale factor $R(t)$ describes evolution of universe.

- r is called the comoving distance.
- $D(t) := r \cdot R(t)$ is called the proper distance,

(e.g., $r \cdot R(t)$ is measured in Mpc)

RW Metric

Other forms of the RW metric are also used:

1. Substitution $S_k(r) \longrightarrow r$ gives

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - R^2(t)\left\{\frac{\mathrm{d}r^2}{1-kr^2} + r^2\,\mathrm{d}\psi^2\right\} \tag{4.28}$$

(i.e., other defi nition of comoving radius r, which is still dimensionless).

2. A metric with a dimensionless scale factor,

$$a(t) := \frac{R(t)}{R(t_0)} = \frac{R(t)}{R_0} \tag{4.29}$$

(where $t_0 =$ today, i.e., $a(t_0) = 1$), gives

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - a^2(t)\left\{\mathrm{d}r^2 + \frac{S_k^2(R_0 r)}{R_0^2}\,\mathrm{d}\psi^2\right\} \tag{4.30}$$

RW Metric

3. Using $a(t)$ *and* the substitution $S_k(r) \longrightarrow r$ is also possible:

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - a^2(t)\left\{\frac{\mathrm{d}r^2}{1 - k\cdot(R_0 r)^2} + r^2\,\mathrm{d}\psi^2\right\} \tag{4.31}$$

The units of $R_0 r$ are Mpc $\Longrightarrow$ *Used for observations!*

4. Replace cosmic time, t, by conformal time, $\mathrm{d}\eta = \mathrm{d}t/R(t)$

$\Longrightarrow$ conformal metric,

$$\mathrm{d}s^2 = R^2(\eta)\left\{\mathrm{d}\eta^2 - \frac{\mathrm{d}r^2}{1 - kr} - r^2\,\mathrm{d}\psi^2\right\} \tag{4.32}$$

Theoretical importance of this metric: For $k = 0$, i.e., a flat space, the RW metric $=$ Minkowski line element $\times\, R^2(\eta) \Longrightarrow$ Equivalence principle!

RW Metric

5. Finally, the metric can also be written in the isotropic form,

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \frac{R(t)}{1+(k/4)r^2}\left\{\mathrm{d}r^2 + r^2\mathrm{d}\psi^2\right\} \quad (4.33)$$

Here, the term in $\{\ldots\}$ is just the line element of a 3d-sphere $\Longrightarrow$ isotropy!

Note: There are as many notations as authors, e.g., some use $a(t)$ where we use $R(t)$, etc. $\Longrightarrow$ Be careful!

Note 2: Local homogeneity and isotropy (i.e., within a Hubble radius, $r = c/H_0$), do not imply *global* homogeneity and isotropy $\Longrightarrow$ Cosmologies with a non-trivial topology are possible (e.g., also with more dimensions. . .).

Hubble's Law

Hubble's Law follows from the variation of $R(t)$:

Small scales $\Longrightarrow$ Euclidean geometry. Then the proper distance between two observers is:

$$D(t) = d \cdot R(t) \tag{4.34}$$

where d: comoving distance.

Expansion $\Longrightarrow$ proper separation changes:

$$\frac{\Delta D}{\Delta t} = \frac{R(t+\Delta t)d - R(t)d}{\Delta t} \quad \Longrightarrow \lim_{\Delta t \to 0} \Longrightarrow \quad v = \frac{\mathrm{d}D}{\mathrm{d}t} = \dot{R}\, d = \frac{\dot{R}}{R} D =: H\, D \tag{4.35}$$

$\Longrightarrow$ Identify local Hubble "constant" as

$$H = \frac{\dot{R}}{R} = \dot{a}(t) \quad (a(t) \text{ from Eq. 4.29, } a(\text{today}) = 1) \tag{4.36}$$

Since $R = R(t) \Longrightarrow H$ is time-dependent!

Redshift, I

The cosmological redshift is a consequence of the expansion of the universe:

The comoving distance is constant, thus in terms of the proper distance:

$$d = \frac{D(t = \text{today})}{R(t = \text{today})} = \frac{D(t)}{R(t)} = \text{const.} \tag{4.37}$$

Set $a(t) = R(t)/R(t = \text{today})$, then eq. (4.37) implies

$$\lambda_{\text{obs}} = \frac{\lambda_{\text{emit}}}{a_{\text{emit}}} \tag{4.38}$$

(λ_{obs}: observed wavelength, λ_{emit}: emitted wavelength)

Thus the observed redshift is

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}} = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}} - 1 = \frac{\nu_{\text{emit}}}{\nu_{\text{obs}}} - 1 \tag{4.39}$$

$$\Longrightarrow \quad 1 + z = \frac{1}{a_{\text{emit}}} = \frac{R(t = \text{today})}{R(t)} = \frac{\nu_{\text{emit}}}{\nu_{\text{obs}}} \tag{4.40}$$

Light emitted at $z = 1$ was emitted when the universe was half as big as today!

z: measure for *relative size* of universe at time the observed light was emitted.

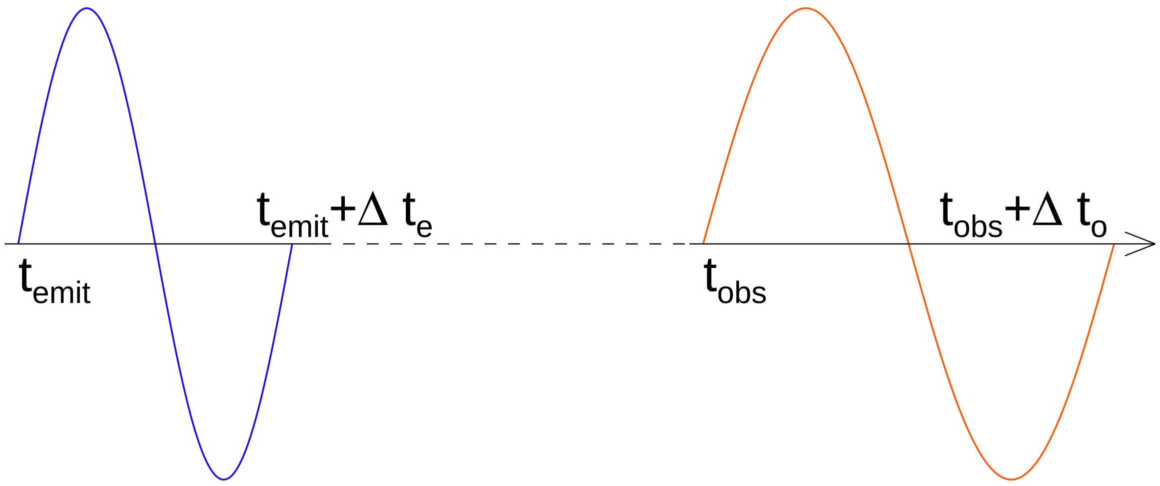
Note that the definition of H allows us to derive Hubble's relation for the case of small v, i.e., $v \ll c$. In this case, the red-shift is

$$z = \frac{v}{c} \quad \Longrightarrow \quad z = \frac{Hd}{c} \tag{4.41}$$

An alternative derivation of the cosmological redshift follows directly from general relativity, using the basic GR fact that for photons $\mathrm{d}s^2 = 0$. Inserting this into the metric, and assuming without loss of generality that $\mathrm{d}\psi^2 = 0$, one finds

$$0 = c^2\,\mathrm{d}t^2 - R^2(t)\,\mathrm{d}r^2 \quad \Longrightarrow \quad \mathrm{d}r = \pm\frac{c\,\mathrm{d}t}{R(t)} \tag{4.42}$$

Since photons travel forward, we choose the $+$-sign.



The *comoving* distance traveled by photons emitted at cosmic times t_{emit} and $t_{\mathrm{emit}} + \Delta t_{\mathrm{e}}$ is

$$r_1 = \int_{t_{\mathrm{emit}}}^{t_{\mathrm{obs}}} \frac{c\,\mathrm{d}t}{R(t)} \quad \text{and} \quad r_2 = \int_{t_{\mathrm{emit}}+\Delta t_{\mathrm{e}}}^{t_{\mathrm{obs}}+\Delta t_{\mathrm{o}}} \frac{c\,\mathrm{d}t}{R(t)} \tag{4.43}$$

But the comoving distances are equal, $r_1 = r_2$! Therefore

$$0 = \int_{t_\mathrm{emit}}^{t_\mathrm{obs}} \frac{c\,\mathrm{d}t}{R(t)} - \int_{t_\mathrm{emit}+\Delta t_\mathrm{e}}^{t_\mathrm{obs}+\Delta t_\mathrm{o}} \frac{c\,\mathrm{d}t}{R(t)} \tag{4.44}$$

$$= \int_{t_\mathrm{emit}}^{t_\mathrm{emit}+\Delta t_\mathrm{e}} \frac{c\,\mathrm{d}t}{R(t)} - \int_{t_\mathrm{obs}}^{t_\mathrm{obs}+\Delta t_\mathrm{o}} \frac{c\,\mathrm{d}t}{R(t)} \tag{4.45}$$

If Δt small $\Longrightarrow R(t) \approx$ const.:

$$= \frac{c\,\Delta t_\mathrm{e}}{R(t_\mathrm{emit})} - \frac{c\,\Delta t_\mathrm{o}}{R(t_\mathrm{obs})} \tag{4.46}$$

For a wave: $c\Delta t = \lambda$, such that

$$\frac{\lambda_\mathrm{emit}}{R(t_\mathrm{emit})} = \frac{\lambda_\mathrm{obs}}{R(t_\mathrm{obs})} \quad \Longleftrightarrow \quad \frac{\lambda_\mathrm{emit}}{\lambda_\mathrm{obs}} = \frac{R(t_\mathrm{emit})}{R(t_\mathrm{obs})} \tag{4.47}$$

From this equation it is straightforward to derive Eq. (4.39).

Redshift, II

Outside of the local universe: Eq. (4.40) only valid interpretation of z.

$\Longrightarrow$ It is common to interpret z as in special relativity:

$$1 + z = \sqrt{\frac{1 + v/c}{1 - v/c}} \qquad (4.48)$$

This is WRONG

Redshift is due to expansion of space, not due to motion of galaxy.

What *is* true is that z is accumulation of many infi nitesimal red-shifts à la Eq. (4.41), see, e.g., Peacock (1999).

Time Dilatation

For light, $D = c\,\Delta t$. Then a consequence of Eq. (4.37) is

$$\frac{c\,\Delta t_{\text{emit}}}{R(t_{\text{emit}})} = \frac{c\,\Delta t_{\text{obs}}}{R(t_{\text{obs}})} \quad \Longrightarrow \quad \frac{\mathrm{d}t}{R} = \text{const.} \tag{4.46}$$

In other words:

$$\frac{\mathrm{d}t_{\text{obs}}}{\mathrm{d}t_{\text{emit}}} = \frac{R(t_{\text{obs}})}{R(t_{\text{emit}})} = 1 + z \tag{4.49}$$

$\Longrightarrow$ Time dilatation of events at large z.

This cosmological time dilatation has been observed in the light curves of supernova outbursts.

All other observables apart from z (e.g., number density $N(z)$, luminosity distance d_{L}, etc.) require explicit knowledge of $R(t)$

$\Longrightarrow$ Need to look at the dynamics of the universe.

Friedmann Equations, I

General relativistic approach: Insert metric into Einstein equation to obtain differential equation for $R(t)$:

Einstein equation:

$$\underbrace{R_{\mu\nu} - \frac{1}{2}\mathscr{R} g_{\mu\nu}}_{G_{\mu\nu}} = \frac{8\pi G}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu} \tag{4.50}$$

where

$g_{\mu\nu}$: Metric tensor ($\mathrm{d}s^2 = g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$)

$R_{\mu\nu}$: Ricci tensor (function of $g_{\mu\nu}$)

$\mathscr{R}$: Ricci scalar (function of $g_{\mu\nu}$)

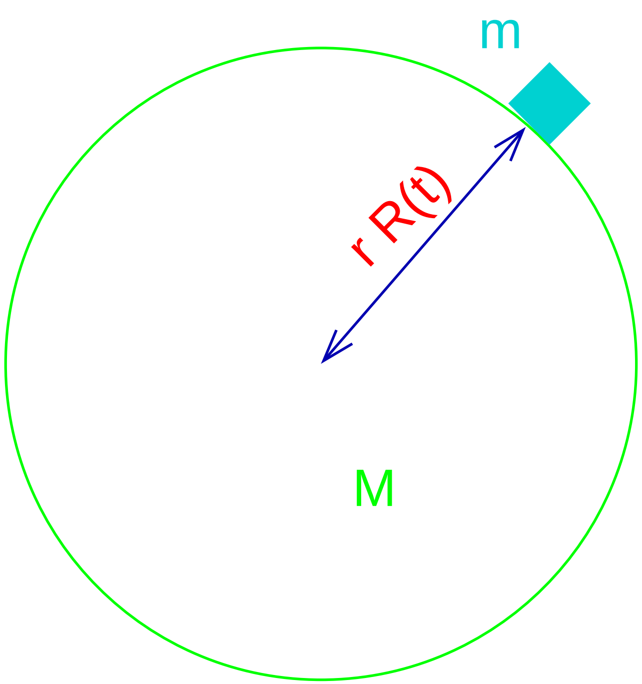
$G_{\mu\nu}$: Einstein tensor (function of $g_{\mu\nu}$)

$T_{\mu\nu}$: Stress-energy tensor, describing curvature of space due to fi elds present (matter, radiation,. . .)

Λ: Cosmological constant

$\Longrightarrow$ Messy, but doable

Friedmann Equations, II



Here, Newtonian derivation of Friedmann equations: Dynamics of a mass element on the surface of sphere of density $\rho(t)$ and comoving radius d, i.e., proper radius $d \cdot R(t)$ (McCrea, 1937)

Mass of sphere:

$$M = \frac{4\pi}{3}(d\,R)^3\rho(t) = \frac{4\pi}{3}d^3\rho_0 \quad \text{where} \quad \rho(t) = \frac{\rho_0}{R(t)^3} \tag{4.51}$$

Force on mass element:

$$m\frac{\mathrm{d}^2}{\mathrm{d}t^2}(d\,R(t)) = -\frac{GMm}{(dR(t))^2} = -\frac{4\pi G}{3}\frac{d\rho_0}{R^2(t)}m \tag{4.52}$$

Canceling $m \cdot d$ gives momentum equation:

$$\ddot{R}(t) = -\frac{4\pi G}{3}\frac{\rho_0}{R(t)^2} = -\frac{4\pi G}{3}\rho(t)R(t) \tag{4.53}$$

Multiplying Eq. (4.53) with $\dot{R}$ and integrating yields the energy equation:

$$\frac{1}{2}\dot{R}(t)^2 = +\frac{4\pi G}{3}\frac{\rho_0}{R(t)} + \text{const.} = +\frac{4\pi G}{3}\rho(t)R^2(t) + \text{const.} \tag{4.54}$$

where the constant can only be obtained from GR.

Friedmann Equations, III

Problems with the Newtonian derivation:

1. Cloud is implicitly assumed to have $r_{\text{cloud}} < \infty$

   (for $r_{\text{cloud}} \to \infty$ the force is undefi ned)

   $\Longrightarrow$ violates cosmological principle.

2. Particles move *through* space

   $\Longrightarrow v > c$ possible

   $\Longrightarrow$ violates SRT.

Why do we get correct result?

GRT $\longrightarrow$ Newton for small scales and mass densities

Since universe is isotropic: scale invariance on Mpc scales

$\Longrightarrow$ Newton suffi cient (classical limit of GR).

(In fact, point 1 above *does* hold in GR: Birkhoff's theorem).

Friedmann Equations, IV

The exact GR derivation of Friedmanns equation gives:

$$\begin{aligned}\ddot{R} &= -\frac{4\pi G}{3}R\left(\rho + \frac{3p}{c^2}\right) + \left[\frac{1}{3}\Lambda R\right] \\ \dot{R}^2 &= +\frac{8\pi G\rho}{3}R^2 - kc^2 + \left[\frac{1}{3}\Lambda c^2 R^2\right]\end{aligned} \tag{4.55}$$

Notes:

1. For $k = 0$: Eq. (4.55) $\longrightarrow$ Eq. (4.54).
2. k determines the curvature of space (and is *not* an integer here!).
3. The density, ρ, includes the contribution of all different kinds of energy (remember mass-energy equivalence!).
4. There is energy associated with the vacuum, parameterized by the parameter Λ.

The evolution of the Hubble parameter is ($\Lambda = 0$):

$$\left(\frac{\dot{R}}{R}\right)^2 = H^2(t) = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2} \tag{4.56}$$

The Critical Density, I

Solving Eq. (4.56) for k:

$$\frac{R^2}{c}\left(\frac{8\pi G}{3}\rho - H^2\right) = k \qquad (4.57)$$

$\Longrightarrow$ Sign of curvature parameter k only depends on density, ρ. With

$$\rho_{\rm c} = \frac{3H^2}{8\pi G} \quad \text{and} \quad \Omega = \frac{\rho}{\rho_{\rm c}} \qquad (4.58)$$

it is easy to see that:

$$\begin{aligned} \Omega > 1 &\Longrightarrow k > 0 \Longrightarrow \text{closed universe} \\ \Omega = 1 &\Longrightarrow k = 0 \Longrightarrow \text{flat universe} \\ \Omega < 1 &\Longrightarrow k < 0 \Longrightarrow \text{open universe} \end{aligned}$$

$\rho_{\rm c}$ is called the critical density

For $\Omega \leq 1$ the universe will expand until ∞,

For $\Omega > 1$ we will see the "big crunch".

Current value of $\rho_{\rm c}$: $\sim 1.67 \times 10^{-24}\,\mathrm{g\,cm^{-3}}$ (3...10 H-atoms $\mathrm{m^{-3}}$).

The Critical Density, II

Ω has a second order effect on the expansion:

Taylor series of $R(t)$ around $t = t_0$:

$$\frac{R(t)}{R(t_0)} = \frac{R(t_0)}{R(t_0)} + \frac{\dot{R}(t_0)}{R(t_0)} (t - t_0) + \frac{1}{2} \frac{\ddot{R}(t_0)}{R(t_0)} (t - t_0)^2 \tag{4.59}$$

The Friedmann equation Eq. (4.53) can be written

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3} \rho = -\frac{4\pi G}{3} \Omega \frac{3H^2}{8\pi G} = -\frac{\Omega H^2}{2} \tag{4.60}$$

Since $H(t) = \dot{R}/R$ (Eq. 4.36), Eq. (4.59) is

$$\frac{R(t)}{R(t_0)} = 1 + H_0 (t - t_0) - \frac{1}{2} \frac{\Omega_0}{2} H_0^2 (t - t_0)^2 \tag{4.61}$$

where $H_0 = H(t_0)$ and $\Omega_0 = \Omega(t_0)$.

The subscript 0 is often omitted in the case of Ω.

Often, Eq. (4.61) is written using the deceleration parameter:

$$q := \frac{\Omega}{2} = -\frac{\ddot{R}(t_0) R(t_0)}{\dot{R}^2(t_0)} \tag{4.62}$$

Equation of state, I

Evolution of the universe determined by three different kinds of equation of state:

1. Matter: Normal (nonrelativistic) particles get diluted by expansion of the universe:

$$\rho_{\rm m} \propto R^{-3} \tag{4.63}$$

   Matter is also often called dust by cosmologists.

2. Radiation: The energy density of radiation decreases because of volume expansion and because of the cosmological redshift (Eq. 4.47: $\lambda_{\rm obs}/\lambda_{\rm emit} = \nu_{\rm emit}/\nu_{\rm obs} = R(t_{\rm obs})/R(t_{\rm emit})$) such that

$$\rho_{\rm r} \propto R^{-4} \tag{4.64}$$

3. Vacuum: The vacuum energy density (=Λ) is independent of R:

$$\rho_{\rm v} = \text{const.} \tag{4.65}$$

Inserting these equations of state into the Friedmann equation and solving with the boundary condition $R(t = 0) = 0$ then gives a specific world model.

Equation of state, II

Current scale factor is determined by H_0 and Ω_0:

Friedmann for $t = t_0$:

$$\dot{R}_0^2 - \frac{8\pi G}{3}\rho R_0^2 = -kc^2 \tag{4.66}$$

Insert Ω and note $H_0 = \dot{R}_0/R_0$

$$\Longleftrightarrow \quad H_0^2 R_0^2 - H_0^2 \Omega_0 R_0^2 = -kc^2 \tag{4.67}$$

And therefore

$$R_0 = \frac{c}{H_0}\sqrt{\frac{k}{\Omega - 1}} \tag{4.68}$$

For $\Omega \longrightarrow 0$, $R_0 \longrightarrow c/H_0$, the Hubble length.
For $\Omega = 1$, R_0 is arbitrary.

We now have everything we need to solve the Friedmann equation and determine the evolution of the universe for $k = 0$, $+1$, and -1.

$k = 0$, Matter dominated

For the matter dominated, flat case (the Einstein-de Sitter case), the Friedmann equation is

$$\dot{R}^2 - \frac{8\pi G}{3}\frac{\rho_0 R_0^3}{R^3}\, R^2 = 0 \tag{4.69}$$

For $k = 0$: $\Omega = 1$ and

$$\frac{8\pi G\rho_0}{3} = \Omega_0 H_0^2 R_0^3 = H_0^2 R_0^3 \tag{4.70}$$

Therefore, the Friedmann eq. is

$$\dot{R}^2 - \frac{H_0^2 R_0^3}{R} = 0 \quad \Longrightarrow \quad \frac{\mathrm{d}R}{\mathrm{d}t} = H_0 R_0^{3/2} R^{-1/2} \tag{4.71}$$

Separation of variables and setting $R(0) = 0$,

$$\int_0^{R(t)} R^{1/2}\,\mathrm{d}R = H_0 R_0^{3/2} t \quad \Longrightarrow \quad \frac{2}{3}R^{3/2}(t) = H_0 R_0^{3/2} t \quad \Longrightarrow \quad R(t) = R_0 \left(\frac{3H_0}{2}\, t\right)^{2/3} \tag{4.72}$$

Therefore, for $k = 0$, the universe expands until ∞, its current age ($R(t_0) = R_0$) is given by

$$t_0 = \frac{2}{3H_0} \tag{4.73}$$

Reminder: The Hubble-Time is $H_0^{-1} = 9.78\,\mathrm{Gyr}/h$.

For the matter dominated, closed case, Friedmanns equation is

$$\dot{R}^2 - \frac{8\pi G}{3}\frac{\rho_0 R_0^3}{R} = -c^2 \quad \Longleftrightarrow \quad \dot{R}^2 - \frac{H_0^2 R_0^3 \Omega_0}{R} = -c^2 \tag{4.74}$$

Inserting R_0 from Eq. (4.68) gives

$$\dot{R}^2 - \frac{H_0^2 c^3 \Omega_0}{H_0^3(\Omega-1)^{3/2}}\frac{1}{R} = -c^2 \tag{4.75}$$

which is equivalent to

$$\frac{\mathrm{d}R}{\mathrm{d}t} = c\left(\frac{\xi}{R}-1\right)^{1/2} \quad \text{with} \quad \xi = \frac{c}{H_0}\frac{\Omega_0}{(\Omega_0-1)^{3/2}} \tag{4.76}$$

With the boundary condition $R(0) = 0$, separation of variables gives

$$ct = \int_0^{R(t)} \frac{\mathrm{d}R}{(\xi/R-1)^{1/2}} = \int_0^{R(t)} \frac{\sqrt{R}\,\mathrm{d}R}{(\xi-R)^{1/2}} \tag{4.77}$$

Integration by substitution gives the “cycloid solution”

$$R = \xi\sin^2\frac{\theta}{2} = \frac{\xi}{2}(1-\cos\theta) \quad \text{and} \quad ct = \frac{\xi}{2}(\theta-\sin\theta) \tag{4.78}$$

where θ is an implicit parameter.

The age of the universe, t_0, is obtained by solving

$$R_0 = \frac{c}{H_0(\Omega_0-1)^{1/2}} = \frac{\xi}{2}(1-\cos\theta_0) = \frac{1}{2}\frac{c}{H_0}\frac{\Omega_0}{(\Omega_0-1)^{3/2}}(1-\cos\theta_0) \tag{4.79}$$

(remember Eq. 4.68!). Therefore

$$\cos\theta_0 = \frac{2-\Omega_0}{\Omega_0} \quad \Longleftrightarrow \quad \sin\theta_0 = \frac{2}{\Omega_0}\sqrt{\Omega_0-1} \tag{4.80}$$

Inserting this into Eq. (4.78) gives

$$t_0 = \frac{1}{2H_0}\frac{\Omega_0}{(\Omega_0-1)^{3/2}}\left[\arccos\left(\frac{2-\Omega_0}{\Omega_0}\right) - \frac{2}{\Omega_0}\sqrt{\Omega_0-1}\right] \tag{4.81}$$

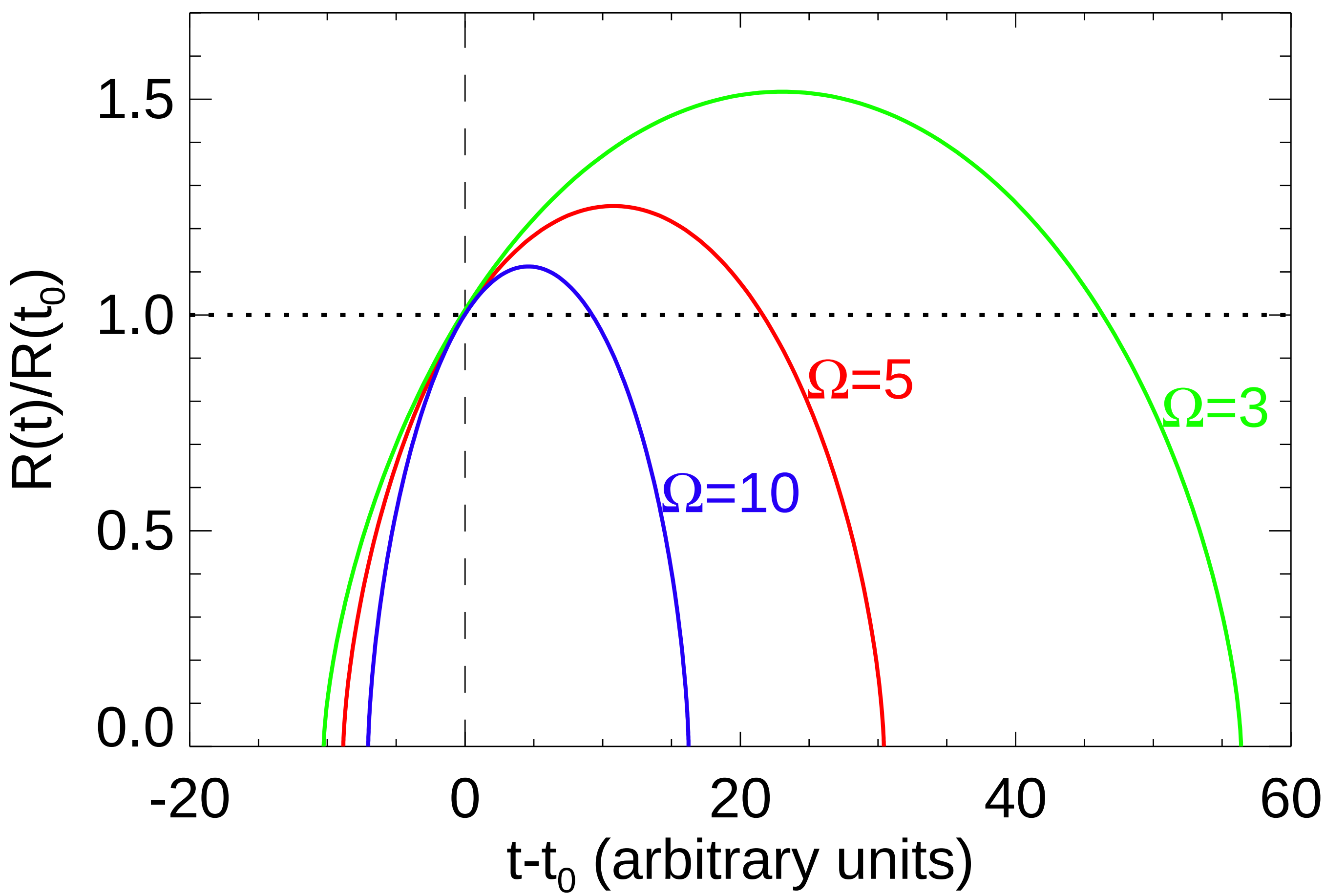
The cycloid solution shows that for $\Omega > 1$, the universe has a finite lifetime, i.e., it expands to a maximum and then becomes smaller and dies in a "big crunch". The max. expansion occurs at $\theta = \pi$, with a maximum scale factor of

$$R_{\mathrm{max}} = \xi = \frac{c}{H_0} \frac{\Omega_0}{(\Omega_0 - 1)^{3/2}} \tag{4.82}$$

The big crunch will happen at $\theta = 2\pi$, such that the lifetime of the closed universe is

$$t_{\mathrm{life}} = \frac{\pi}{H_0} \frac{\Omega_0}{(\Omega_0 - 1)^{3/2}} \tag{4.83}$$

$k = +1$, Matter dominated, I



For the closed universe, one finds

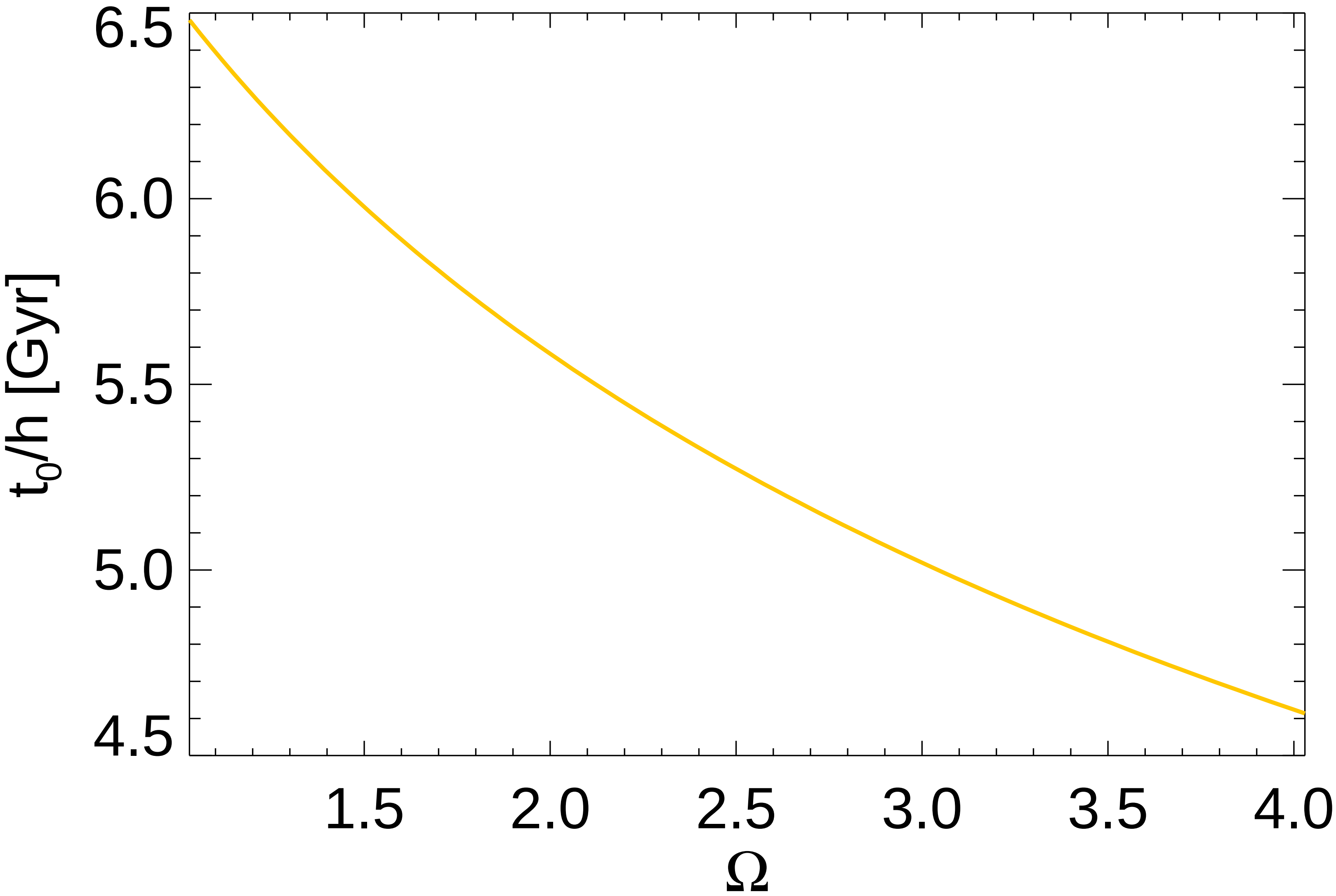
$$R = \frac{\xi}{2}(1 - \cos\theta)$$
$$ct = \frac{\xi}{2}(\theta - \sin\theta) \qquad (4.78)$$

Note that R is a cyclic function

$\Longrightarrow$ The closed universe has a finite lifetime, given by

$$t_{\text{life}} = \frac{\pi}{H_0} \frac{\Omega_0}{(\Omega_0 - 1)^{3/2}} \qquad (4.83)$$

$k = +1$, Matter dominated, II



Age of a closed and matter dominated universe.

$k = -1$, Matter dominated, I

Finally, the matter dominated, open case. This case is very similar to the case of $k = +1$:

For $k = -1$, the Friedmann equation becomes

$$\frac{\mathrm{d}R}{\mathrm{d}t} = c\left(\frac{\zeta}{R} + 1\right)^{1/2} \tag{4.84}$$

where

$$\zeta = \frac{c}{H_0}\,\frac{\Omega_0}{(1-\Omega_0)^{3/2}} \tag{4.85}$$

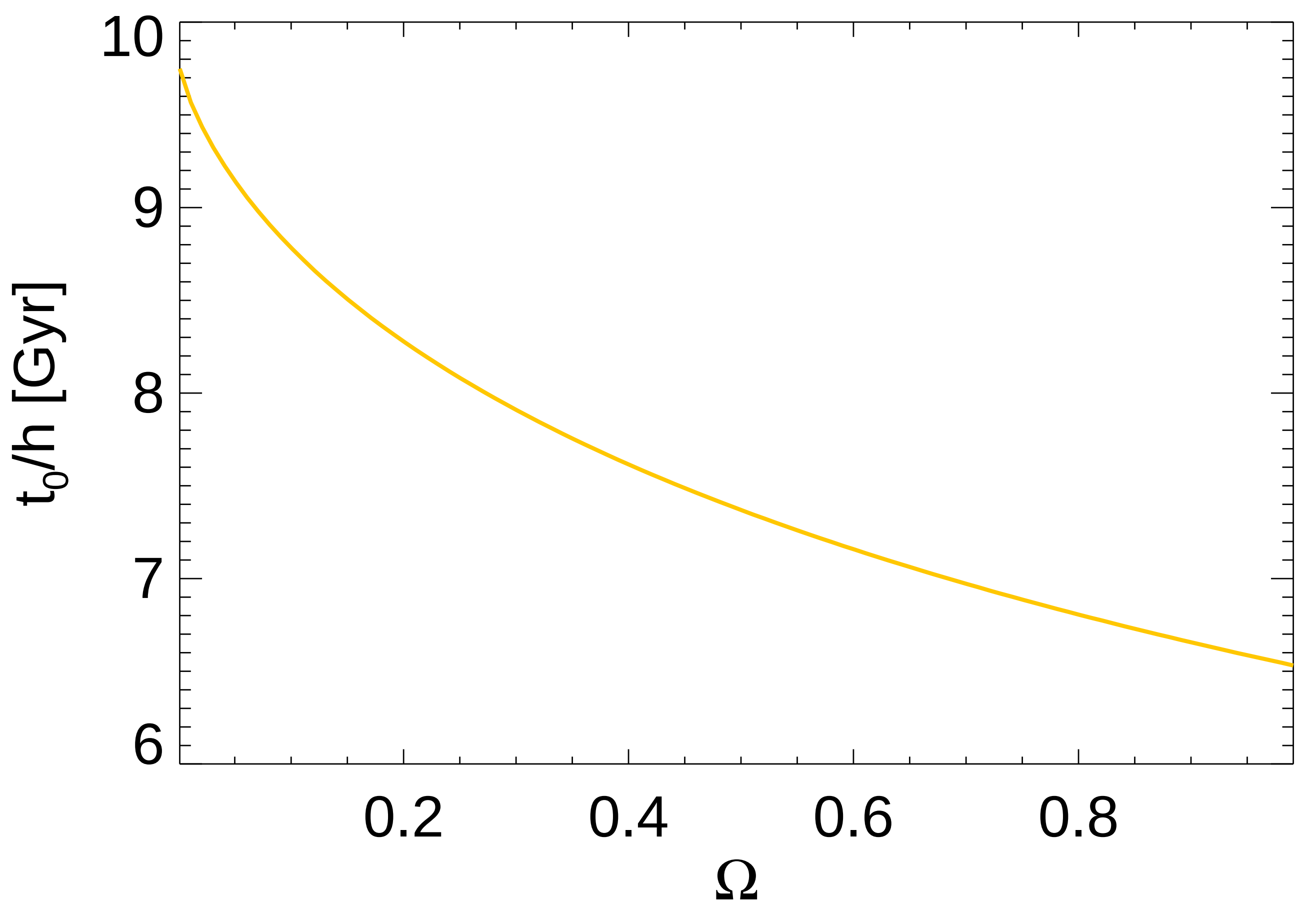
Separation of variables gives after a little bit of algebra

$$\begin{aligned} R &= \frac{\zeta}{2}(\cosh\theta - 1) \\ ct &= \frac{\zeta}{2}(\sinh\theta - 1) \end{aligned} \tag{4.86}$$

where the integration was again performed by substitution.

Note: θ here has *nothing* to do with the coordinate angle θ!

$k = -1$, Matter dominated, II



To obtain the age of the universe, note that at the present time,

$$\cosh\theta_0 = \frac{2-\Omega_0}{\Omega_0}$$
$$\sinh\theta_0 = \frac{2}{\Omega_0}\sqrt{1-\Omega_0} \qquad (4.87)$$

(identical derivation as that leading to Eq. 4.79)

therefore,

$$t_0 = \frac{1}{2H_0}\,\frac{\Omega_0}{(1-\Omega_0)^{3/2}}\cdot\left\{\frac{2}{\Omega_0}\sqrt{1-\Omega_0} - \ln\left(\frac{2-\Omega_0+2\sqrt{1-\Omega_0}}{\Omega_0}\right)\right\} \qquad (4.88)$$

Summary

For the matter dominated case, our results from Eqs. (4.78), and (4.86) can be written with the functions S_k and C_k (Eq. 4.24) in form of the cycloid solution:

$$\begin{aligned} R &= k\mathscr{R}\,(1 - C_k(\theta)) \\ ct &= k\mathscr{R}\,(\theta - S_k(\theta)) \end{aligned} \tag{4.89}$$

with

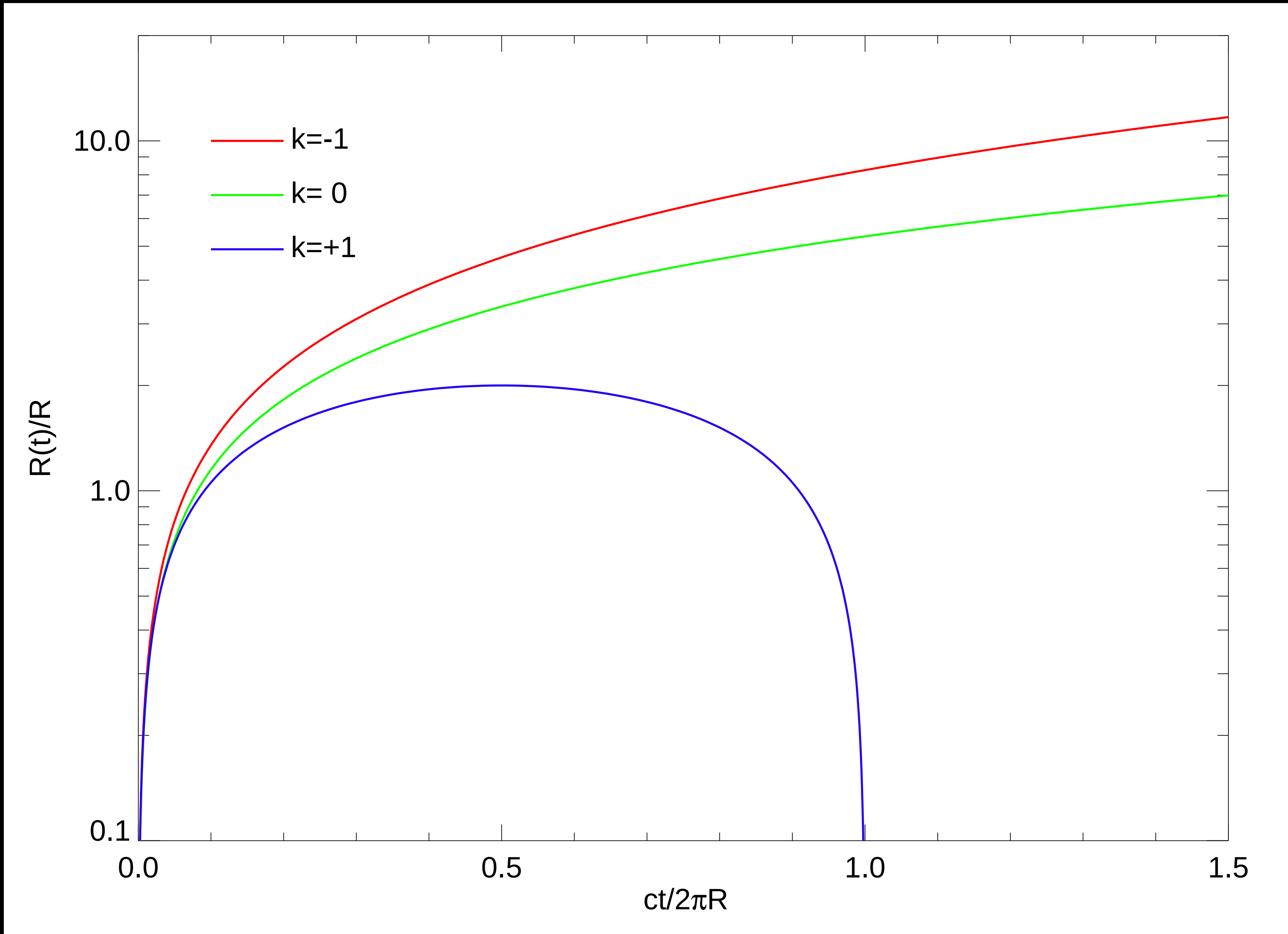
$$S_k(\theta) = \begin{cases} \sin\theta \\ \theta \\ \sinh\theta \end{cases} \quad \text{and} \quad C_k(\theta) = \begin{cases} \cos\theta & \text{for } k = +1 \\ 1 & \text{for } k = 0 \\ \cosh\theta & \text{for } k = -1 \end{cases} \tag{4.24}$$

and where the characteristic radius, $\mathscr{R}$, is given by

$$\mathscr{R} = \frac{c}{H_0} \frac{\Omega_0/2}{(k\,(\Omega_0 - 1))^{3/2}} \tag{4.90}$$

Notes:

1. Eq. (4.89) can also be derived as the result of the Newtonian collapse/expansion of a spherical mass distribution.
2. θ is called the development angle, it is equal to the *conformal time* (Eq. (4.32)).



k=-1
k= 0
k=+1
10.0
1.0
0.1
R(t)/R
0.0
0.5
1.0
1.5
ct/2πR

McCrea, W. H., & Milne, E. A., 1934, Quart. J. Math. (Oxford Series), 5, 73

Silk, J., 1997, A Short History of the Universe, Scientifi c American Library 53, (New York: W. H. Freeman)

Classical Cosmology

Classical Cosmology

To understand what universe we live in, we need to determine observationally the following numbers:

1. The Hubble constant, H_0
   $\Longrightarrow$ Requires distance measurements.
2. The current density parameter, Ω_0
   $\Longrightarrow$ Requires measurement of the mass density.
3. The cosmological constant, Λ
   $\Longrightarrow$ Requires acceleration measurements.
4. The age of the universe, t_0, for consistency checks
   $\Longrightarrow$ Requires age measurements.

The determination of these numbers is the realm of classical cosmology.

First part: Distance determination and H_0!

Introduction, I

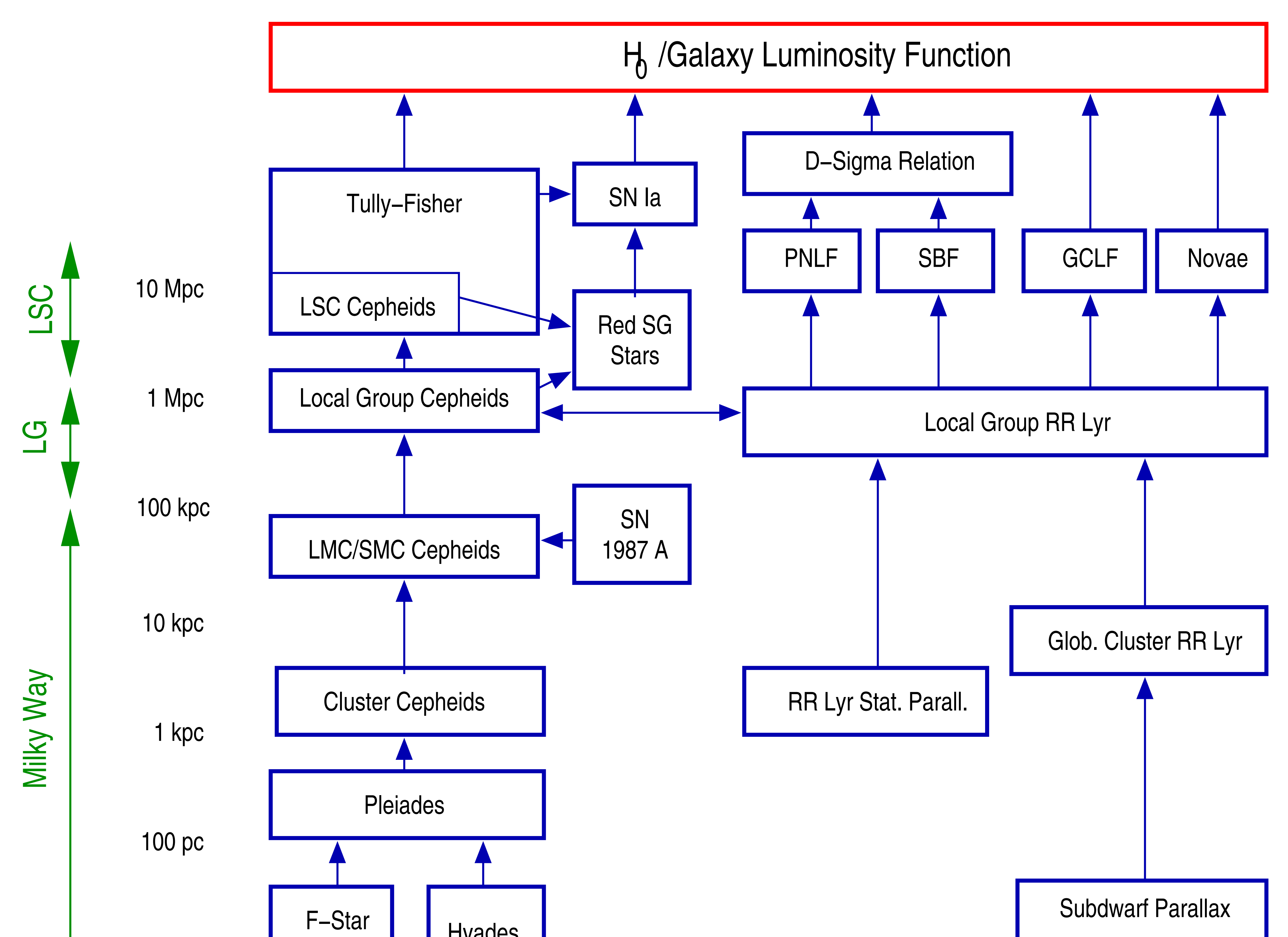
Distances are required for determination of H_0.

$\Longrightarrow$ Need to measure distances out to $\sim$200 Mpc to obtain reliable values.

To get this far: cosmological distance ladder.

1. Trigonometric Parallax and Moving Cluster
2. Main Sequence Fitting
3. RR Lyr
4. Baade-Wesselink
5. Cepheids
6. (Light echos)
7. Brightest Stars
8. Type Ia Supernovae
9. Tully-Fisher
10. D_n-σ for ellipticals
11. Brightest Cluster Galaxies
12. Gravitational Lenses

Still the best reference on this subject is Rowan-Robinson, M., 1985, The Cosmological Distance Ladder, New York: Freeman.



H_0 /Galaxy Luminosity Function
Tully–Fisher
LSC Cepheids
SN Ia
Red SG Stars
D–Sigma Relation
PNLF
SBF
GCLF
Novae
Local Group Cepheids
Local Group RR Lyr
LMC/SMC Cepheids
SN 1987 A
Cluster Cepheids
RR Lyr Stat. Parall.
Glob. Cluster RR Lyr
Pleiades
F–Star
Hyades
Subdwarf Parallax
LSC
LG
Milky Way
10 Mpc
1 Mpc
100 kpc
10 kpc
1 kpc
100 pc

Units

Basic unit of length in astronomy: Astronomical Unit (AU).

Colloquial Definition: 1 AU $=$ mean distance Earth–Sun.

Measurement: (Venus) radar ranging, interplanetary satellite positions, χ^2 minimization of N-body simulations of solar system

$$1\,\text{AU} \sim 149.6 \times 10^6\,\text{km}$$

In the astronomical system of units (IAU 1976), the AU is defined via Gaussian gravitational constant (k), where the acceleration

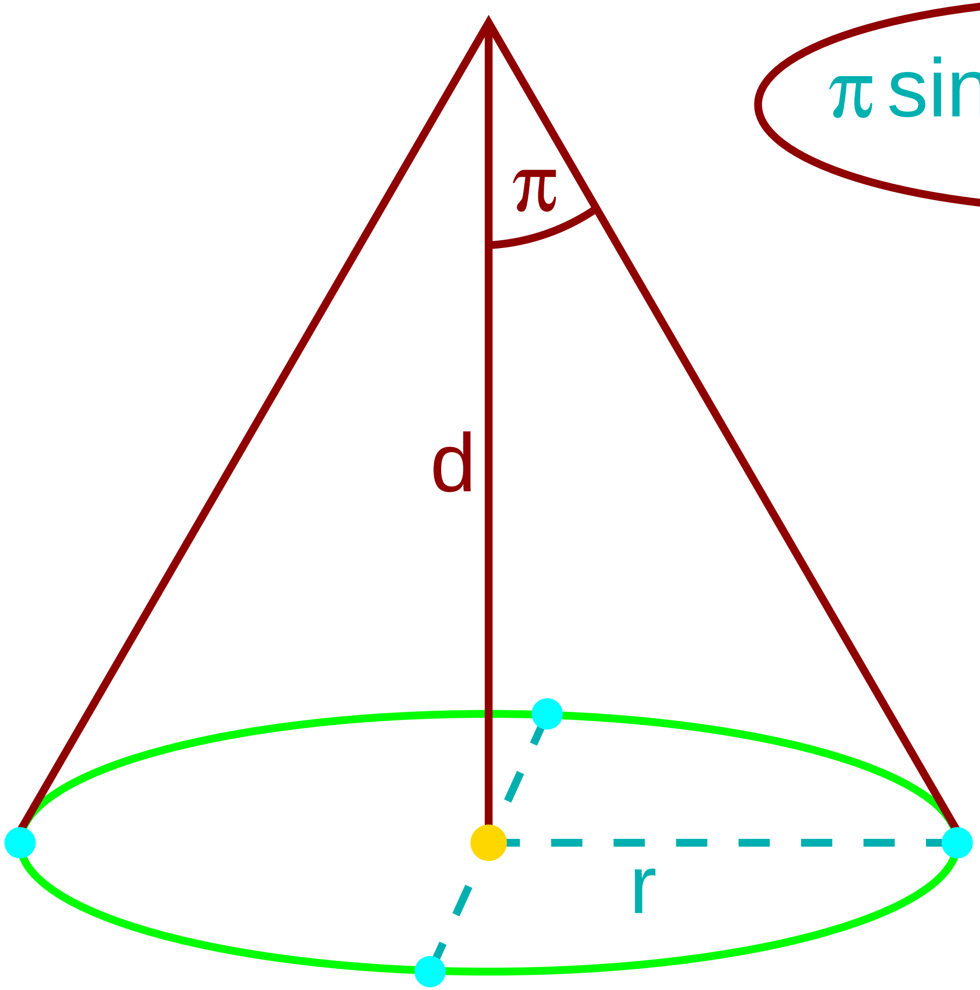
$$\ddot{\boldsymbol{r}} = -\frac{k^2(1+m)\boldsymbol{r}}{r^3}$$

where $k := 0.01720209895$, leading to $a_{\oplus} = 1.00000105726665$, and 1 AU=$1.4959787066 \times 10^{11}$ m (Seidelmann, 1992).

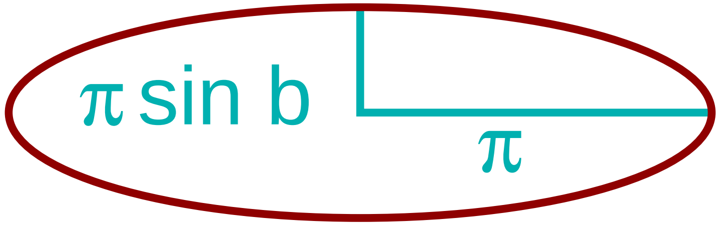
Reason for this definition: k much better known than G.

(2006 CODATA: $G = 6.67428(67) \times 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$, so only known to 4 signifi cant digits)

Trigonometric Parallax, I



after Rowan-Robinson (1985, Fig. 2.1)



Motion of Earth around Sun $\Longrightarrow$ Parallax produces apparent motion by amount

$$\tan\pi \sim \pi = r_{\oplus}/d \qquad (5.1)$$

π is called the trigonometric parallax, and *not* 3.141!

If star is at ecliptic latitude b, then ellipse with axes π and $\pi \sin b$.

Measurement difficult: $\pi \lesssim 0.76''$ (α Cen).

Define unit for distance:

Parsec: Distance where 1 AU has $\pi = 1''$. 1 pc = 206265 AU = 3.08×10^{18} cm = 3.26 ly

Trigonometric Parallax, II

Best measurements to date: Hipparcos satellite (1989–1993)

- systematic error of position: $\sim$0.5 mas for stars brighter 9 mag
- effective distance limit: 1 kpc
- standard error of proper motion: $\sim$1 mas yr^{-1}
- broad band photometry
- narrow band: $B - V$, $V - J$
- magnitude limit: 12 mag
- complete to mag: 7.3–9.0

Results available at

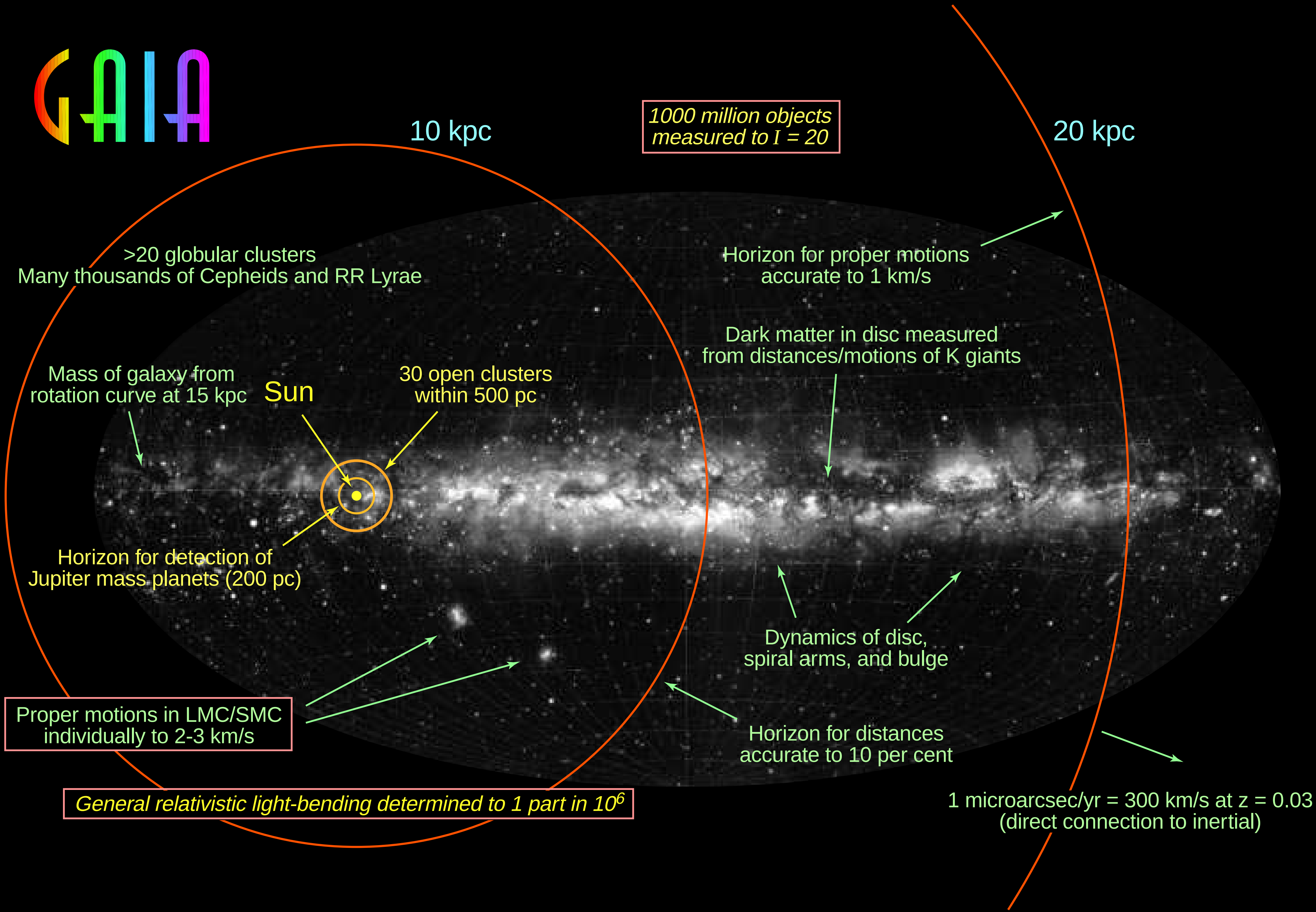
`http://www.rssd.esa.int/index.php?project=HIPPARCOS`

Hipparcos catalogue: 118 218 objects with milliarcsecond precision.

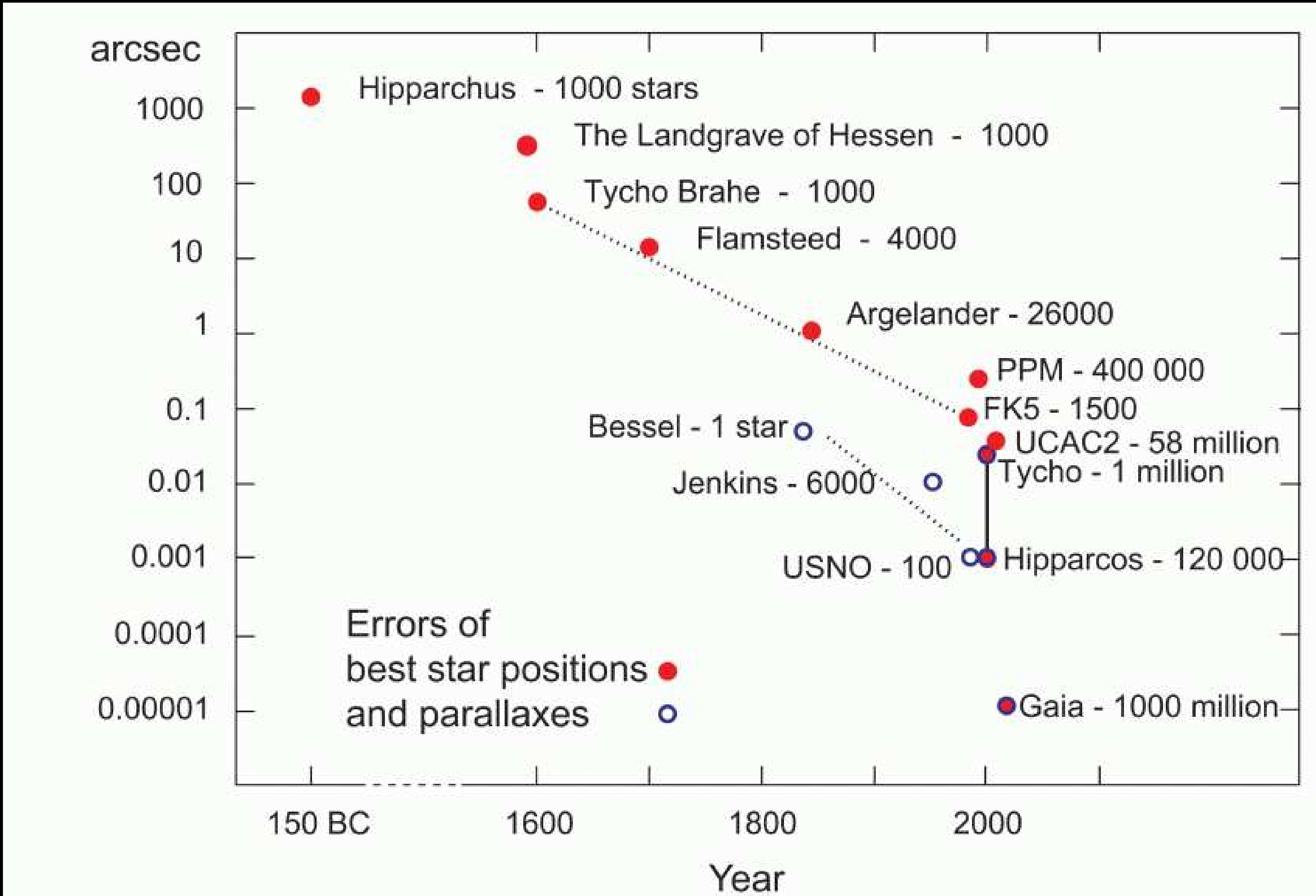
Tycho catalogue: 2 539 913 stars with 20–30 mas precision, two-band photometry (99% complete down to 11 mag)

Revised Hipparcos calibration: see van Leeuwen (2007).

GAIA (ESA mission, to be launched 2011 Dec on Soyuz from Kourou):

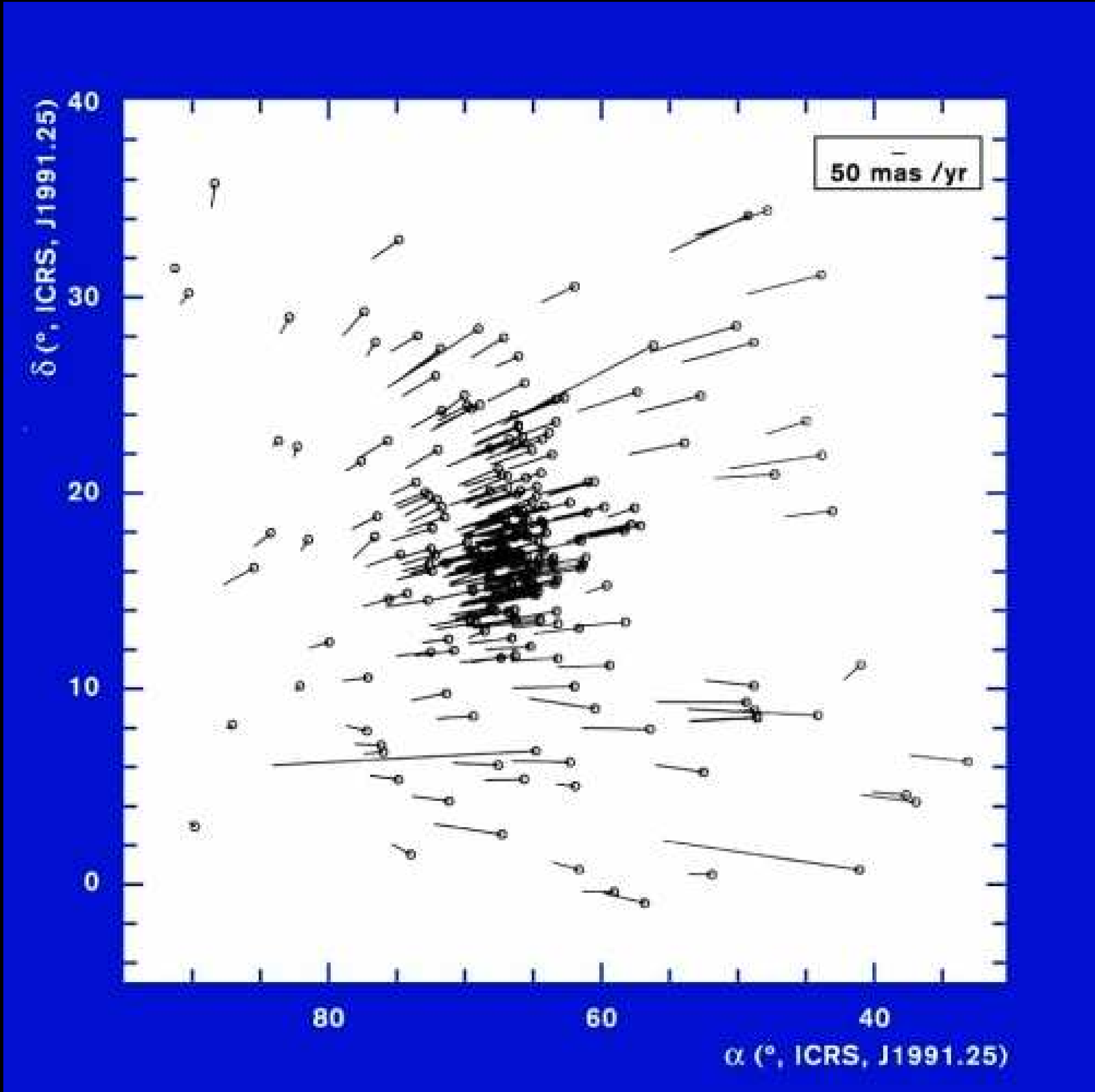


GAIA: $\sim$ 4μarcsec precision, 4 color to $V = 20$ mag, 10^9 objects.



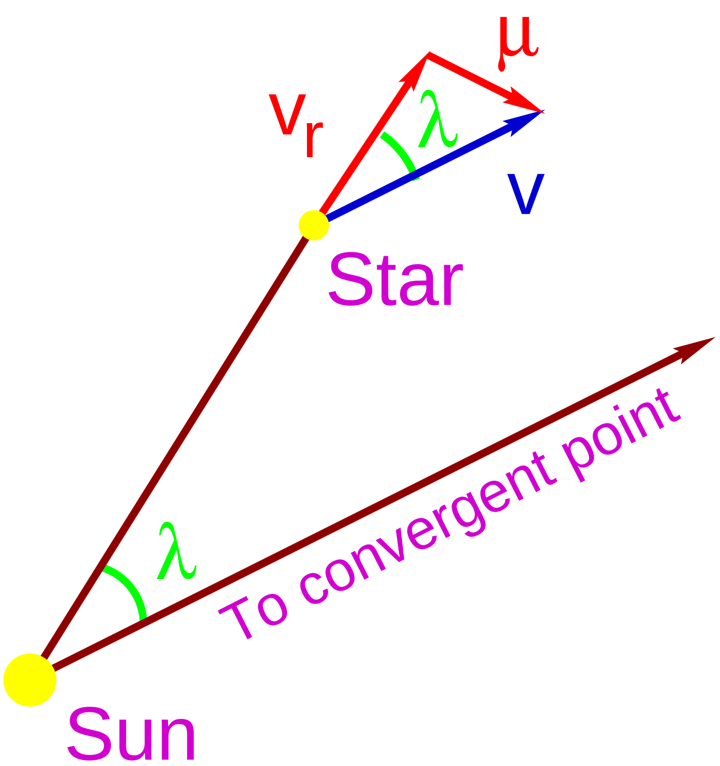
ESA/M. Perryman

Development of the precision of astronomical position measurements



Source: ESA

Moving Cluster



Perspective effect of spatial motion towards convergent point:

$$\tan\lambda = \frac{v_t}{v_r} = \frac{\mu d}{v_r} \tag{5.2}$$

or

$$\frac{d}{1\,\mathrm{pc}} = \frac{v_r/(1\,\mathrm{km/s})\tan\lambda}{4.74\,\mu/(1''/\mathrm{a})} \tag{5.3}$$

Problem: determination of convergent point

Less error prone: moving cluster method = rate of variation of angular diameter of cluster:

$$\dot{\theta} d = \theta v_r \tag{5.4}$$

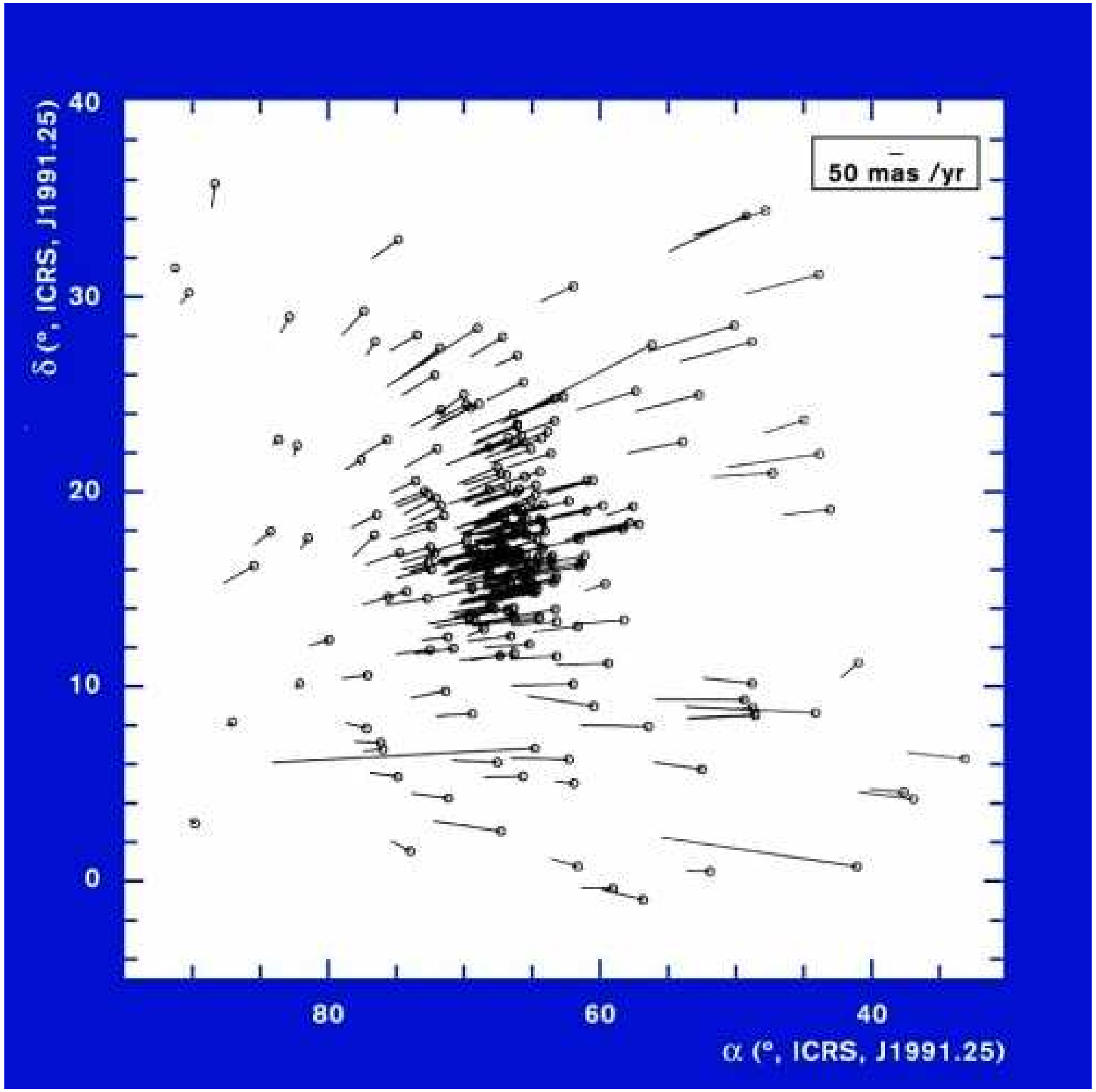
Observation of proper motions gives

$$\frac{\dot{\theta}}{\theta} = \frac{\mathrm{d}\mu_\alpha}{\mathrm{d}\alpha} = \frac{\mathrm{d}\mu_\delta}{\mathrm{d}\delta} \tag{5.5}$$

where $\mu_{\alpha,\delta}$ proper motion in α and δ. Therefore, from Eq. (5.4),

$$d = v_r\,\frac{\dot{\theta}}{\theta} \tag{5.6}$$

Moving Cluster



Application: Distance to Hyades.

Tip of "arrow": Position of stars in 100000 years.

Hanson (1980) finds from this a distance of 46 pc

However: *Hipparcos:* geometric distance to Hyades is $d = 46.34 \pm 0.27$ pc from parallax measurements.

$\Longrightarrow$ Moving cluster method only of historic interest.

Source: ESA

Interlude

Parallax and Moving Cluster: geometrical methods.

All other methods (exception: light echoes): standard candles.

Requirements for standard candles (Mould, Kennicutt, Jr. & Freedman, 2000):

1. Physical basis should be understood.
2. Parameters should be measurable objectively.
3. No corrections (“fudges”) required.
4. Small intrinsic scatter ($\Longrightarrow$ requiring small number of measurements!).
5. Wide dynamic range in distance.

Magnitudes

Assuming isotropic emission, distance and luminosity are related ("inverse square law") $\Longrightarrow$ luminosity distance:

$$F = \frac{L}{4\pi d_{\mathrm{L}}^2} \tag{5.7}$$

where F is the measured flux (erg cm^{-2} s^{-1}) and L the luminosity (erg s^{-1}).

Definition also true for flux densities, I_ν (erg cm^{-2} s^{-1} Å^{-1}).

The magnitude is defined by

$$m = A - 2.5\log_{10} F \tag{5.8}$$

where A is a constant used to define the zero point (defined by $m = 0$ mag for Vega).

For a filter with transmission function ϕ_ν,

$$m_i = A_i - 2.5\log \int \phi_\nu F_\nu \, \mathrm{d}\nu \tag{5.9}$$

where, e.g., $i =$ U, B, V.

Magnitudes

To enable comparison of luminosities: defi ne

absolute magnitude M = magnitude at distance 10 pc

Thus, since $m = A - 2.5\log(L/4\pi d^2)$,

$$M = m - 5\log\left(\frac{d_\mathrm{L}}{10\,\mathrm{pc}}\right) \tag{5.10}$$

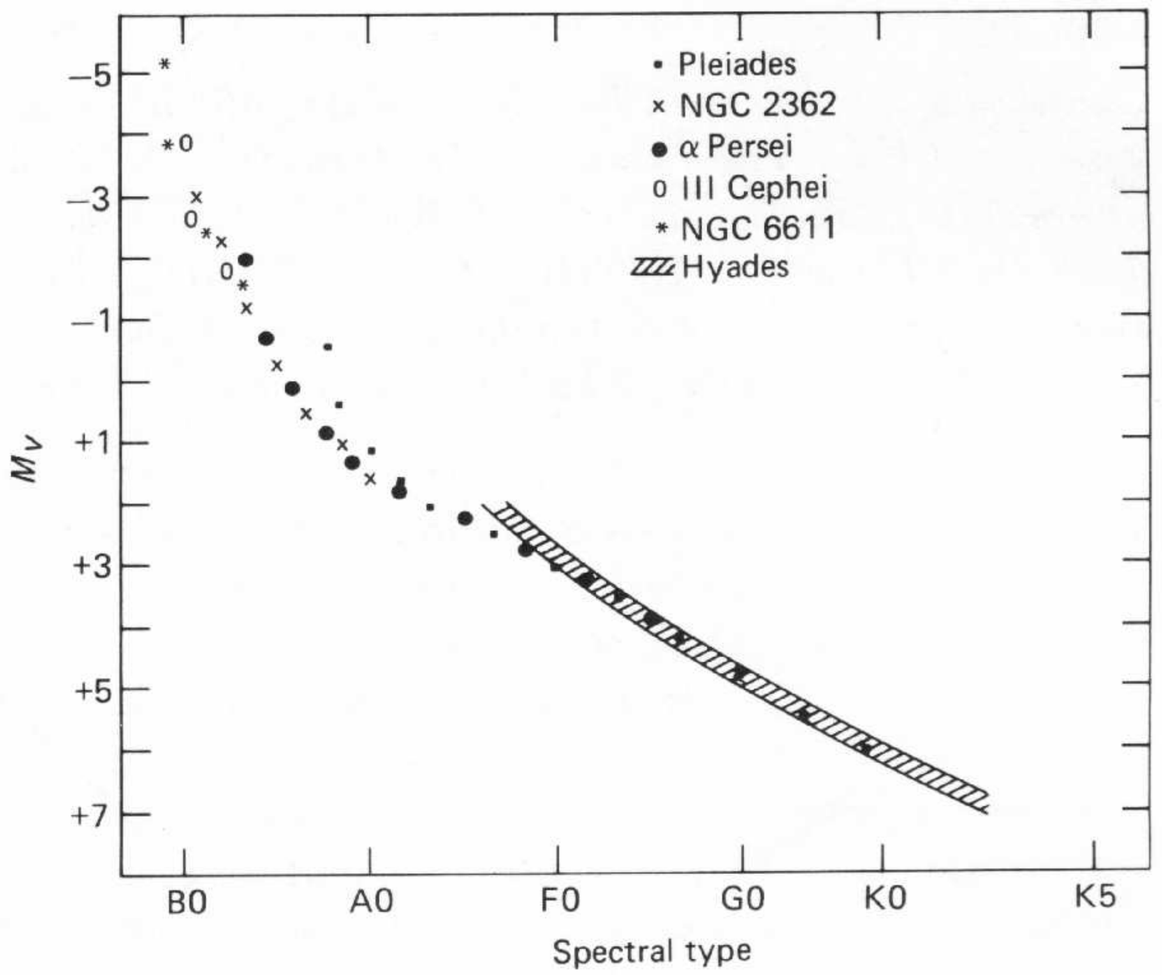
The difference $m - M$ is called the distance modulus, μ_0:

$$\mu_0 = \mathrm{DM} = m - M = 5\log\left(\frac{d_\mathrm{L}}{10\,\mathrm{pc}}\right) \tag{5.11}$$

Often, distances are given in terms of $m - M$, and not in pc.

DM [mag]	3	5	10	15	20	25	30
d	40 pc	100 pc	1 kpc	10 kpc	100 kpc	1 Mpc	10 Mpc

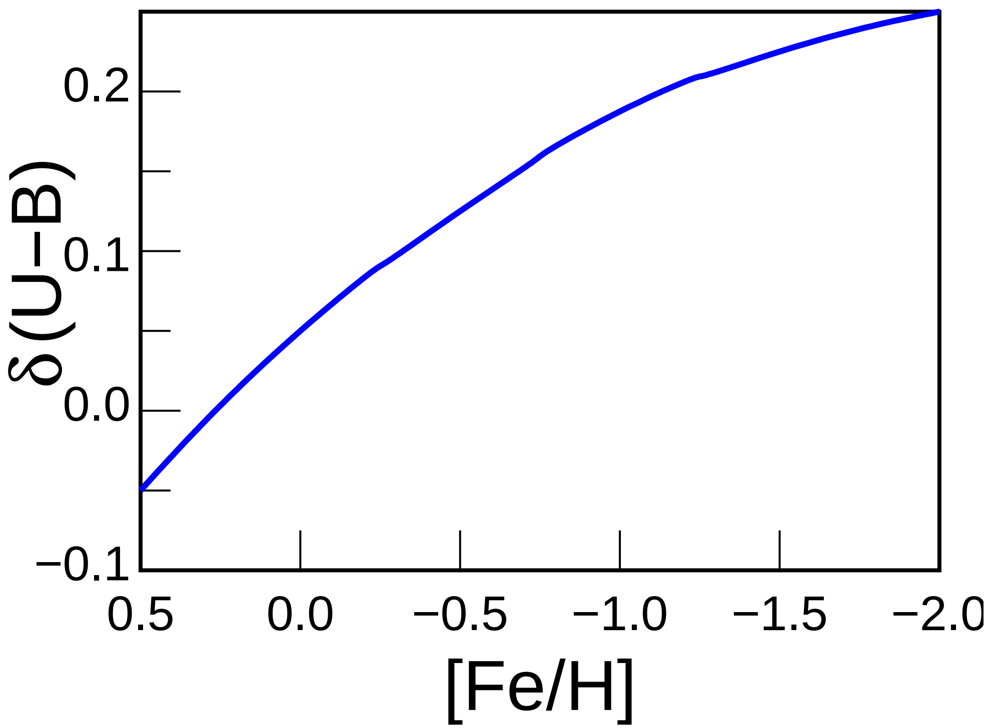
Main Sequence Fitting, I



after Rowan-Robinson (1985, Fig. 2.11)

All open clusters are comparably young
⟹ Hertzsprung Russell Diagram (HRD) dominated by Zero Age Main Sequence (ZAMS).
⟹ Measure HRD (or Color Magnitude Diagram; CMD), shift magnitude scale until main sequence aligns
⟹ distance modulus.

Main Sequence Fitting, II



(after Rowan-Robinson, 1985, Fig. 2.12)

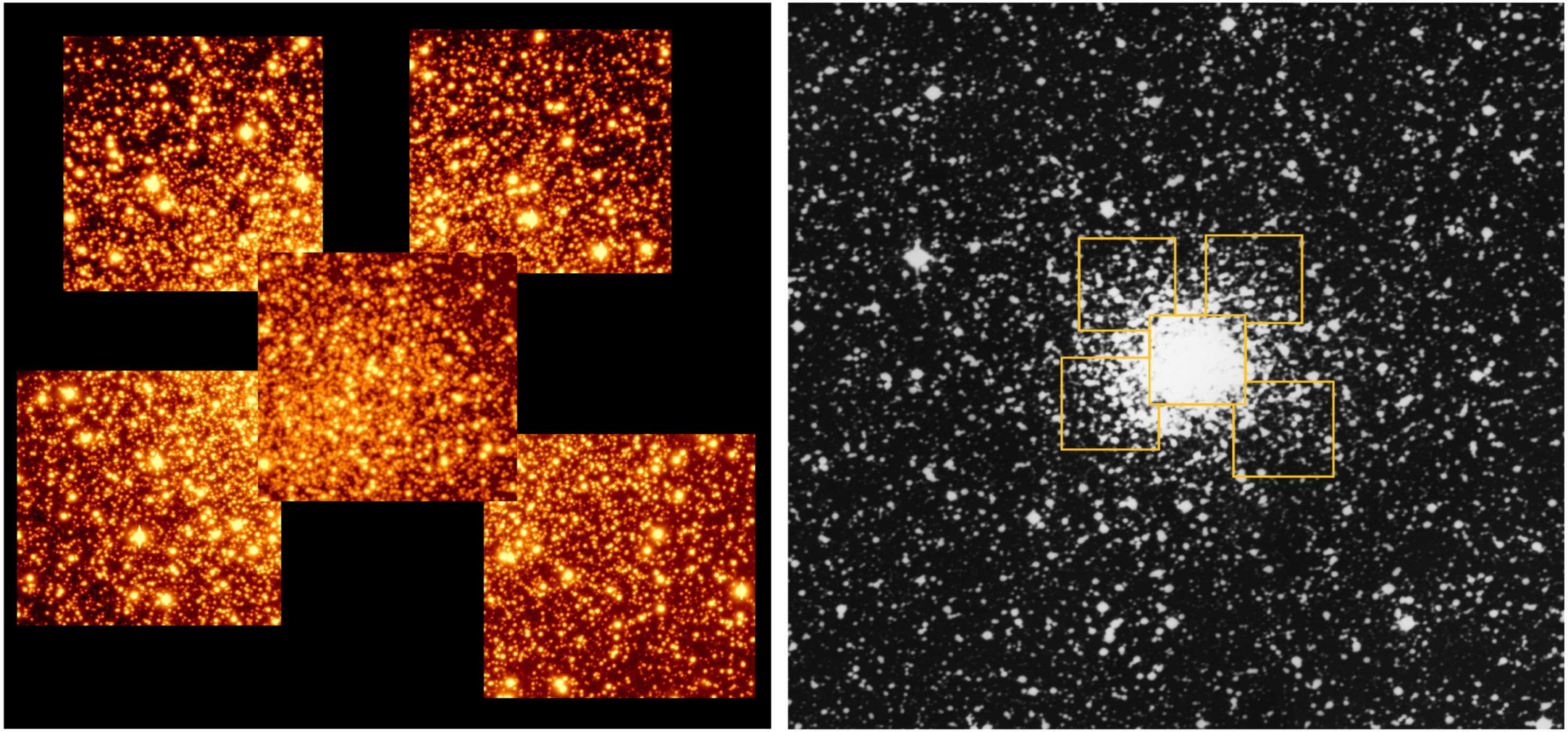
Caveats:

1. Location of ZAMS more age dependent than expected (van Leeuwen, 1999).
2. interstellar extinction
   $\Longrightarrow \mu_0 = \mu_{\mathrm{V}} - A_{\mathrm{V}}$, where μ_{V}, A_{V} DM/extinction measured in V-band.
3. metals: line blanketing (change in stellar continuum due to metal absorption lines, see figure)
   $\Longrightarrow$ Changes color
   $\Longrightarrow$ horizontal shift in CMD.

van den Bergh (1977): $Z_{\mathrm{Hyades}} \sim 1.6 Z_{\odot}$, while other open clusters have solar metallicity $\Longrightarrow$ Cepheid DM were overestimated by 0.15 mag.

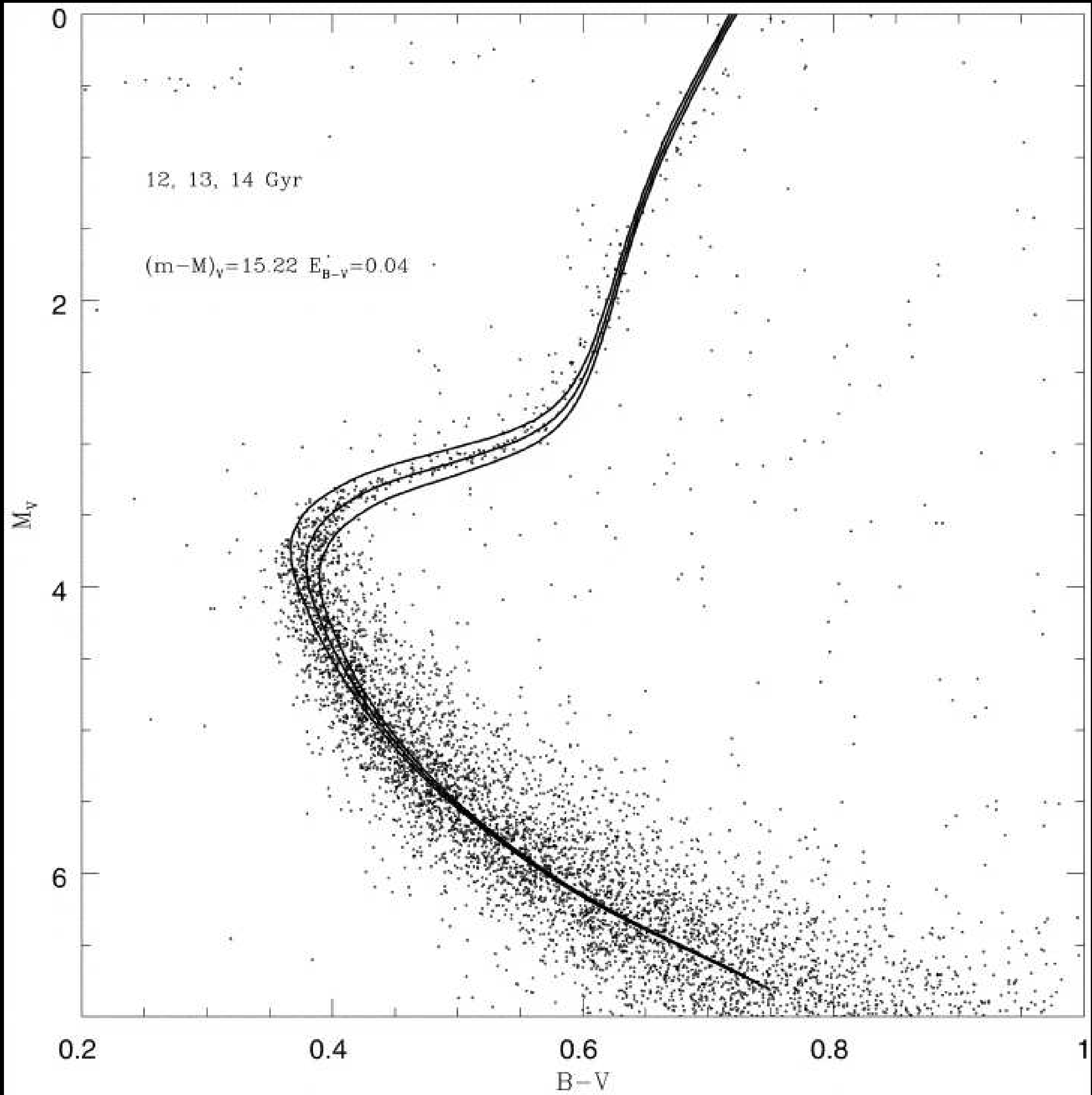
4. identification of unevolved stars crucial (evolution to larger magnitudes on MS during stellar life).

Currently: distances to ~200 open clusters known (Fenkart & Binggeli, 1979), limit ~7 kpc.



Globular Cluster NGC 6712

ESO PR Photo 06a/99 (18 February 1999)



(M68, Straniero, Chieffi & Limongi, 1997, Fig. 11)

Globular clusters: HRD different from open clusters:

- population II
  $\Longrightarrow Z \ll Z_\odot$
- evolved

Use theoretical HRDs (isochrones) to obtain distance.

For distant clusters: MS unobservable
$\Longrightarrow$ position of horizontal branch.

Baade-Wesselink

Basic principle (Baade, 1926): Assume black body

$\Longrightarrow$ Use color/spectrum to get $kT_{\rm eff}$

$\Longrightarrow$ Emitted intensity is Planckian, B_ν

$\Longrightarrow$ Observed Intensity is $I_\nu \propto \pi R_*^2 \cdot B_\nu$.

Radius from integrating velocity profi le of spectral lines:

$$R_2 - R_1 = p \int_1^2 v \, \mathrm{d}t \tag{5.12}$$

(p: projection factor between velocity vector and line of sight).

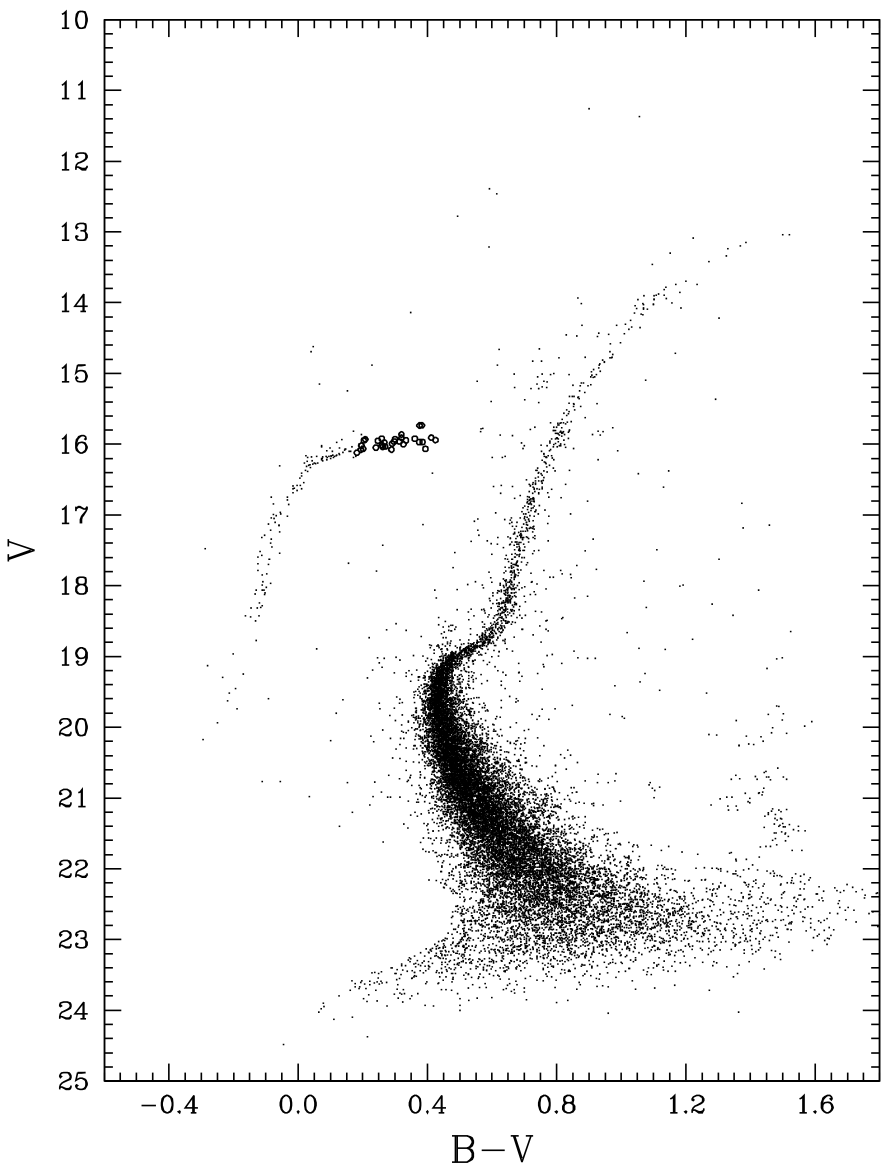
Wesselink (1947): Determine brightness for times of same color

$\Longrightarrow$ rather independent of knowledge of stellar spectrum (deviations from B_ν).

Stars: Calibration using interferometric diameters of nearby giants.

Baade-Wesselink works for pulsating stars such as RR Lyr, Cepheids, Miras, and expanding supernova remnants.

RR Lyr



RR Lyrae variables: Stars crossing instability strip in HRD

$\Longrightarrow$ Variability ($P \sim 0.2 \ldots 1$ d)

$\Longrightarrow$ RR Lyr gap (change in color!).

Absolute magnitude of RR Lyr gap:

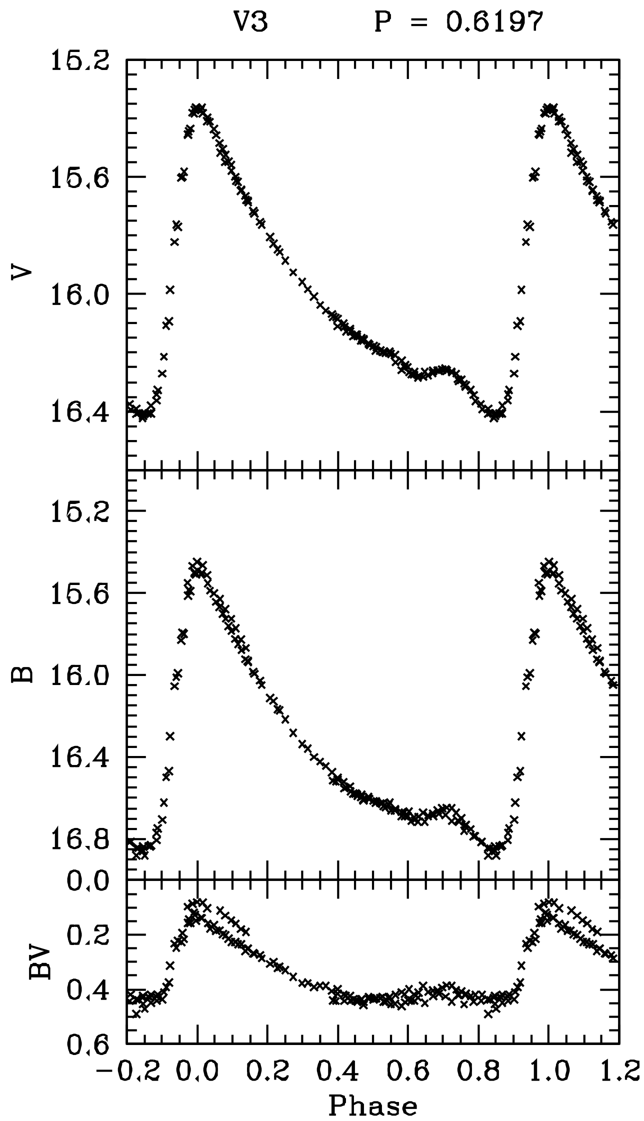
$M_V = 0.6$, $M_B = 0.8$ mag, i.e.,

$L_{RR} \sim 50\,L_\odot$.

M determined from ZAMS fi tting, statistical parallax, and Baade-Wesselink method.

M2: Lee & Carney (1999, Fig. 2)

RR Lyr



Lightcurve shows characteristic color variations over pulsation (temperature change!), and a fast rise, slow decay behavior.

RR Lyr in GCs show bimodal number distribution due to a metallicity effect:

- RRab with $P > 0.5$ d and most probable period of $P_{\text{ab}} \sim 0.7$ d, and
- RRc, with $P < 0.5$ d and $P_{\text{c}} \sim 0.3$ d.

M is larger for higher Z, i.e., metal-rich RR Lyr are *fainter*

$\Longrightarrow$ difference in RR Lyr from population I and II.

RR Lyr work out to LMC and other dwarf galaxies of local group, however, used mainly for globular clusters.

(Lee & Carney, 1999, Fig. 5)

Interlude

Previous methods: Selection of methods for distances within Milky Way (and Magellanic Clouds): Basis for extragalactic distance scale.

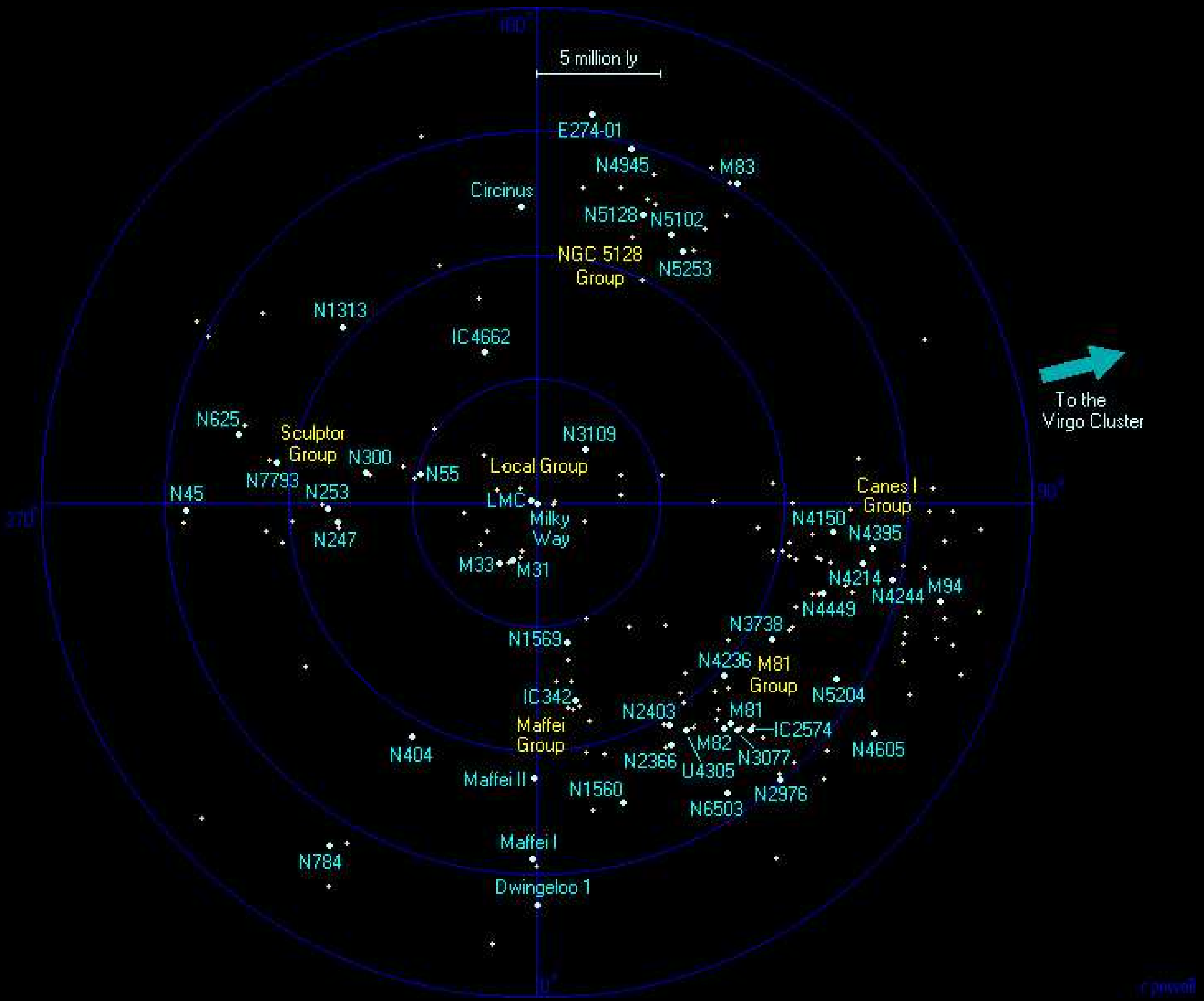
Primary extragalactic distance indicators: Distance can be calibrated from observations *within* milky way or from theoretical grounds.

Primary indicators usually work within our neighborhood (i.e., out to Virgo cluster at 15–20 Mpc).

Examples: Cepheids, light echos,. . .

Secondary extragalactic distance indicators: Distance calibrated from primary distance indicators.

Examples: Type Ia SNe, methods based on integral galaxy properties.



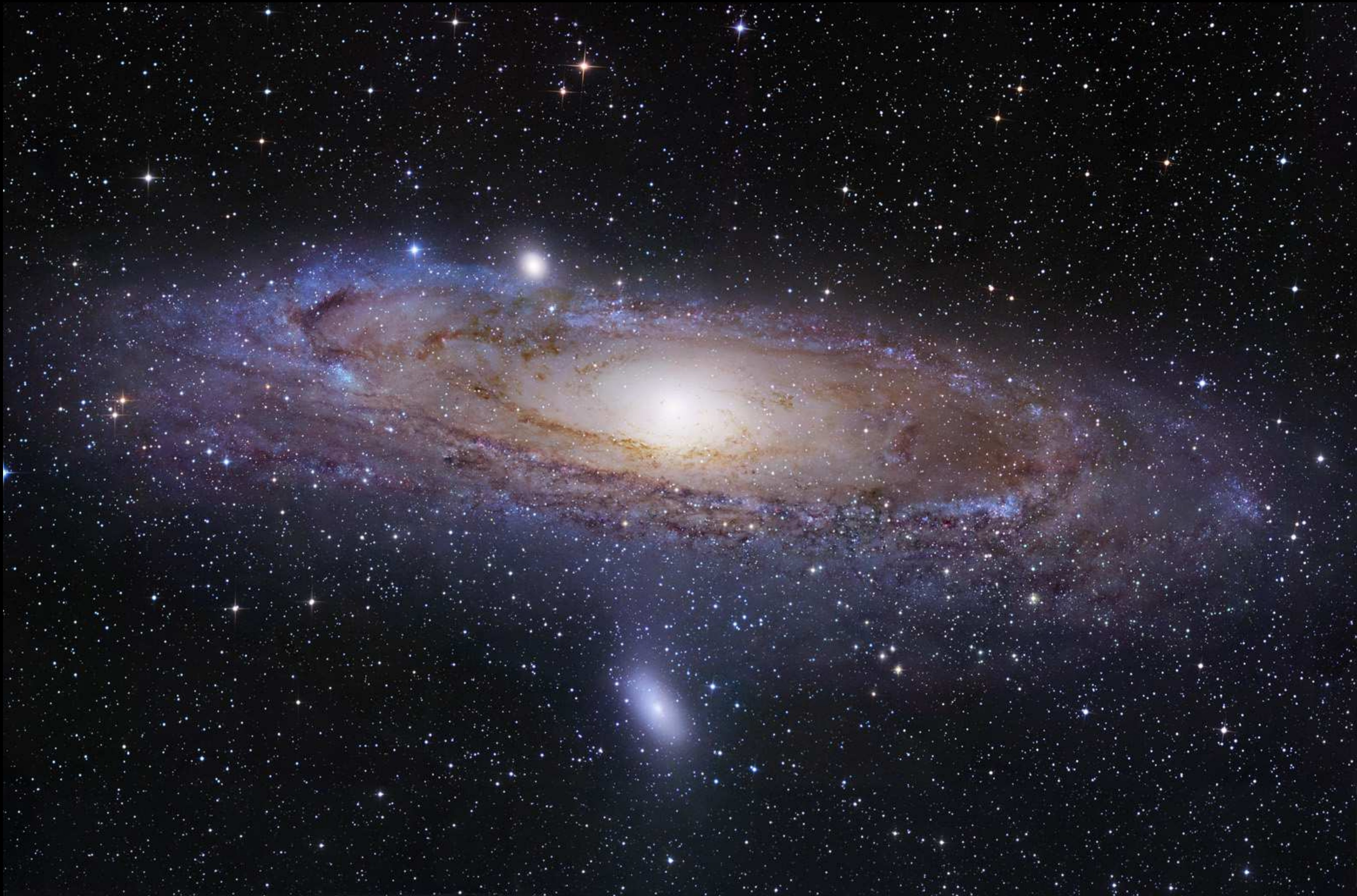
source: `http://www.atlasoftheuniverse.com/galgrps.html`

To get a feel for the distances in our "neighborhood":

50 kpc: LMC, SMC, some other dwarf galaxies

Loke Kun Tan

700 kpc: M31 (Andromeda)



Robert Gendler

the largest astronomical picture ever taken, 21904 $\times$ 14454 pixels

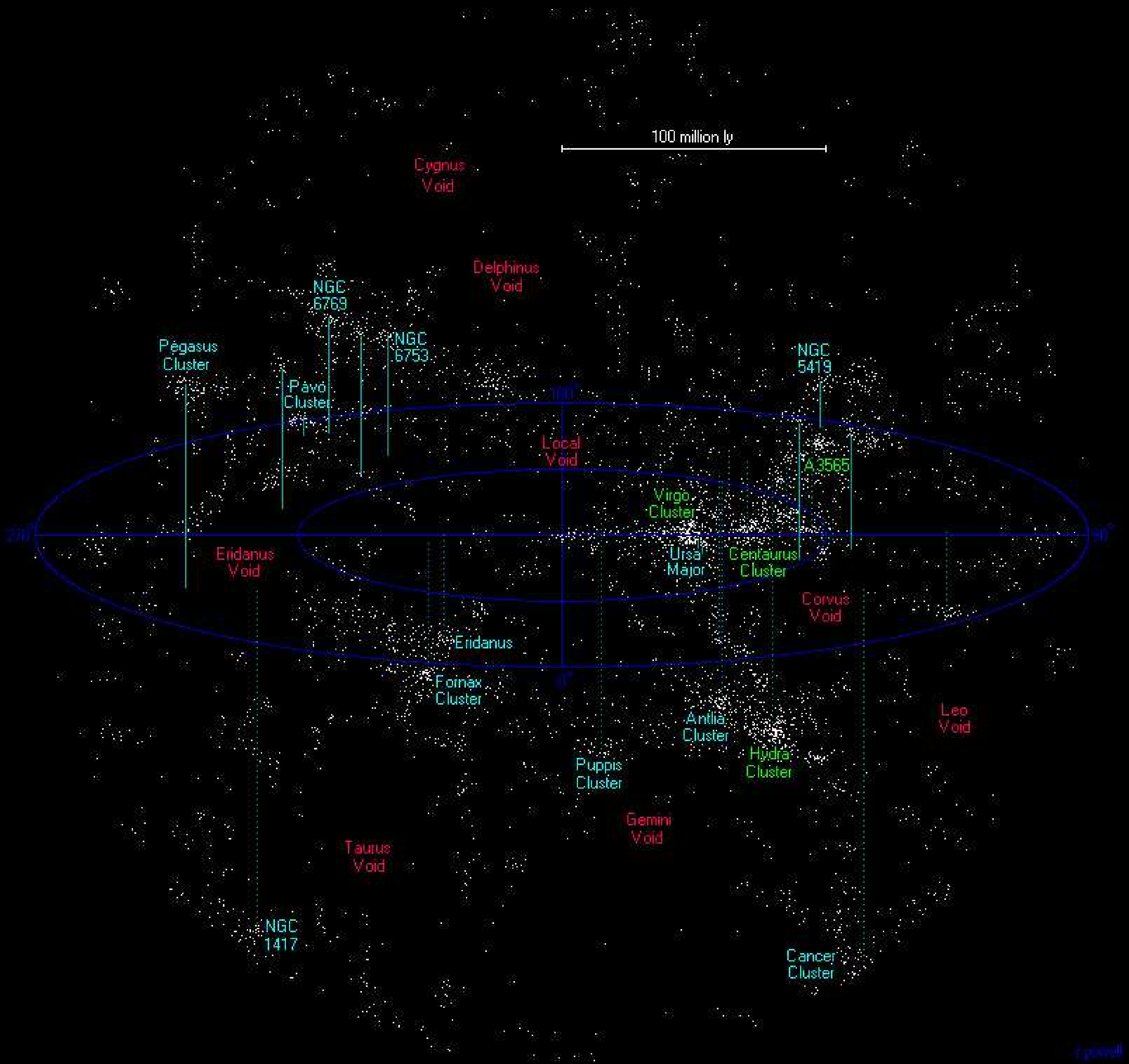
2–3 Mpc: Sculptor and M81 group

(groups similar to local group: a few large spirals, plus smaller stuff).

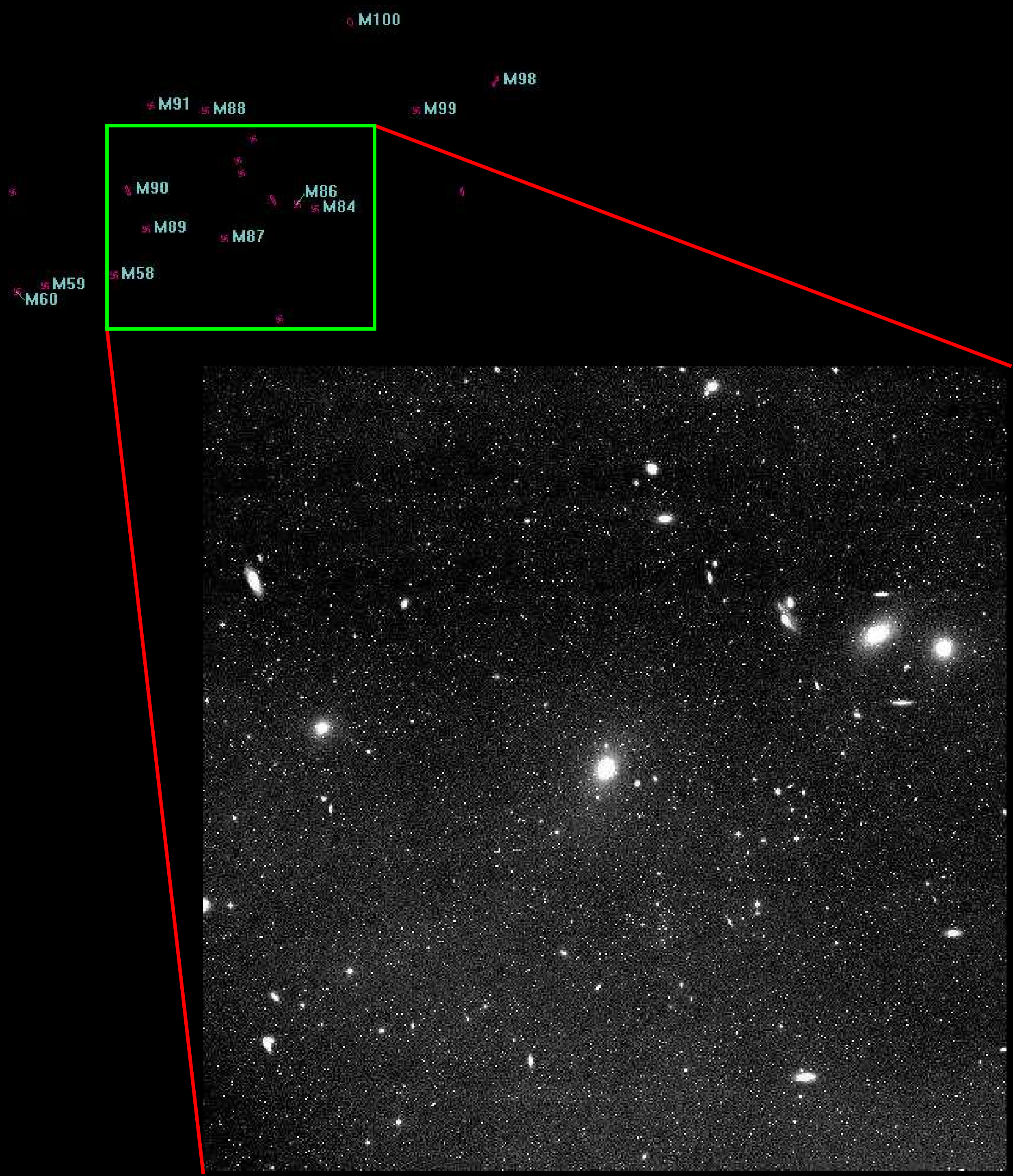
NGC 300 (Sculptor; Laustsen, Madsen, West, 1991)

5–7 Mpc: M101 group (“pinwheel galaxy”). Important because of high L.

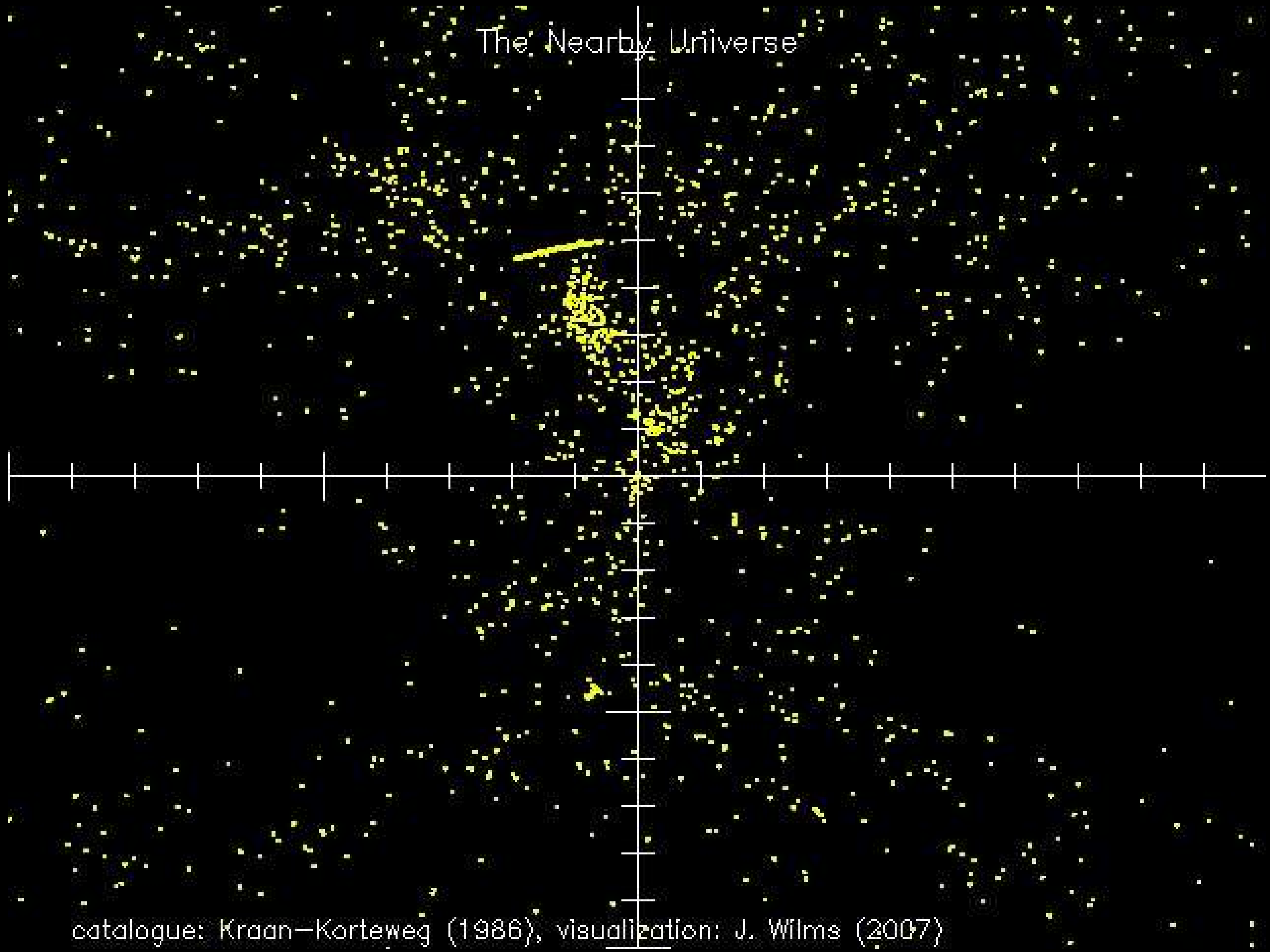
Adam Block/NOAO/AURA/NSF



source: http://www.atlasoftheuniverse.com/200mill.html

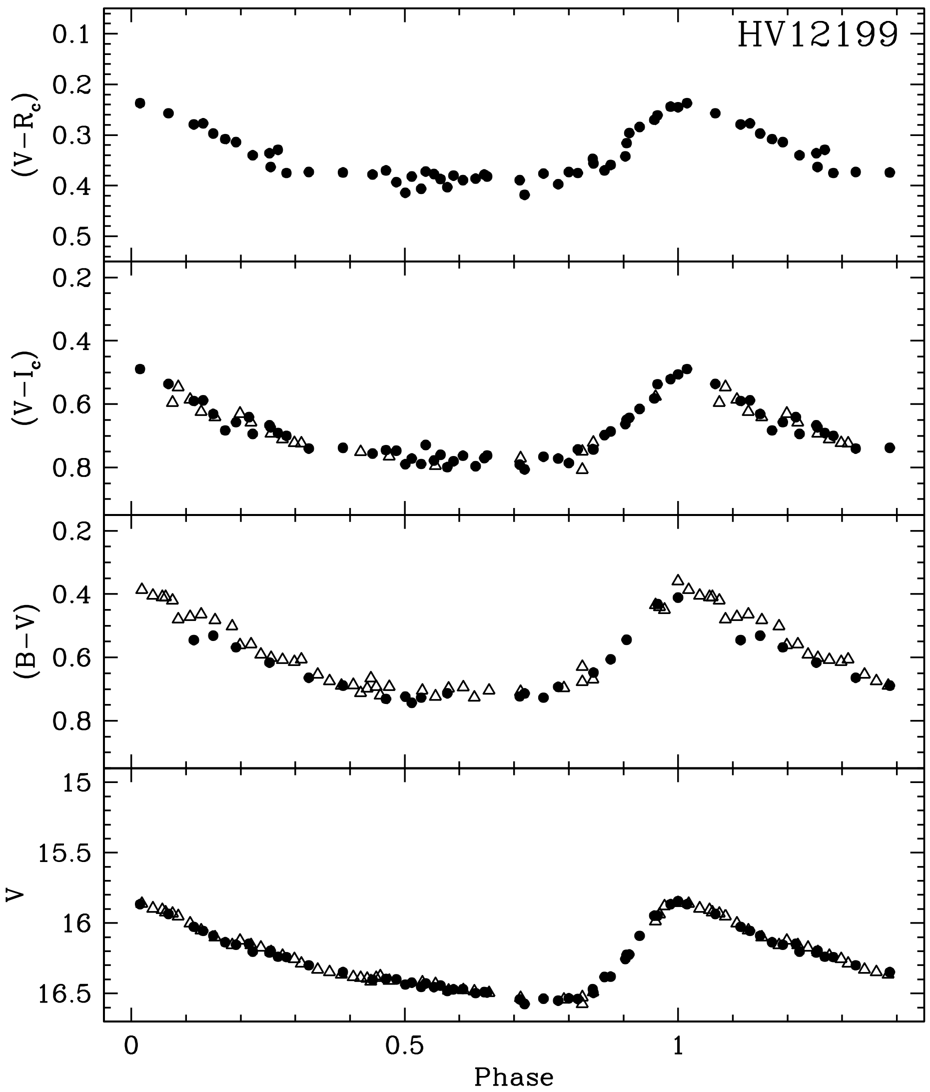


15–20 Mpc: Virgo cluster.



The Nearby Universe
catalogue: Kraan-Korteweg (1986), visualization: J. Wilms (2007)

Cepheids, I

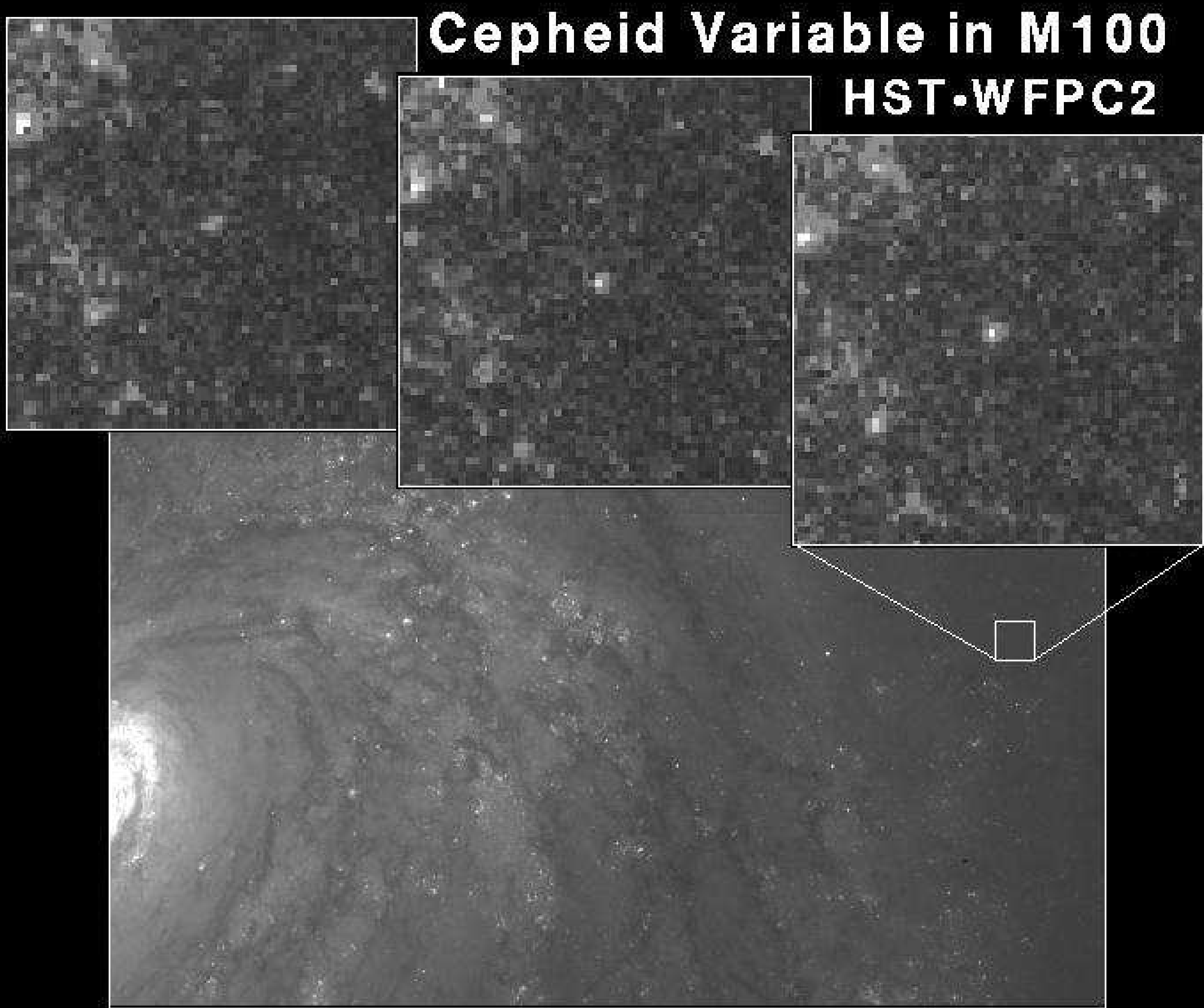


Cepheids:

- Luminous stars ($L \sim 1000\,L_\odot$) in instability strip
  (He II–He III ionization)
- large intensity amplitude variation,
- $P \sim 2 \ldots 150\,\mathrm{d}$ (easily measurable).

Review: Feast (1999).

(Gieren et al., 2000, Fig. 3)


Cepheid Variable in M100
HST•WFPC2

STScI

Cepheids, IV

© ASP

Henrietta Leavitt (1868–1921):

- Graduated from Radcliffe College
- from 1895: volunteer at Harvard Observatory
- was ill, and partially deaf as a result
- 1902: back at Harvard Obs
- discovered 1777 variable stars in LMC
- 1912: discovered Period-Luminosity relation of Cepheids in SMC, but was not allowed to follow this up
- later: defi ned Harvard photographic magnitude system
- died of cancer in 1921

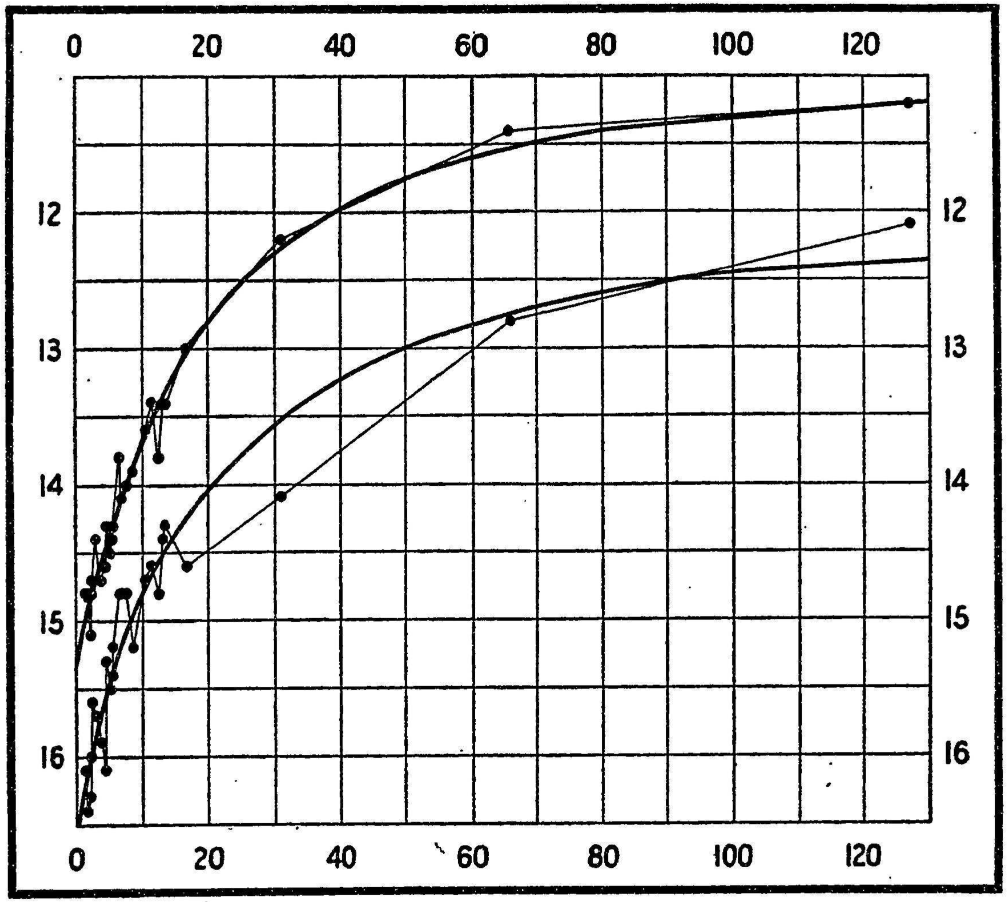


FIG. 1.

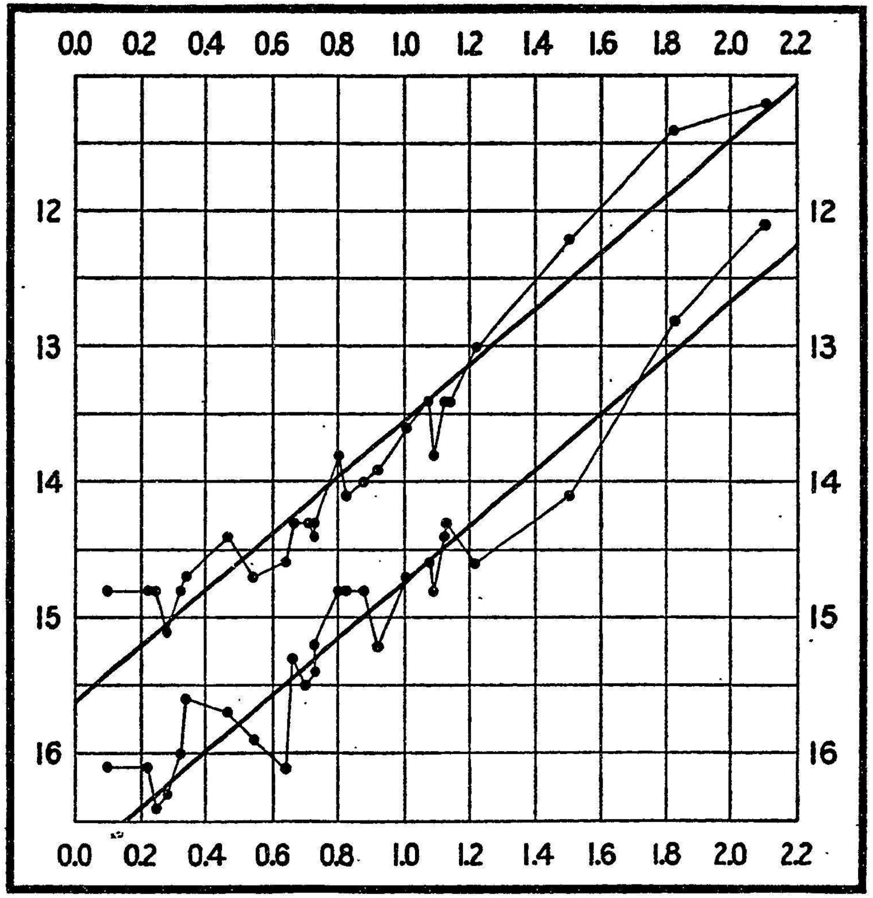
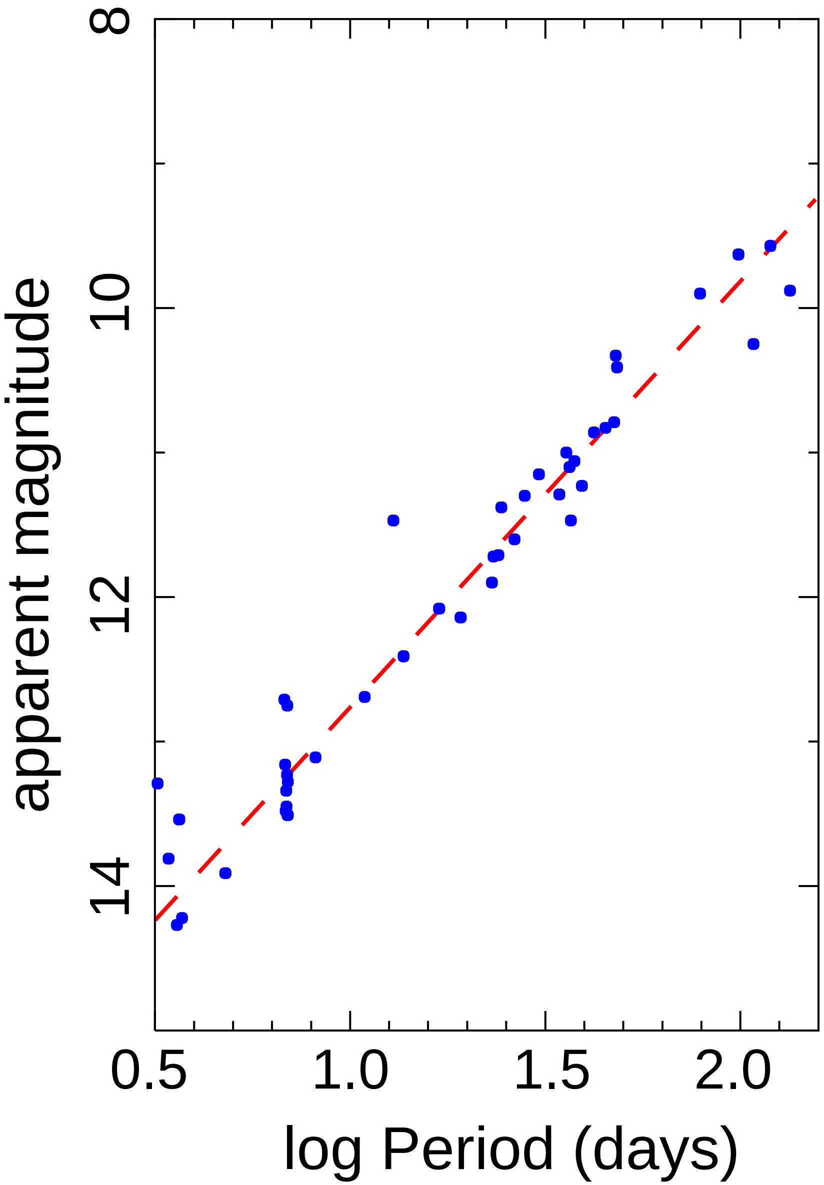


FIG. 2.

X-axis: period in days, Y-axis: magnitude

Leavitt & Pickering, 1912, Periods of 25 Variable Stars in the Small Magellanic Cloud, Harvard College Observatory Circular, vol. 173, pp. 1–3

Cepheids, VI



Period-Luminosity (PL) relation:
$M_{\mathrm{V}} \propto -2.76 \log P$.

Low luminosity Cepheids have lower periods.

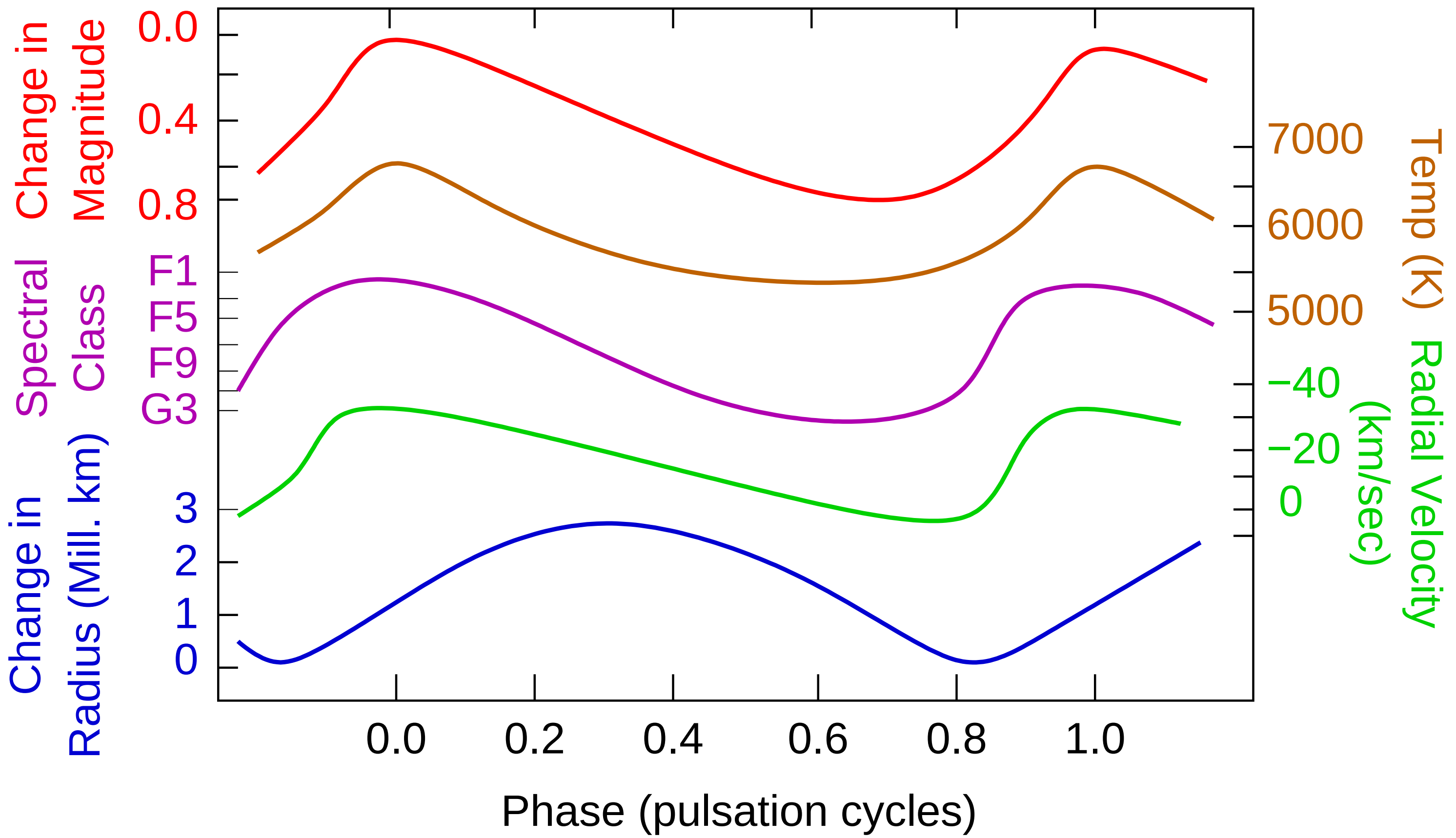
There are indications that there is also an influence of the color

$\Longrightarrow$ Period-Luminosity-Color (PLC) relation

Note: W Vir stars, also called type II Cepheids = "little brother of Cepheids" (present in globular clusters). Less luminous than normal Cepheids, similar PLC relation, fi rst confused with Cepheids $\Longrightarrow$ Cause for early thoughts of much smaller universe.

PL relation for the LMC Cepheids (after Mould, Kennicutt, Jr. & Freedman, 2000, Fig. 2).

Cepheids, VII



after `http://csep10.phys.utk.edu/astr162/lect/index.html`

Typical variation of measurable parameters over one pulsation.

Cepheids, VIII

Physics of Period-Luminosity-Color relation:

Star pulsates such that outer parts remain bound:

$$\frac{1}{2}\left(\frac{R}{P}\right)^2 \lesssim \frac{GM}{R} \quad \Longrightarrow \quad \frac{M}{R^3} \propto P^{-2} \tag{5.13}$$

where P period. Therefore:

$$P \propto \rho^{-1/2} \quad \Longleftrightarrow \quad P\rho^{1/2} = Q \tag{5.14}$$

(Q: pulsational constant, $\rho \propto MR^{-3}$ mean density). But Radius R related to luminosity L:

$$L = 4\pi R^2 \sigma T^4 \quad \Longrightarrow \quad R \propto L^{1/2} T^{-2} \tag{5.15}$$

Inserting everything into Eq. (5.14) gives:

$$PL^{-3}T^3 = \text{const.} \Longleftrightarrow \quad \log P - 3\log L + 3\log T = \text{const.} \tag{5.16}$$

But: *bolometric magnitude:* $M_{\text{bol}} \propto -\log L$, and *colors:* $\text{B} - \text{V} \propto \log T$ such that

$$c_1 \log P + c_2 M_{\text{bol}} + c_3(\text{B} - \text{V}) = \text{const.} \tag{5.17}$$

where $c_{1,2,3}$ calibration constants.

Cepheids, IX

Calibration: Need slope and zero point of PLC.

Slope: Observations of nearby galaxies (e.g., open clusters in LMC)

Zero point is difficult:

- Cepheids in galactic clusters, distance to these via ZAMS fitting
  $\Longrightarrow$ problematic due to age dependency of ZAMS.
- Hipparcos: geometrical distances
  $\Longrightarrow$ problematic due to low SNR (resulting in 9% systematic error.
- Baade-Wesselink using IR info (low metallicity dependence).

Typical relations (Mould et al., 2000, 32 Cepheids):

$$\begin{aligned} M_\mathrm{V} &= -2.76 \log P - 1.40 + C(Z) \\ M_\mathrm{I} &= -3.06 \log P - 1.81 + C(Z) \end{aligned} \tag{5.18}$$

The metallicity (color) dependence is roughly

$$(m - M)_\mathrm{true} = (m - M)_\mathrm{PL} - \gamma \log Z/Z_\mathrm{LMC} \tag{5.19}$$

where $\gamma = -0.11 \pm 0.03\,\mathrm{mag/dex}$ (Z: metallicity) (=Cepheids with larger Z are fainter).

Cepheids, X

Notes:

1. Is the pulsational constant a constant? (or is $Q = Q(\rho, P)$?):

   $\Longrightarrow$ possible deviation from PLC, especially at high luminosity

   $\Longrightarrow$ adds uncertainty at large distances.

2. M_V depends on metallicity

   $\Longrightarrow$ LMC Cepheids *are* bluer [$Z_\mathrm{LMC} < Z_\odot$]), but the exact value of γ in Eq. (5.19) is very uncertain.

   For V and I magnitudes, most probably $\delta(m - M)_0/\delta[\mathrm{O/H}] \lesssim -0.4$ mag dex^{-1}, however, others find $+0.75$ mag dex^{-1}, see Ferrarese et al. (2000) for details...

3. Stellar evolution unclear (multiple crossings of instability strip are possible).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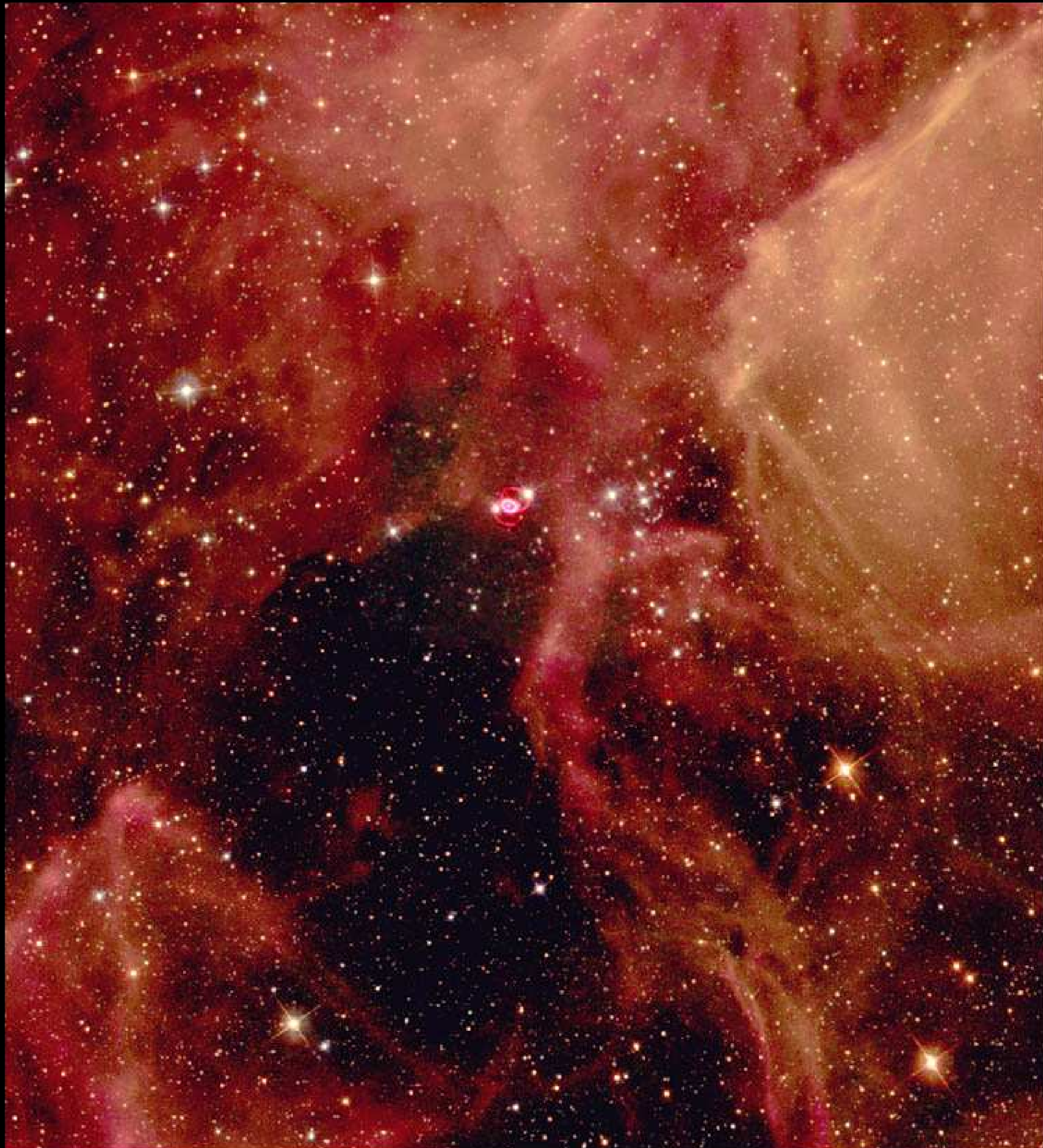
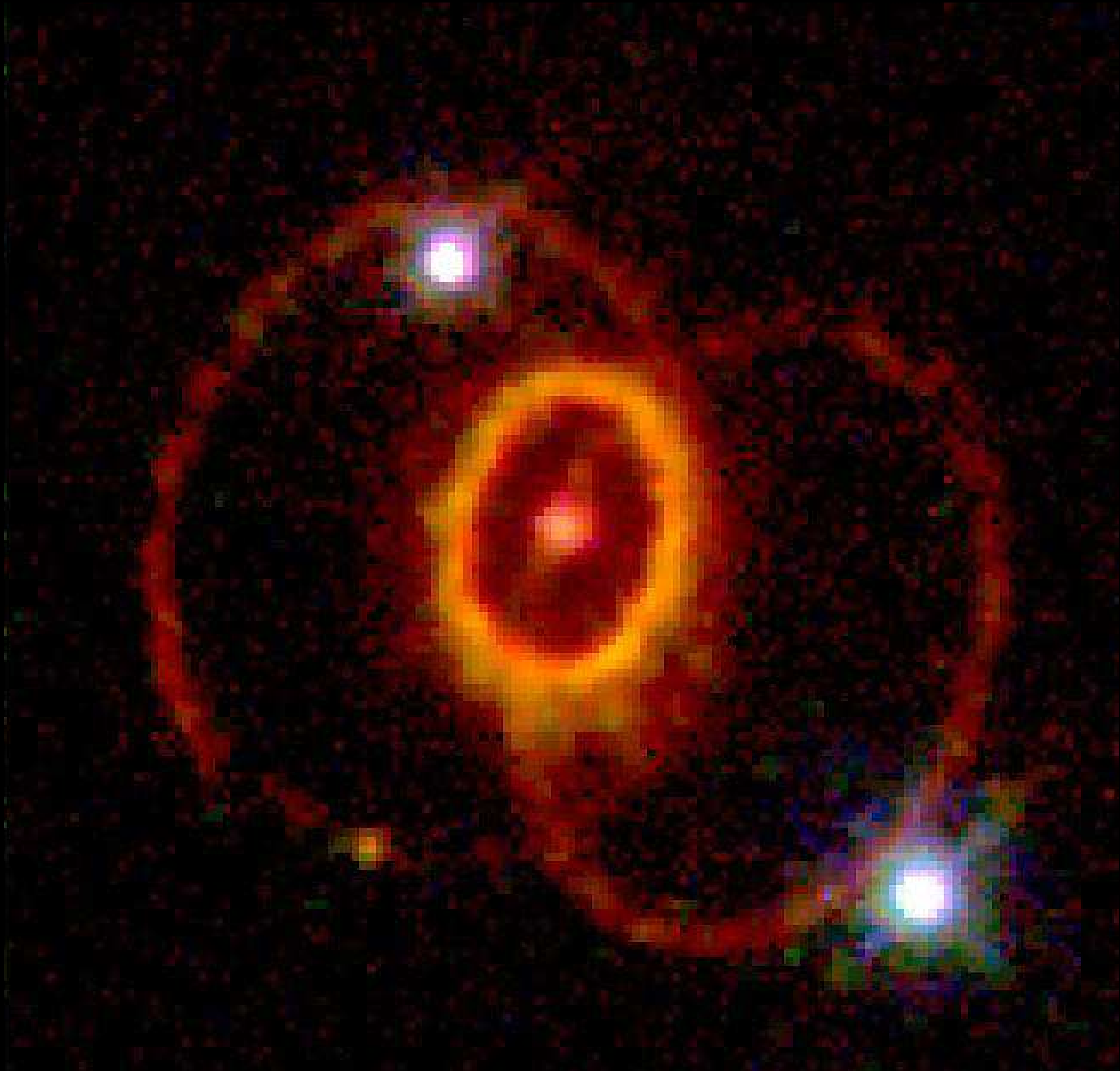
1987 February: Supernova in Large Magellanic Cloud.



HST/NASA/ESA/STScI



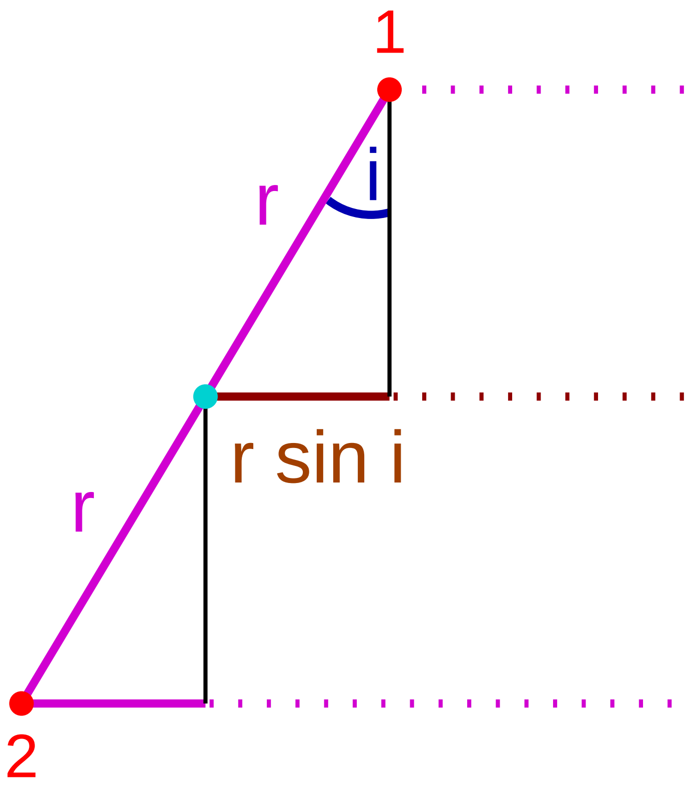
STScI PR94-22

87 d after explosion: Ring ($1.66'' \times 1.21''$) of ionized C and N around SN

$\Longrightarrow$ Excitation of C, N in ring-like shell (ejecta from red giant phase of progenitor?): “light echo”

Light echos, V

Light echo: direct geometrical determination of distance to LMC possible:



Time delay SN: close side of ring:

$$ct_1 = r(1 - \sin i) = 86 \pm 6\,\mathrm{d} \tag{5.20}$$

Time delay SN: far side of ring:

$$ct_2 = r(1 + \sin i) = 413 \pm 24\,\mathrm{d} \tag{5.21}$$

The ring radius is:

$$r = c\,\frac{t_1 + t_2}{2} = 250 \pm 12\,\mathrm{lt\,d} \tag{5.22}$$

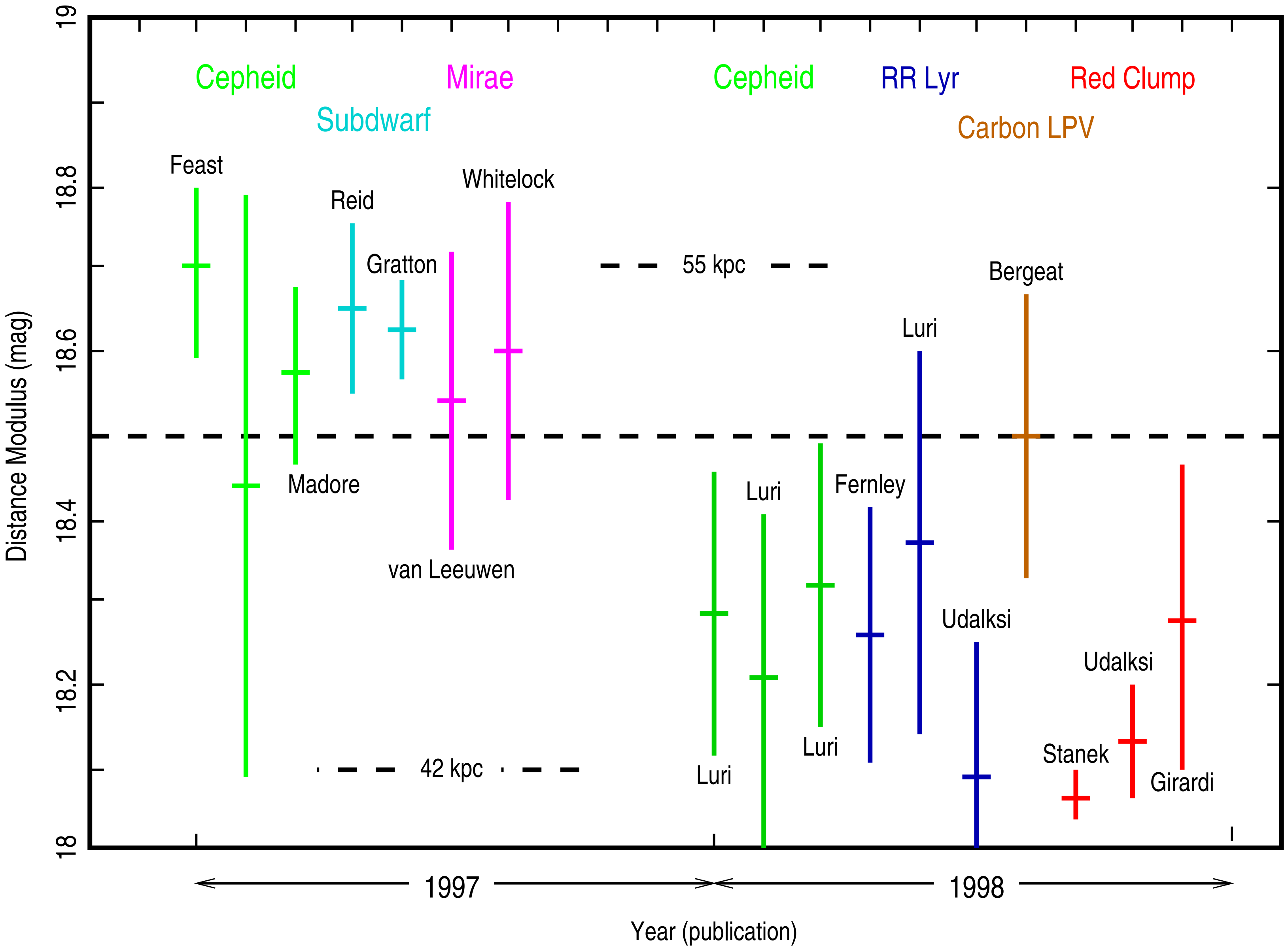
and the inclination is:

$$\sin i = \frac{t_2 - t_1}{t_1 + t_2} \quad\Longrightarrow\quad i \sim 41^\circ \tag{5.23}$$

(From ring-geometry: $\cos i = 1''\!.21/1''\!.66 \Longrightarrow i \sim 43^\circ$)

Thus from angular size of ring:

$$1''\!.66 = \frac{r\cos i}{d} \quad\Longrightarrow\quad d = 52 \pm 3\,\mathrm{kpc} \tag{5.24}$$



LMC distance: "anchor point" of extragalactic distance scale.

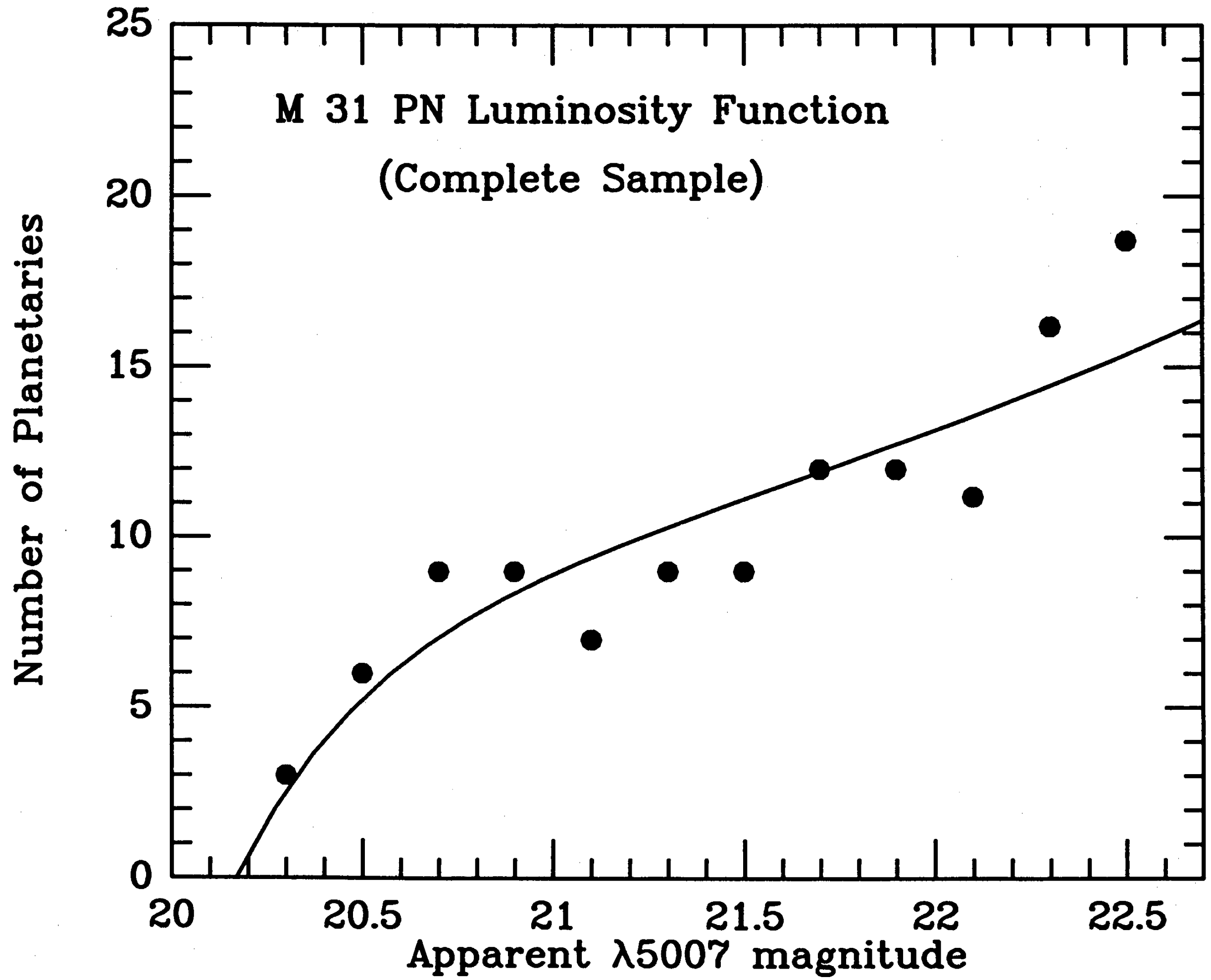
After Gaia Science Workgroup

Strong dependence on Hipparcos calibration.

DM ranges between 18.7 ± 0.1 mag (Feast & Catchpole) and 18.57 ± 0.11 mag (Madore & Freedman)

Currently, the distance to the LMC is less well known than desirable.

PN Luminosity Function, I



Planetary Nebulae have empirical universal luminosity function.

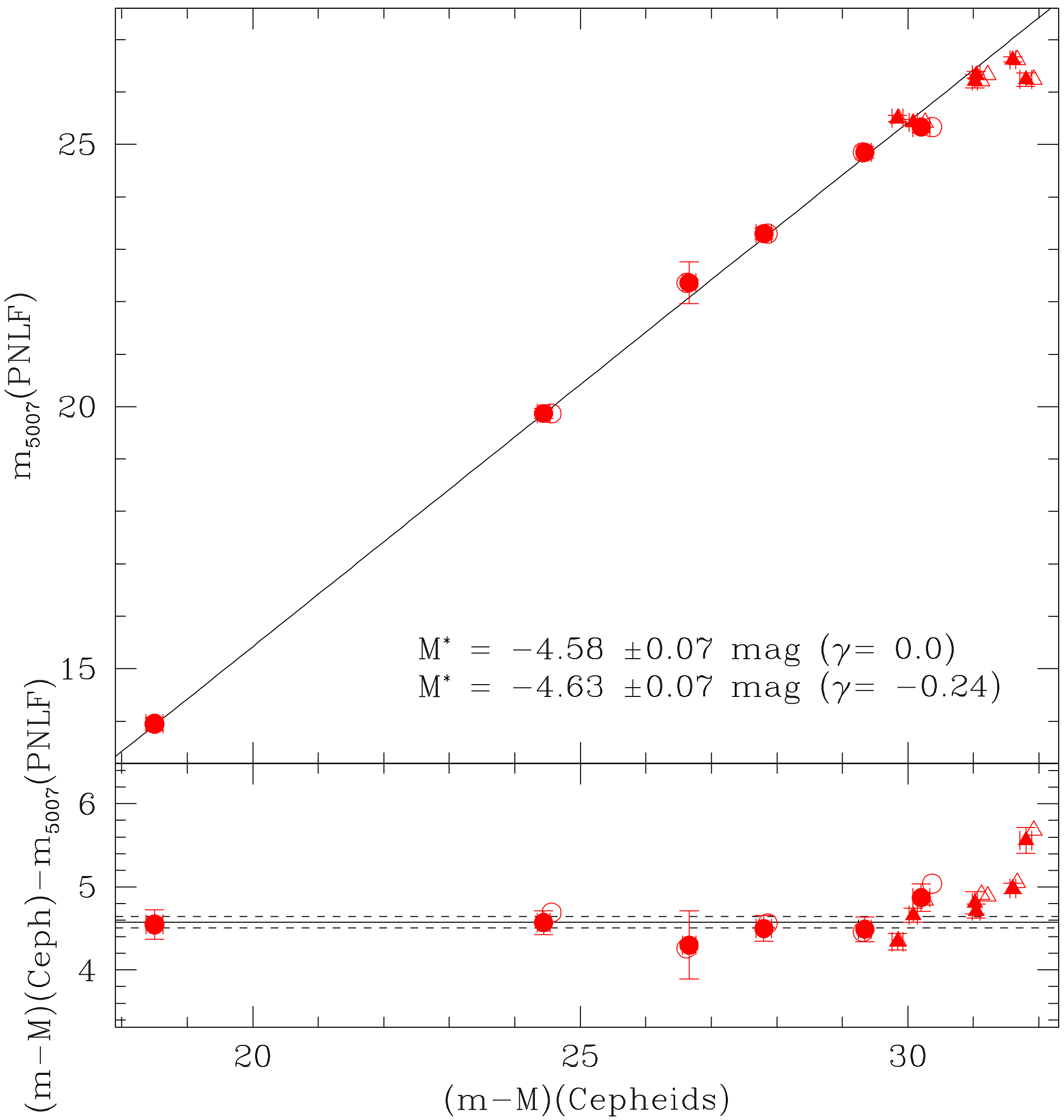
Measurement of "cutoff magnitude" M_{PN}
$\Longrightarrow$ Standard candle!

PN detection with narrow band filter of $\text{O[III]}\lambda 5007\text{Å}$.

(Ciardullo et al., 1989, Fig. 4)

$$N(M) \propto \mathrm{e}^{0.307M} \left(1 - \mathrm{e}^{3(M_{\text{PN}}-M)}\right) \tag{5.25}$$

PN Luminosity Function, II



Result of calibration using Cepheid distances (Ferrarese et al., 2000):

Cutoff of luminosity function:

$$M_{\mathrm{PN}} = -4.58 \pm 0.13\,\mathrm{mag} \tag{5.26}$$

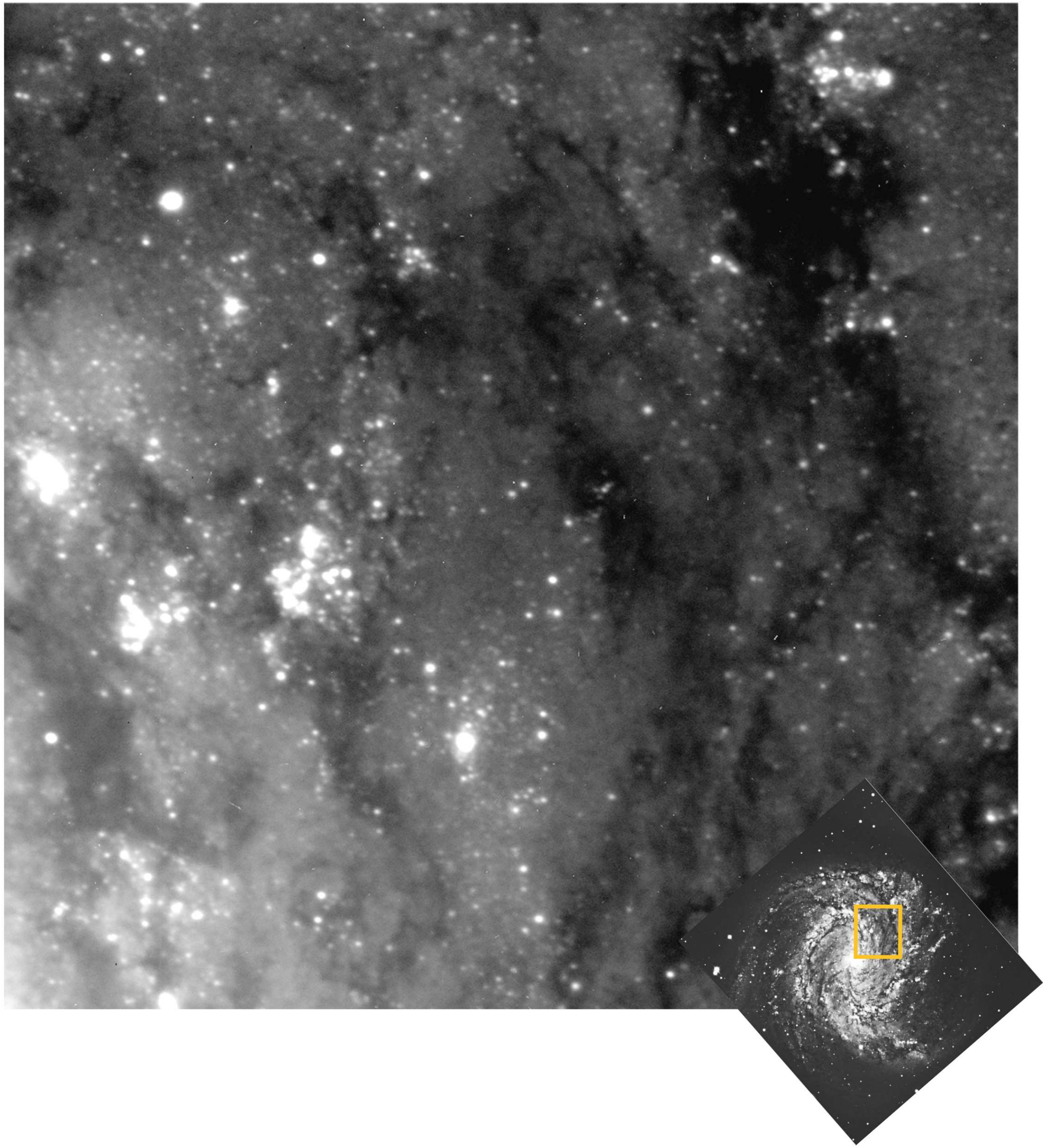
Works out to ∼40 Mpc with 8 m class telescope.

(Ferrarese et al., 2000, Fig. 3), left to right: LMC, M31, NGC 300, M81, M101, NGC 3368, and several galaxy groups.

PN Luminosity Function, III

Caveats: Effects of metallicity, population age, parent galaxy most probably small, but

- Contamination by H II regions (but distinguish using Hα/[O III] ratio.
- Background emission-line galaxies at $z = 3.1$
- intracluster PNe (i.e., PNe outside galaxies)



The VLT Looks Deep into a Spiral Galaxy

ESO PR Photo 20/98 (23 June 1998)

Brightest Stars, II

Brightest Stars= O, B, A supergiants, absolute magnitudes usable in local group, although there is a large scatter.

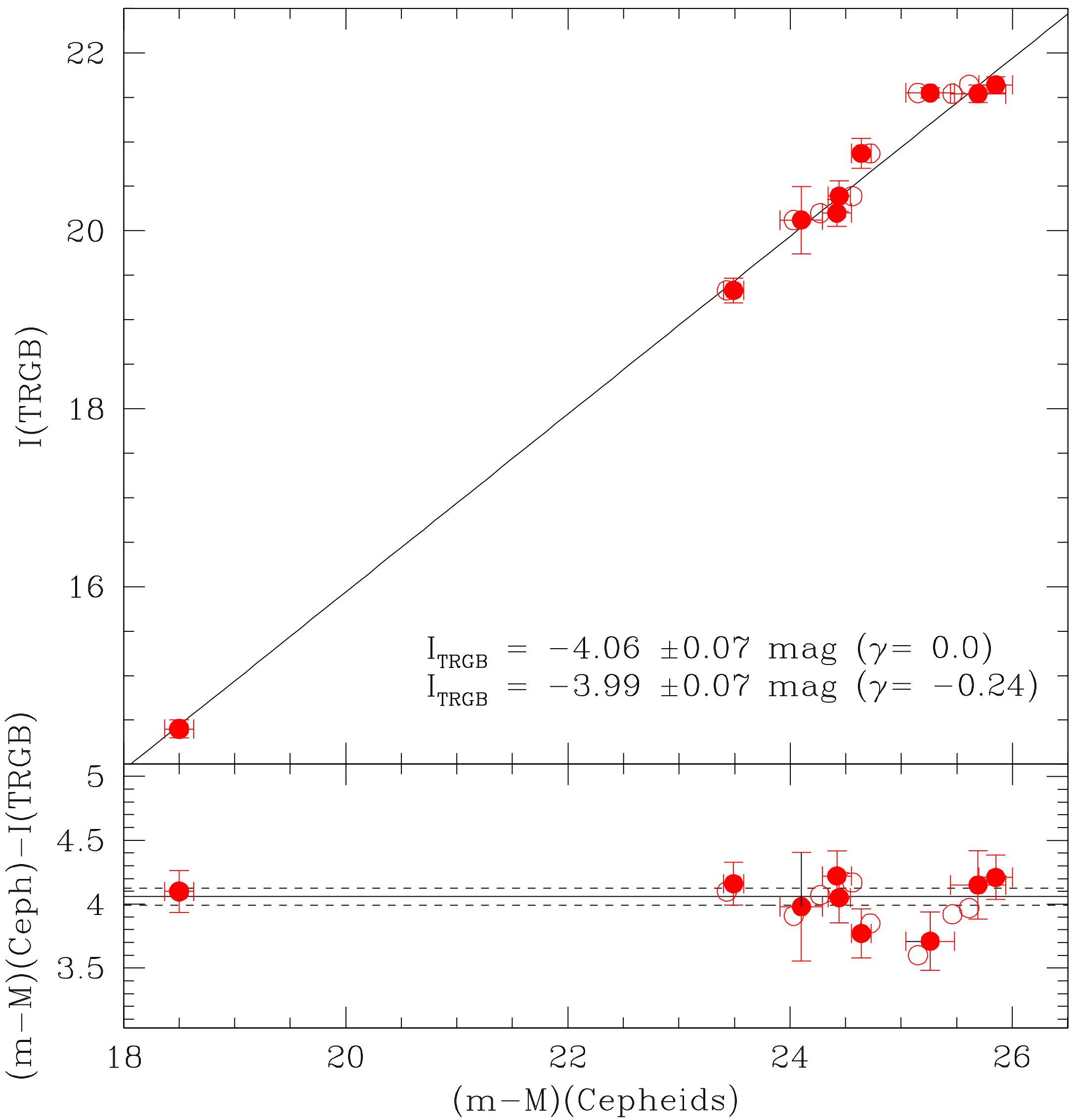
Reason: there is an upper limit to stellar luminosity due to mass loss in supergiants.

Possible Improvement: Strength of Balmer series lines. Hα and Hβ appear biased (class of supergiants with anomalously strong Balmer lines?).

Problems:

- Contamination by foreground halo stars
  $\Longrightarrow$ Choose stars with unusual color (rare, i.e. less foreground contamination): B $-$ V $<$ 0.4 or B $-$ V $>$ 2.0 $\Longrightarrow$ Tip of Red Giant Branch
- Internal extinction.
- Scatter in max. L
  $\Longrightarrow$ Average over brightest N stars (Sandage, Tammann: $N = 3$).
- Metallicity dependence.

Brightest Stars, III



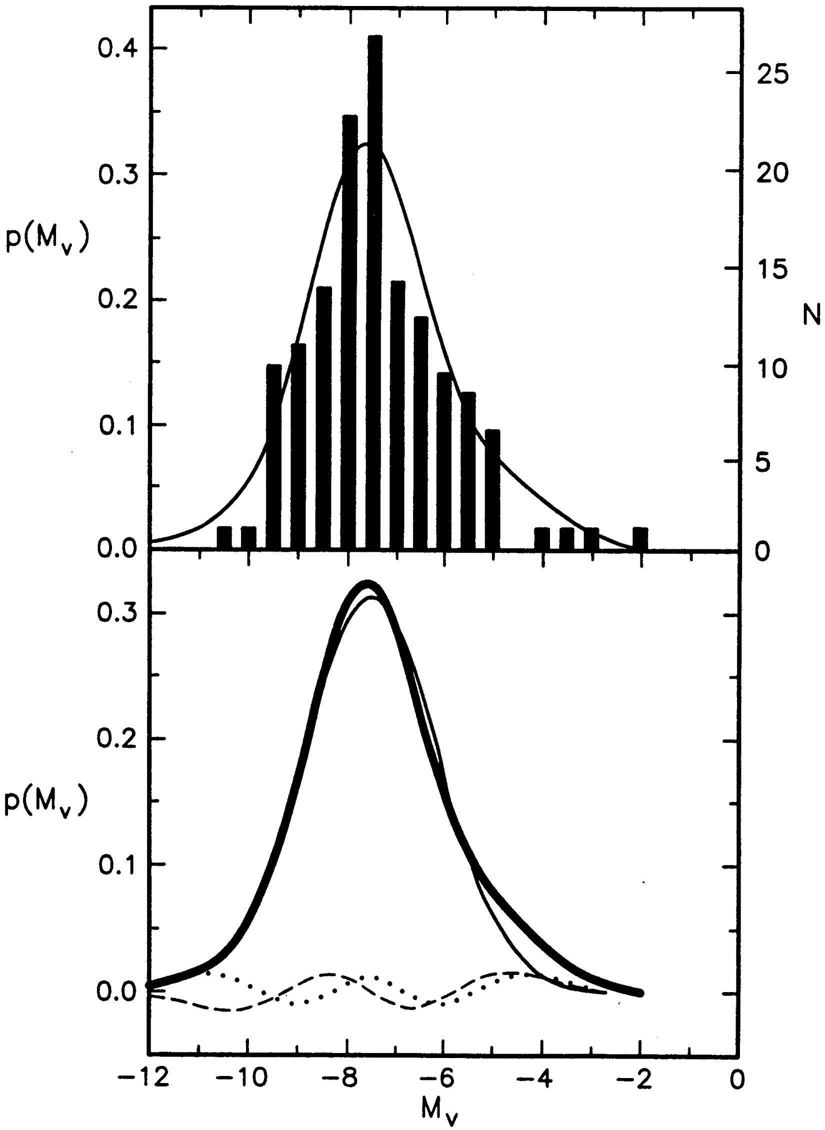
Tip of Red Giant Branch: Usable within local group, possibly out to Virgo.

Calibration:

$$M_{\mathrm{I}} = -4.06 \pm 0.13\,\mathrm{mag} \quad (5.27)$$

(Ferrarese et al., 2000, Fig. 1)

Globular Clusters



Globular Cluster Luminosity Function is ∼Gaussian
$\Longrightarrow$ Use maximum of distribution ("turnover magnitude", M_T) as standard candle.
From Virgo and Fornax Cepheid distances (Ferrarese et al., 2000):

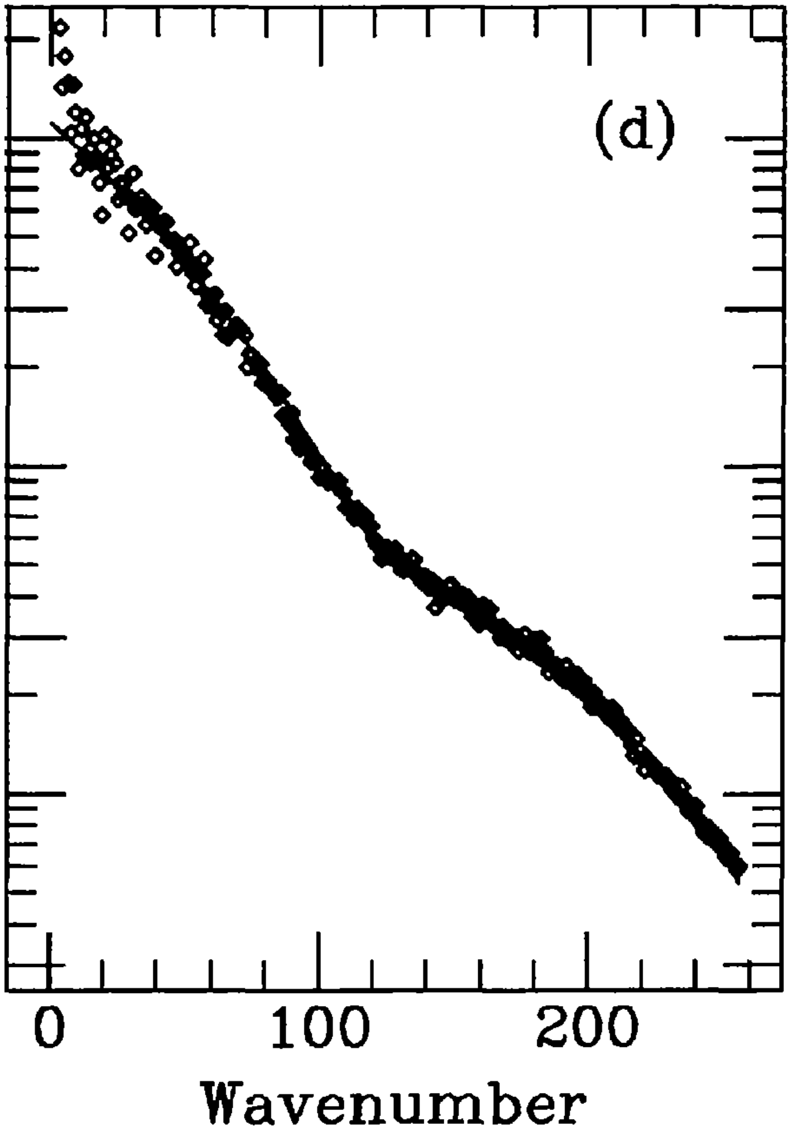
$$M_\mathrm{T,\,V} = -7.60 \pm 0.25\,\mathrm{mag} \qquad (5.28)$$

Caveats:

1. M_T depends on luminosity and type of host galaxy (GC of dwarf galaxies weaker by ~ 0.3 in V).
2. Metallicity of galaxy cluster influences M_T.
3. Measurement diffi cult (need the weak GCs!).
4. Large scatter in data $\Longrightarrow$ Method rather unreliable.

(MW GCs, Abraham & van den Bergh, 1995, Fig. 1)

Surface Brightness Fluctuations, I



For early type galaxies: Assume N stars in picture element (pixel), with average flux f each.

$$\Longrightarrow \text{Mean pixel intensity:} \quad \mu = Nf \tag{5.29}$$

independent of distance, since $N \propto r^2$ and $f \propto r^{-2}$.
Standard deviation between pixels (Poisson!):

$$\sigma = \sqrt{N} f \propto r^{-1} \tag{5.30}$$

and therefore

$$f = \frac{\sigma^2}{\mu} = \frac{L}{4\pi r^2} \tag{5.31}$$

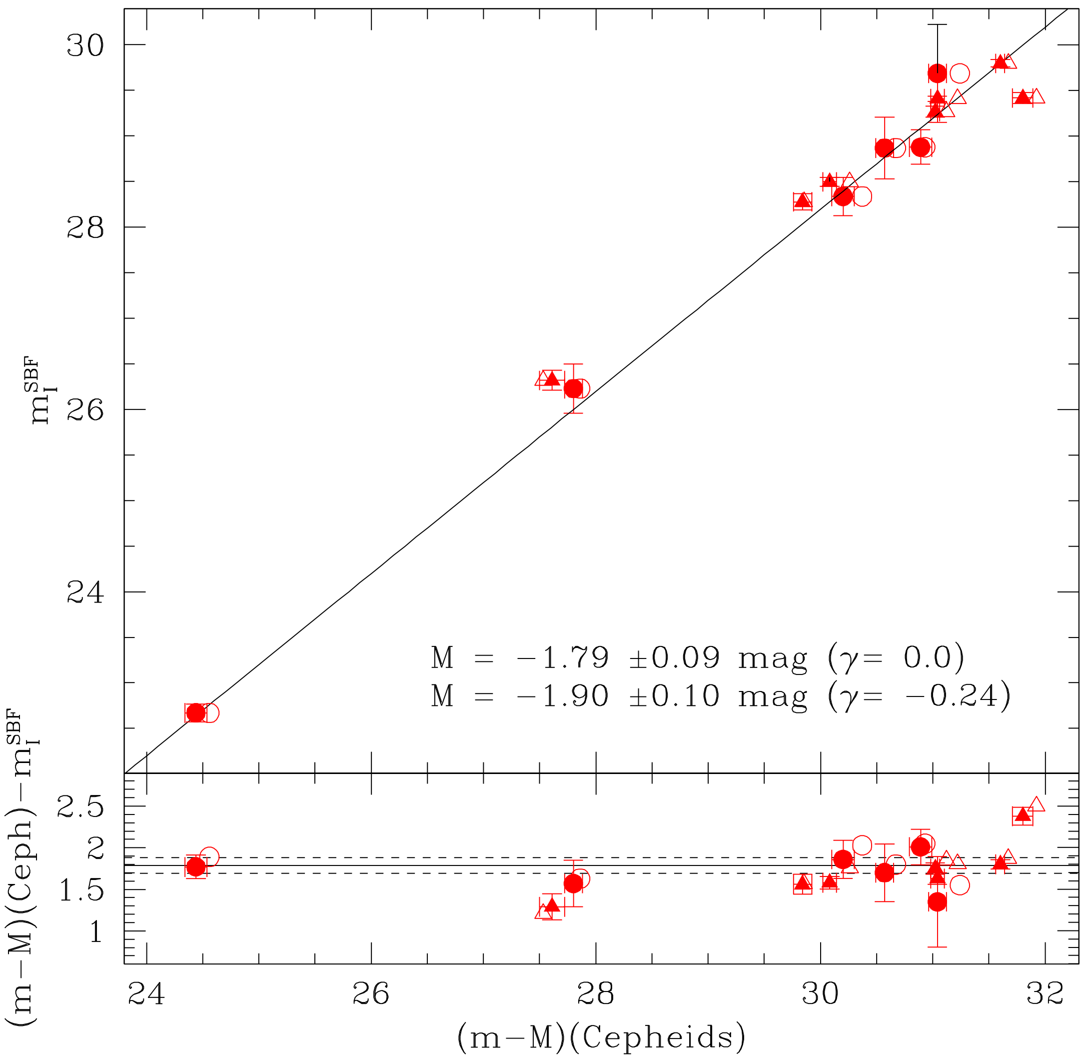
which gives the distance r.

Review: Blakeslee, Ajhar & Tonry (1999).

Complication: Adjacent pixels not independent (point spread function of telescope!)
$\Longrightarrow$ Use radial power spectrum to obtain σ^2 and μ.

(Ajhar et al., 1997, Fig. 3d)

Surface Brightness Fluctuations, II



Luminosity of galaxy dominated by Red Giant Branch stars

⟹ Strong wavelength and color dependence

⟹ Primary calibration: I-band plus broad-band color dependency to give standard candle.

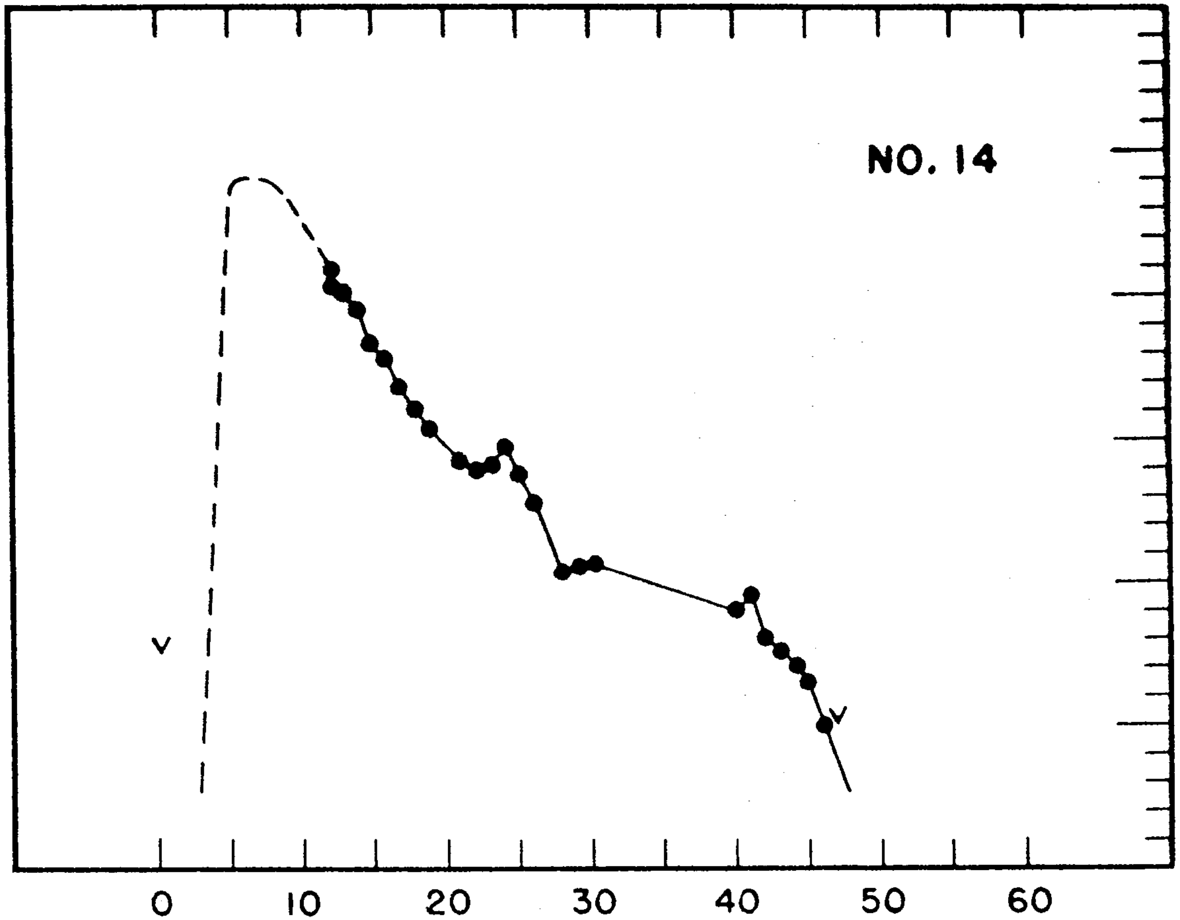
Often also used: HST WFPC2 plus F814W filter (close to I-band),

$$M_{\mathrm{F814W}} = (-1.70 \pm 0.16) + (4.5 \pm 0.3)\left[(\mathrm{V}-\mathrm{I})_0 - 1.15\right] \quad (5.32)$$

Works out to $\sim$ 70 Mpc with HST.

(Ferrarese et al., 2000, Fig. 5)

Novae, I



"classical nova" = explosion on surface of white dwarf

Novae only in binary systems

$\Longrightarrow$ slow accretion of material onto WD

$\Longrightarrow$ outer skin reaches M_{crit} for fusion

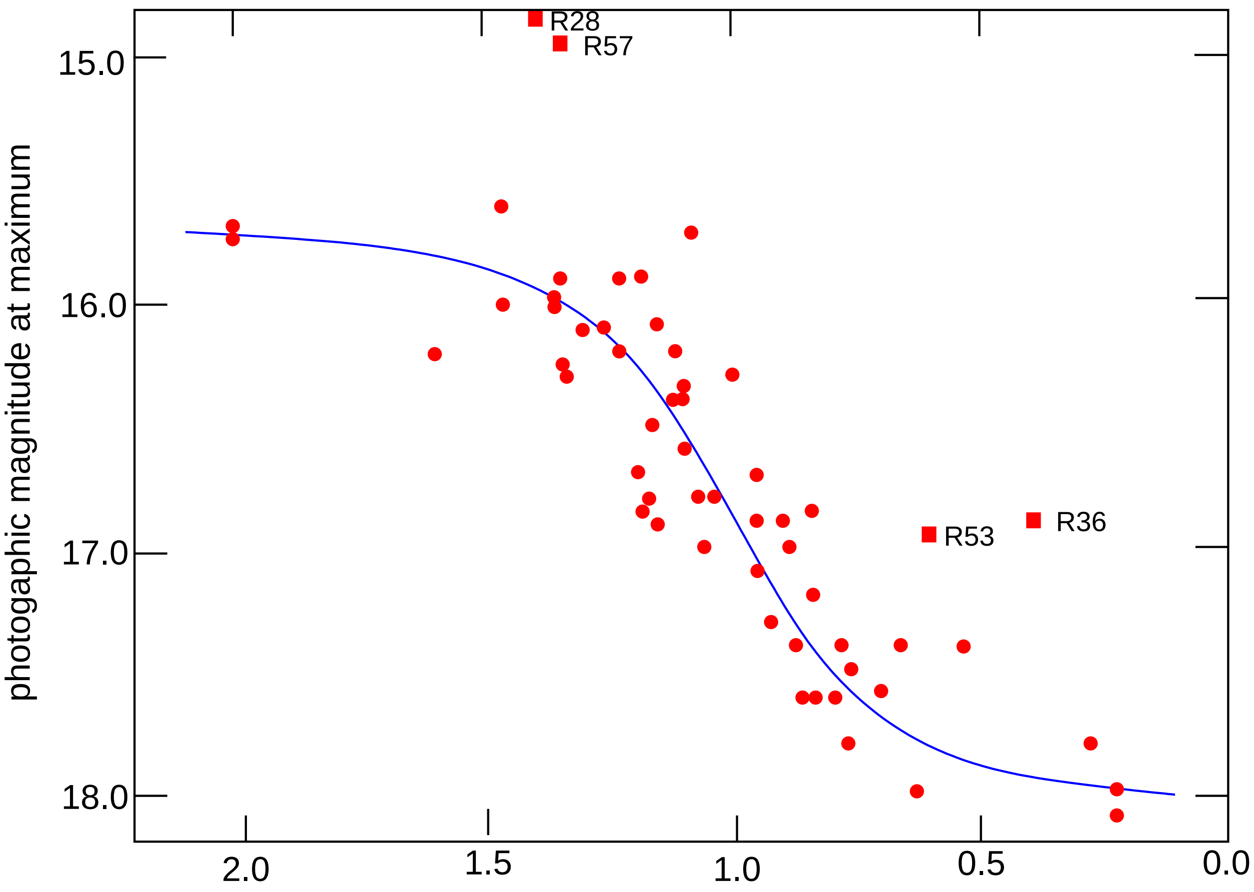
$\Longrightarrow$ explosion

$\Longrightarrow$ ejection of $10^{-6} \dots 10^{-4}\,M_\odot$ with $v \sim 500\,\text{km}\,\text{s}^{-1}$

Explosion produces characteristic lightcurve.

(Nova in M31, Arp, 1956, p. 18)

Novae, II



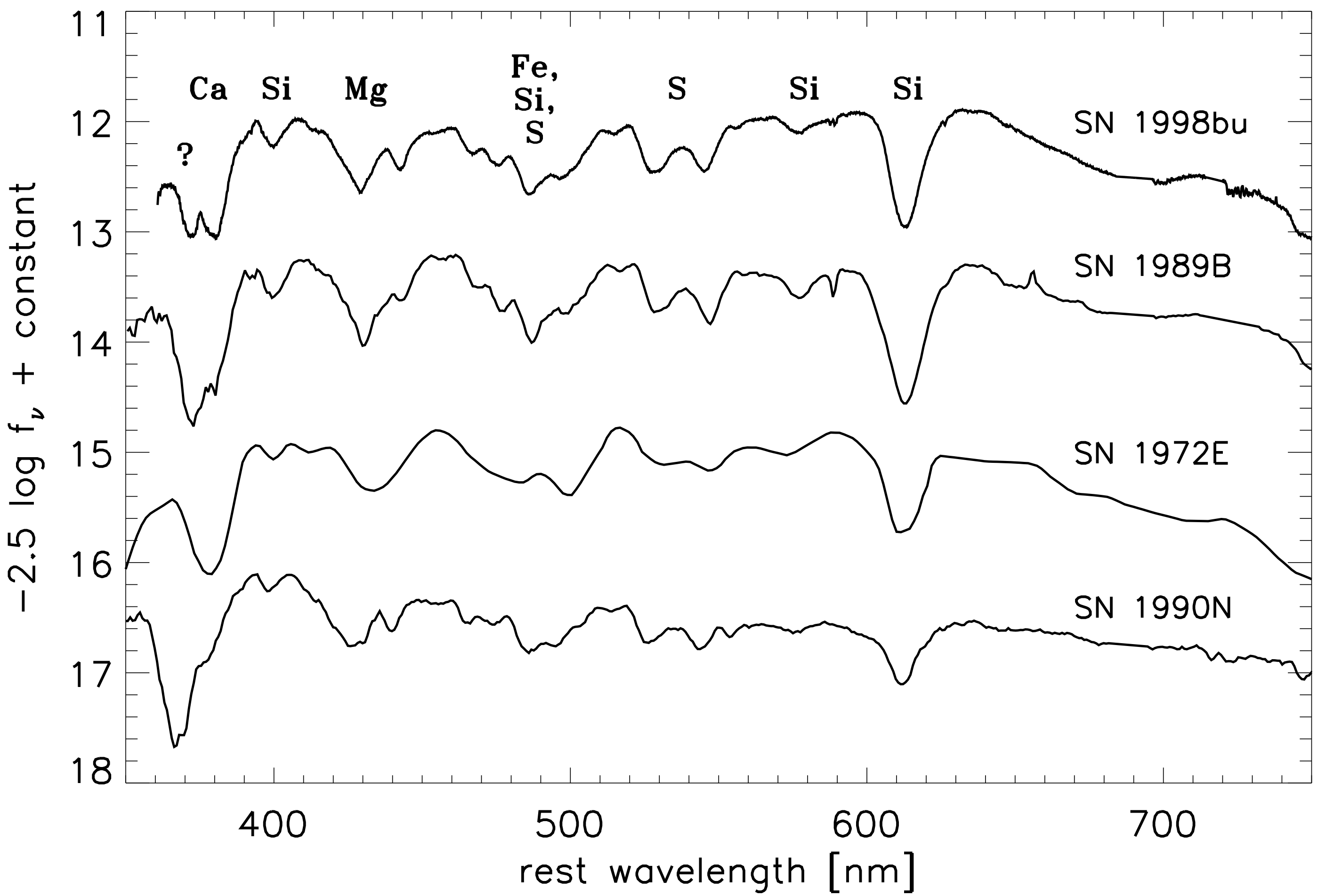
(after van den Bergh & Pritchet, 1986, Fig. 1).

Strong scatter in lightcurves (higher $L_{\text{max}} \Longrightarrow$ faster decline, but typically $\sim 3\times$ brighter than Cepheids), but good Correlation luminosity vs. decline timescale (t_i, time to reach $m(t_i) = m_{\text{max}} + i$). *Calibration:* galactic novae.

Supernovae have luminosities comparable to whole galaxies: $\sim 10^{51}\,\mathrm{erg\,s^{-1}}$ in light, $100\times$ more in neutrinos.

SN1994d (HST WFPC)

Type Ia Supernovae, II

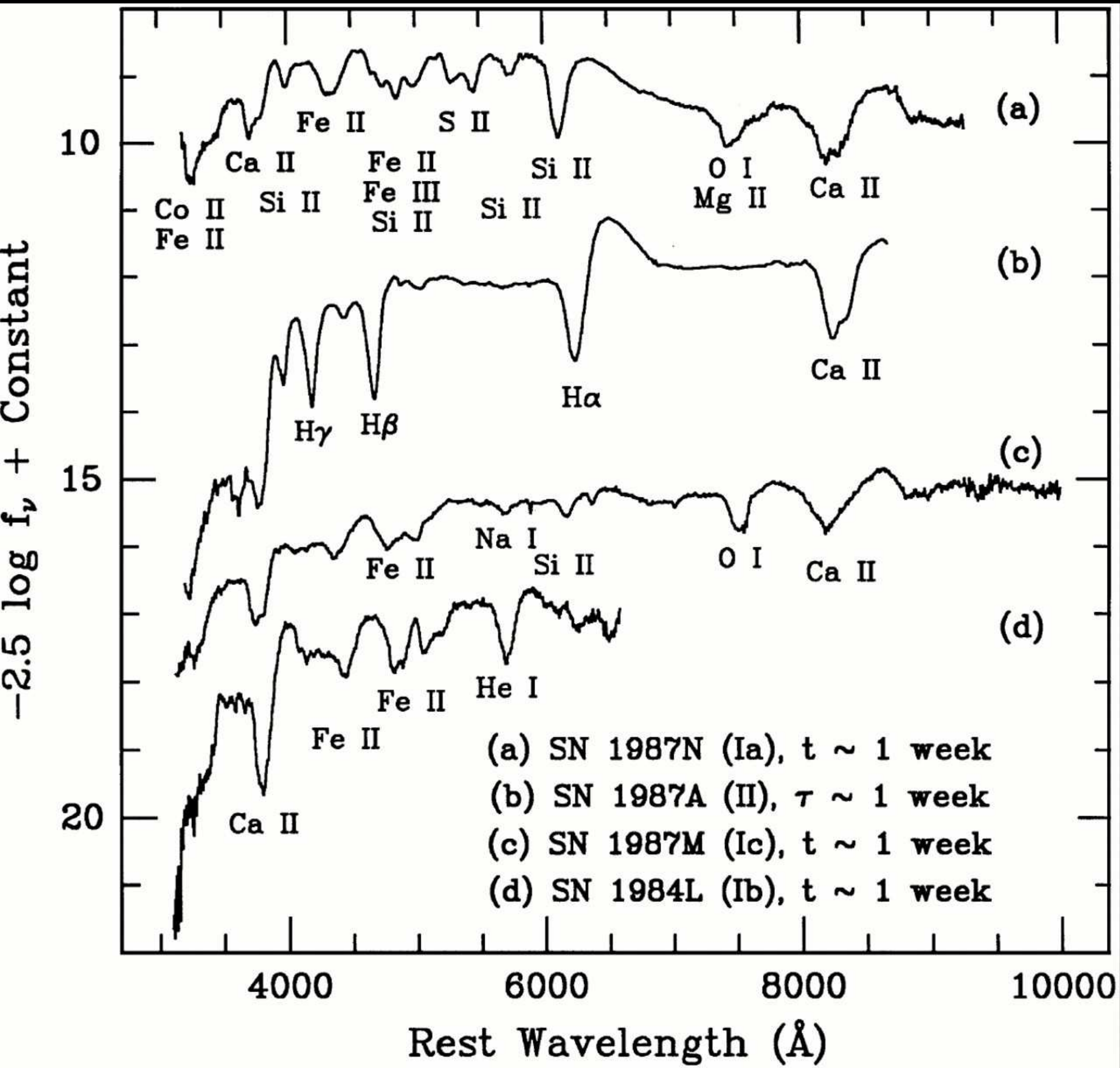


(Spectra of several SNe at maximum light Jha et al., 1999, Fig. 6)

Different supernovae can have very similar spectra.

$\Longrightarrow$

Allows their classifi cation.



(Filippenko, 1997, Fig. 1); t: time after maximum light; τ: time after explosion;

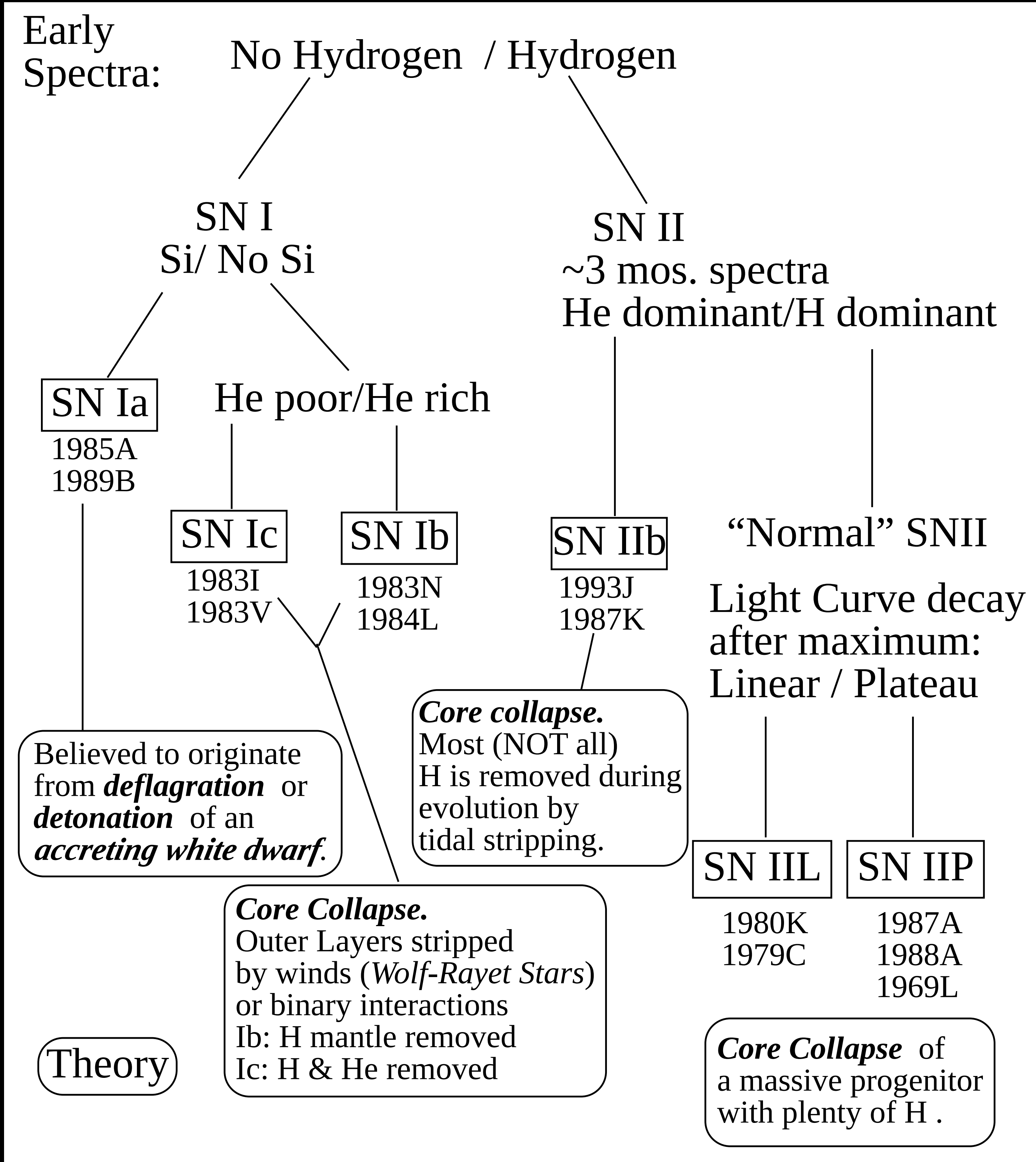
P Cyg profi les give $v \sim 10000\,\mathrm{km\,s^{-1}}$

Rough classifi cation (Minkowski, 1941):

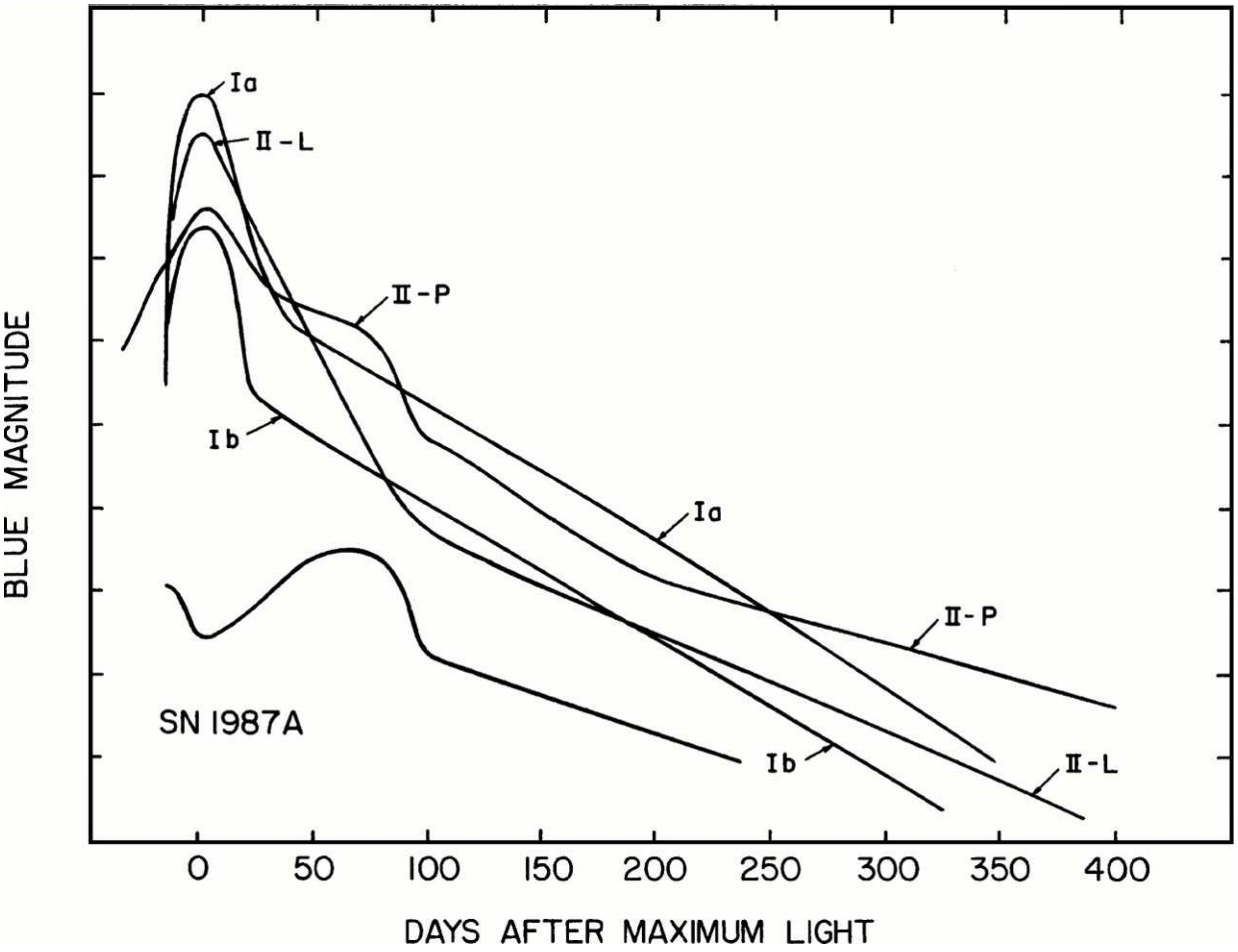
Type I: no hydrogen in spectra; subtypes Ia, Ib, Ic

Type II: hydrogen present, subtypes II-L, II-P

Note: pre 1985 subtypes Ia, Ib had different defi nition than today $\Longrightarrow$ beware when reading older texts.


Early Spectra:
No Hydrogen / Hydrogen
SN I
Si/ No Si
SN II
~3 mos. spectra
He dominant/H dominant
SN Ia
1985A
1989B
He poor/He rich
SN Ic
1983I
1983V
SN Ib
1983N
1984L
SN IIb
1993J
1987K
"Normal" SNII
Light Curve decay after maximum:
Linear / Plateau
Believed to originate from deflagration or detonation of an accreting white dwarf.
Core collapse. Most (NOT all) H is removed during evolution by tidal stripping.
SN IIL
1980K
1979C
SN IIP
1987A
1988A
1969L
Core Collapse. Outer Layers stripped by winds (Wolf-Rayet Stars) or binary interactions
Ib: H mantle removed
Ic: H & He removed
Theory
Core Collapse of a massive progenitor with plenty of H .

Type Ia Supernovae, V

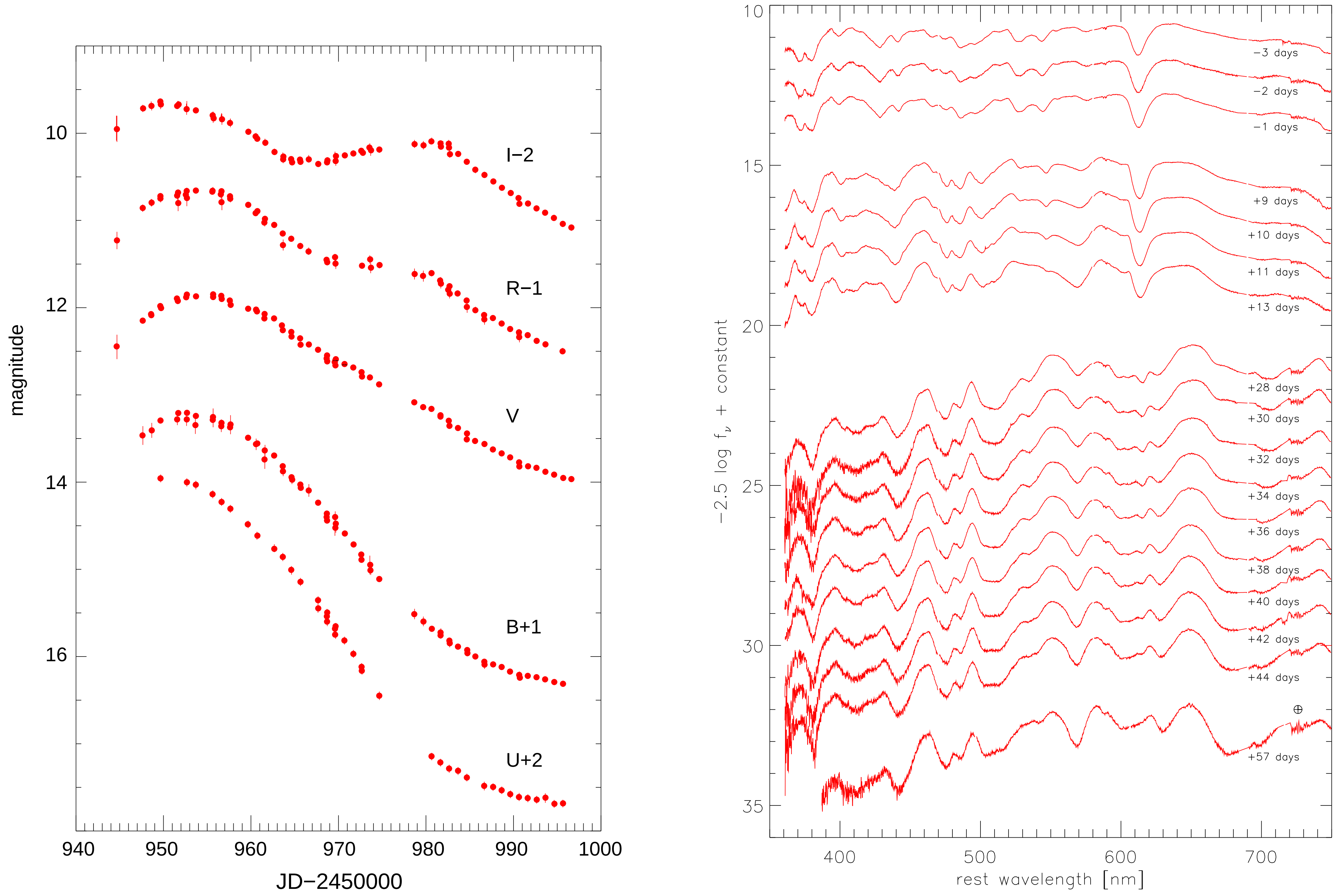


(Filippenko, 1997, Fig. 3)

Light curves of SNe I all very similar, SNe II have much more scatter.

SNe II-L (“linear”) resemble SNe I

SNe II-P (“plateau”) have const. brightness to within 1 mag for extended period of time.



(SN 1998bu in M96, Jha et al., 1999, Figs. 2 and 4)

Type Ia Supernovae, VII

Clue on origin from supernova statistics:

- SNe II, Ib, Ic: never seen in ellipticals; rarely in S0; generally associated with spiral arms and H II regions.

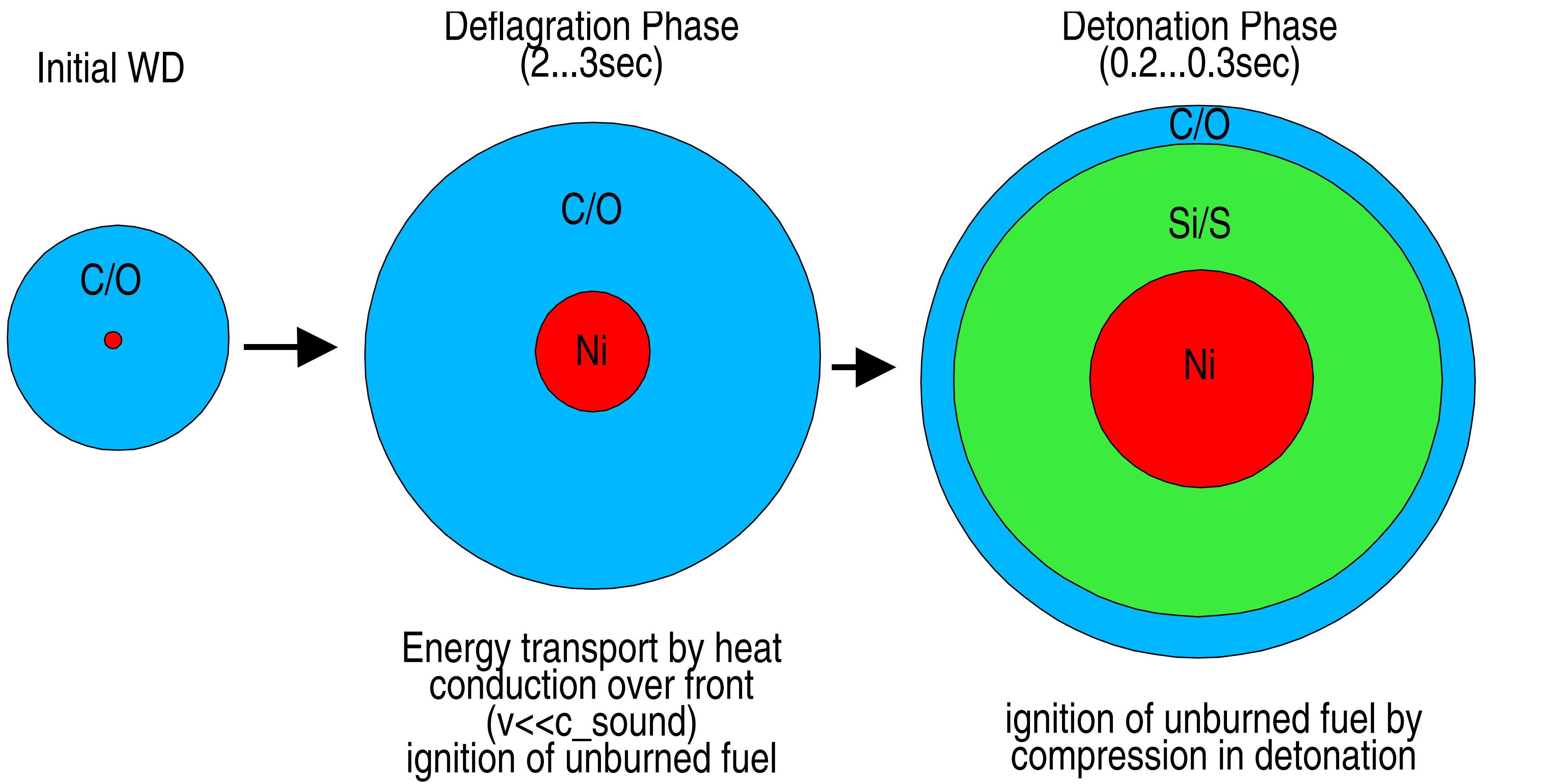
$\Longrightarrow$ *progenitor of SNe II, Ib, Ic:* massive stars ($\gtrsim 8\,M_\odot$) $\Longrightarrow$ core collapse

- SNe Ia: all types of galaxies, no preference for arms, almost no scatter in lightcurves

$\Longrightarrow$ *progenitor of SNe Ia:* accreting carbon-oxygen white dwarfs, undergoing thermonuclear runaway

Rule of thumb: 1...3 SNe per galaxy and per century

Type Ia Supernovae, VIII



after P. Höflich

Type Ia Supernovae, IX

SN Ia = Explosion of CO white dwarf when pushed over Chandrasekhar limit (1.4 $M_\odot$) (via accretion?).

$\Longrightarrow$ Always similar process

$\Longrightarrow$ Very characteristic light curve: fast rise, rapid fall, exponential decay ("FRED") with half-time of 60 d.

60 d time scale from radioactive decay $Ni^{56} \rightarrow Co^{56} \rightarrow Fe^{56}$ ("self calibration" of lightcurve if same amount of Ni^{56} produced everywhere).

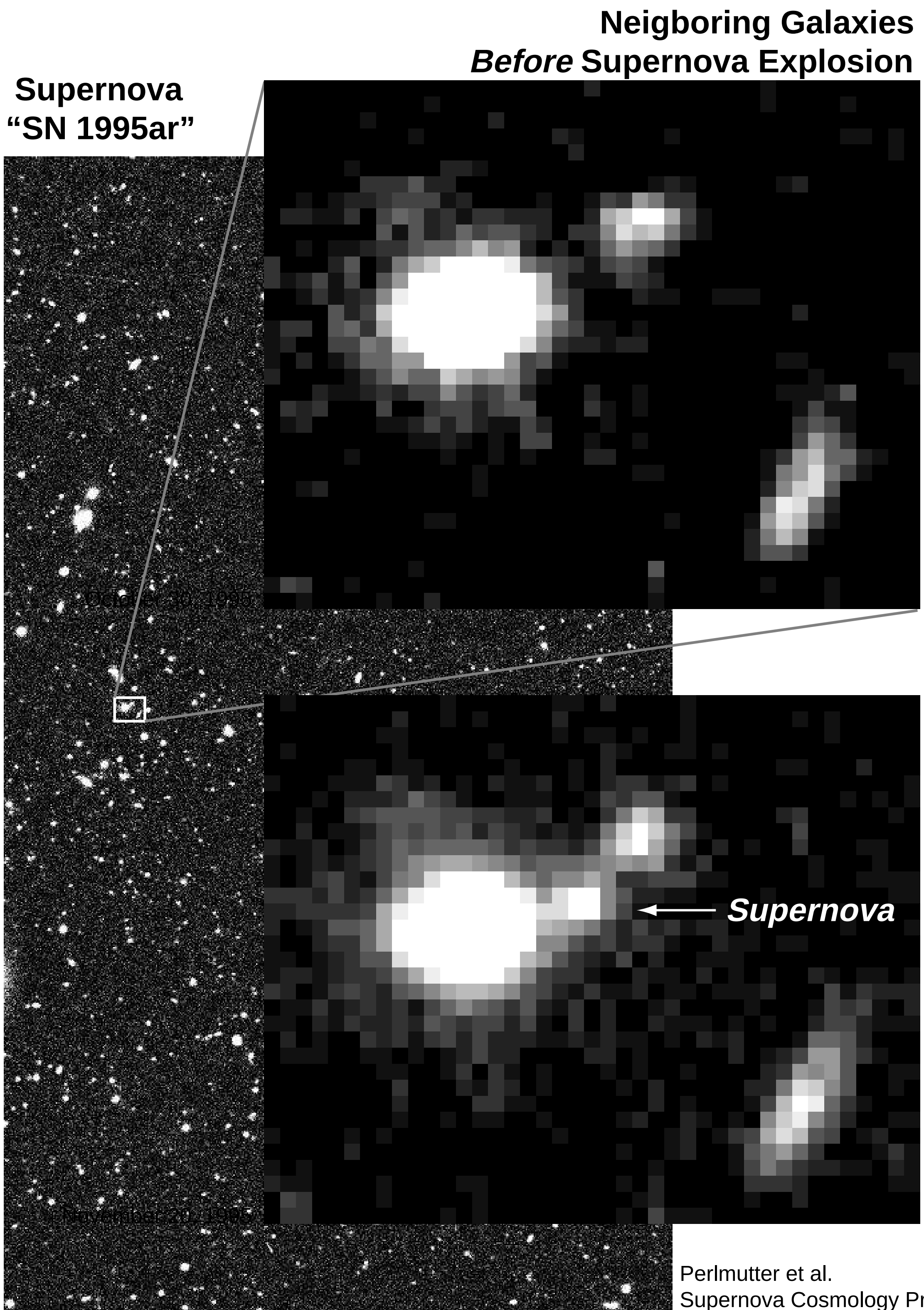
Calibration: SNe Ia in nearby galaxies where Cepheid distances known.

At maximum light:

$$M_\mathrm{B} = -18.33 \pm 0.11 + 5 \log h_{100} \quad (L \sim 10^{9\ldots10}\,L_\odot) \tag{5.33}$$

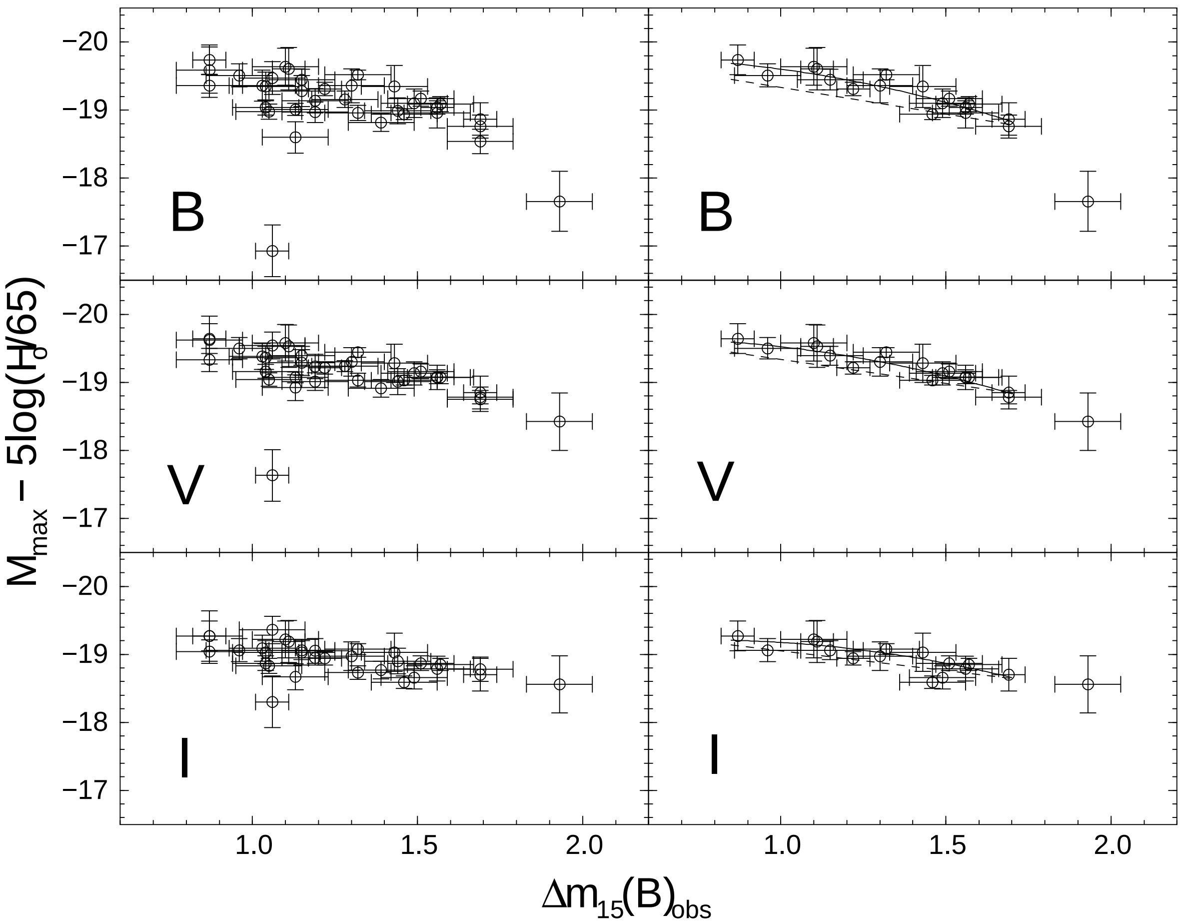
Intrinsic dispersion: $\lesssim$0.25 mag (possibly due to size of clusters analyzed?!?)

Observable out to 1000 Mpc



Perlmutter et al.
Supernova Cosmology Project

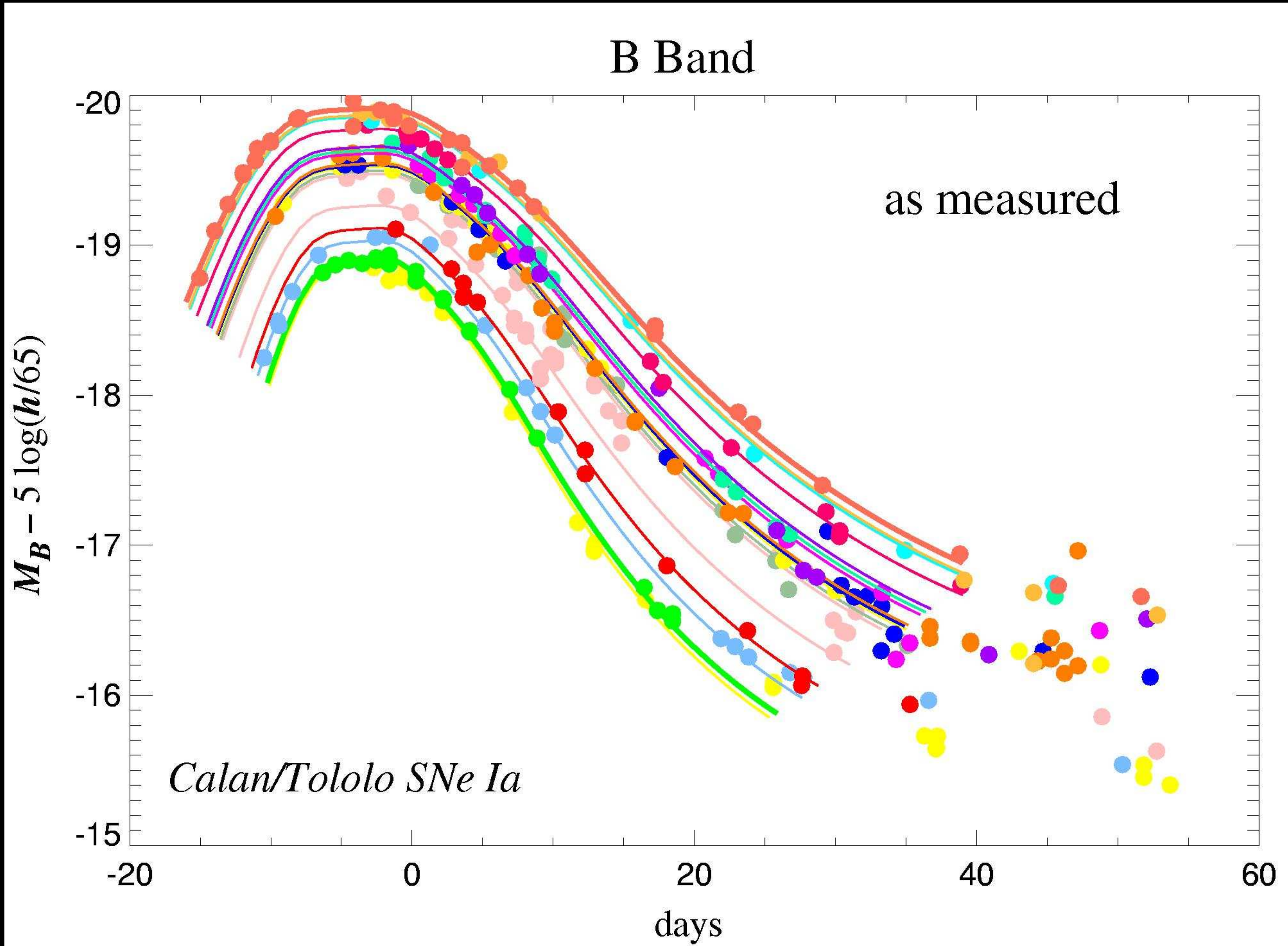
Type Ia Supernovae, XI



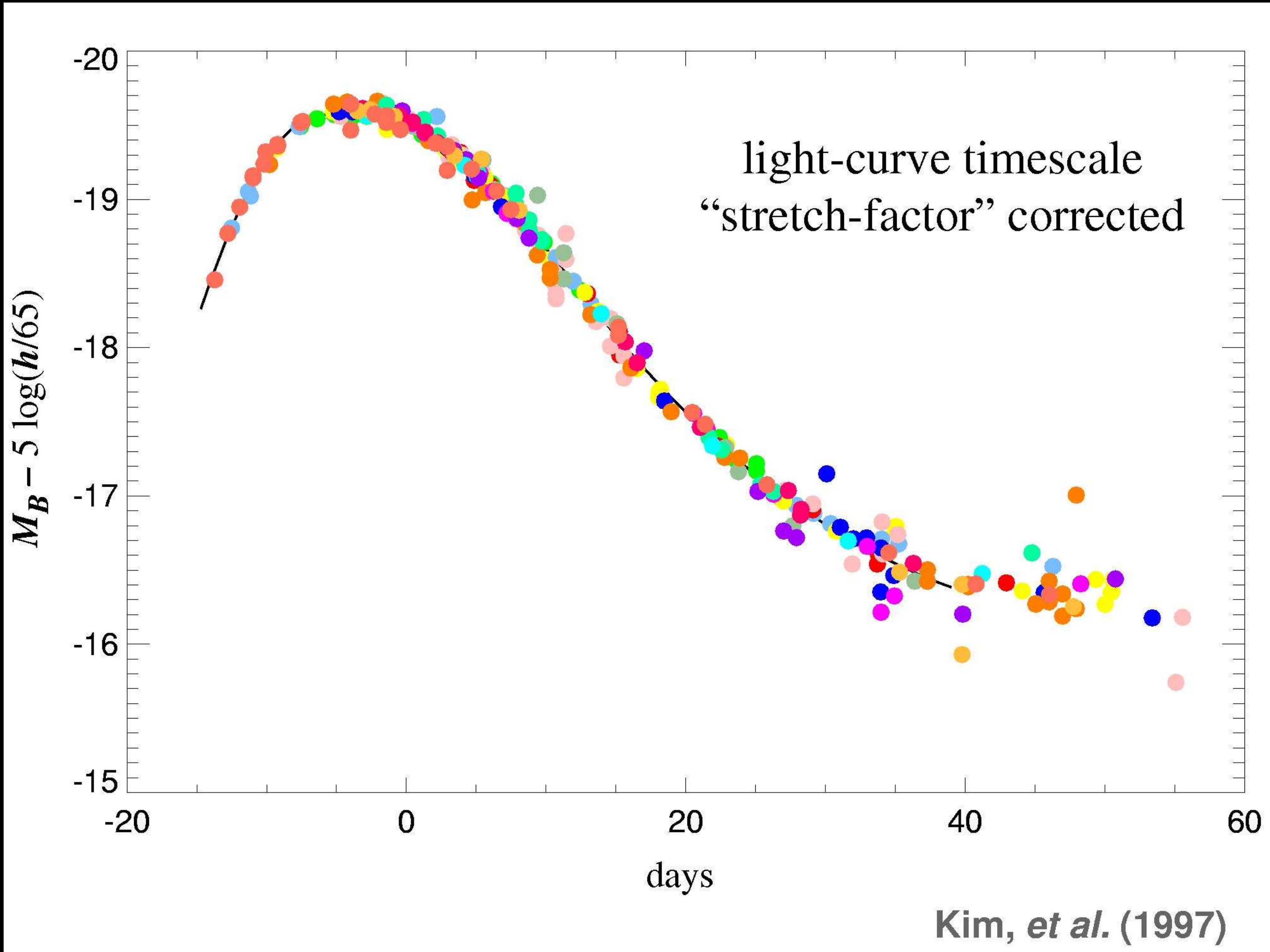
(Phillips et al., 1999, Fig. 8)

Caveats:

1. Are they *really* identical?
   $\Longrightarrow$ history of pre-WD star?
2. Correction for extinction in parent galaxy difficult.
3. Baade-Wesselink for calibration Eq. (5.33) depends crucially on assumed $(\mathrm{B}-\mathrm{V})$-T_{eff} relation.
4. Some SN Iae spectroscopically peculiar $\Longrightarrow$ Do not use these!
5. Decline rate and color vary, but max. brightness and decline rate correlate (see figure).



Lightcurves of Hamuy et al. SN Ia sample (18 SNe discovered within 5 d past maximum, with 3.6 $< \log cz <$ 4.5, i.e., $z <$ 0.1)



Lightcurves of Hamuy et al. SN Ia sample (18 SNe discovered within 5 d past maximum, with 3.6 $< \log cz <$ 4.5, i.e., $z <$ 0.1), after correction of systematic effects and time dilatation (Kim et al., 1997).

Type Ia Supernovae, XIV

Recalibration of SN Ia distances with Cepheids gives (Gibson et al., 2000):

$$\begin{aligned}\log H_0 = 0.2\Big\{M_\mathrm{B}^\mathrm{max} - 0.720(\pm 0.459) \\ \cdot [\Delta m_{\mathrm{B},15,t} - 1.1] - 1.010(\pm 0.934) \\ \cdot [\Delta m_{\mathrm{B},15,t} - 1.1]^2 + 28.653(\pm 0.042)\Big\} \quad (5.34)\end{aligned}$$

where

$$\Delta m_{\mathrm{B},15,t} = \Delta m_{\mathrm{B},15} + 0.1E(\mathrm{B}-\mathrm{V}) \quad (5.35)$$

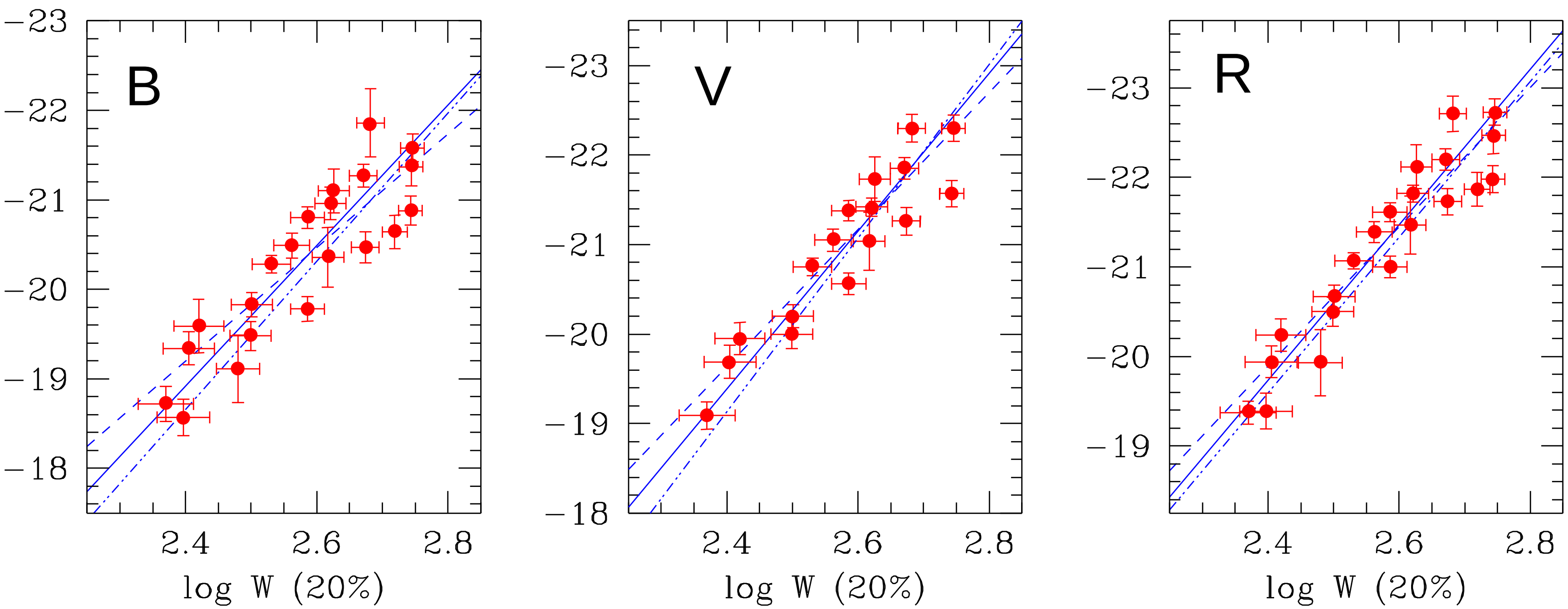
where

$\Delta m_{\mathrm{B},15}$: observed 15 d decline rate,
$E(\mathrm{B}-\mathrm{V})$: total extinction (galactic+intrinsic).

Eq. (5.34) valid for B-band, equivalent formulae exist for V and I.

Overall, the calibration is good to better than 0.2 mag in B.

Tully-Fisher, I



(after Sakai et al., 2000, Fig. 1)

Tully-Fisher relation for spiral galaxies: Width of 21 cm line of H correlated with galaxy luminosity:

$$M = -a \log\left(\frac{W_{20}}{\sin i}\right) - b \qquad (5.36)$$

where W_{20}: 20% line width (km s^{-1}; typically $W_{20} \sim 300$ km s^{-1}), i inclination angle.

For the B- and I-Bands (Sakai et al., 2000):

	B	I
a	7.97± 0.72	9.24± 0.75
b	19.80± 0.11	21.12± 0.12

Tully-Fisher, II

Qualitative Physics: Line width related to mass of galaxy: $W/2 \sim V_{\max}$, where $V_{\max}$ max. velocity of rotation curve

$\Longrightarrow$ Assume $M/L =$ const. (good assumption)

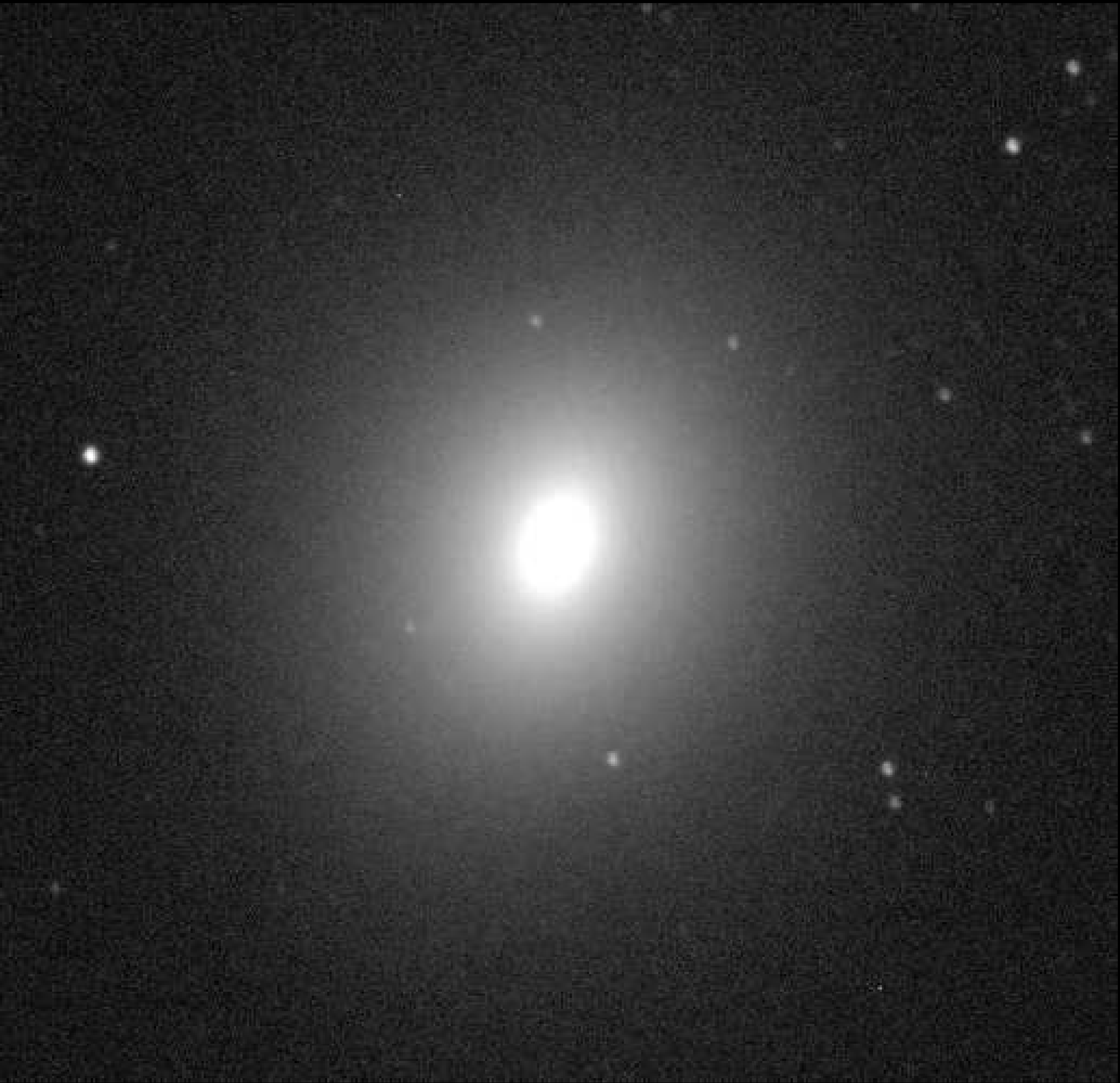
$\Longrightarrow$ width related to luminosity.

Detailed physical basis unknown. Might be related to galaxy formation ("hierarchical clustering", see later).

I-band is better (less internal extinction).

Caveats:

1. Determination of inclination i.
2. Influence of turbulent motion within galaxy.
3. Constants dependent on galaxy type (Sa and Sb similar, Sc more luminous by factor of $\sim$2).
4. Optical extinction.
5. Intrinsic dispersion $\sim$0.2 mag.
6. Barred Galaxies problematic.



"Faber-Jackson" law for elliptical galaxies:

The luminosity L of an elliptical galaxy scales with its intrinsic velocity dispersion, σ, as $L \propto \sigma^4$.

Note that ellipticals have virtually no Hydrogen

$\Longrightarrow$ cannot use 21 cm.

M32 (companion of Andromeda), courtesy W. Keel

Ellipticals: $$M_\mathrm{B} = -19.38 \pm 0.07 - (9.0 \pm 0.7)(\log \sigma - 2.3) \quad (5.37)$$

Lenticulars (Type S0): $$M_\mathrm{B} = -19.65 \pm 0.08 - (8.4 \pm 0.8)(\log \sigma - 2.3) \quad (5.38)$$

D_n-σ

The Faber-Jackson law is a specialized case of the more general D_n–σ-relation:

The intensity profi le of an elliptical galaxy is given by de Vaucouleurs' $r^{1/4}$ law:

$$I(r) = I_0 \exp\left(-(r/r_0)^{1/4}\right) \quad \Longrightarrow \quad L = \int I \propto I_0 r_0^2 \tag{5.39}$$

Because of the virial theorem ($E_{\mathrm{kin}} = -E_{\mathrm{pot}}/2$):

$$\frac{1}{2} m\sigma^2 = G\frac{mM}{r_0} \quad \Longleftrightarrow \quad \sigma^2 \propto \frac{M}{r_0} \tag{5.40}$$

where σ: velocity dispersion.

Assume a mass-to-light ratio

$$M/L \propto M^\alpha \tag{5.41}$$

($\alpha \sim 0.25$). and use r_0 from Eq. (5.39) to obtain

$$L^{1+\alpha} \propto \sigma^{4-4\alpha} I_0^{\alpha-1} \tag{5.42}$$

This is called the "fundamental plane" relationship (Dressler et al., 1987).

D_n-σ

Observational version of the fundamental plane relationship: Instead of inserting r_0 and I_0, measure diameter D_n of aperture to reach some mean surface brightness (typically sky brightness, $20.75\,\mathrm{mag\,arcsec}^{-2}$ in B), and use calibration.

Note: Assumptions are

1. M/L same everywhere.
2. ellipticals have same stellar population everywhere

Calibration paper: Kelson et al. (2000).

Path to H_0

To obtain H_0, we need distances, and redshifts.

Redshifts: Trivial

Distances: Hubble Space Telescope Key Project on Extragalactic Distance Scale.

Summary paper: Freedman et al. (2001), there are a total of 29 papers on the HST key project!

Strategy:

1. Use high-quality candles: Cepheid variables as primary distance calibrator.
2. Calibrate secondary calibrators that work out to $cz = 10000\,\mathrm{km\,s^{-1}}$:
   - Tully-Fisher,
   - Type Ia Supernovae,
   - Surface Brightness Fluctuations,
   - Fundamental-plane for Ellipticals.
3. Combine uncertainties from these methods.

Velocity Field, I

Before determining H_0: correct for influence of velocity field (cluster motion with respect to comoving coordinates).

The observed redshift is given by

$$1 + z = (1 + z_\mathrm{R}) \left(1 - \frac{v_0}{c} + \frac{v_\mathrm{G}}{c}\right) \tag{5.43}$$

where

v_0: observer's radial velocity in direction of galaxy

v_G: radial velocity of the galaxy, difficult to find

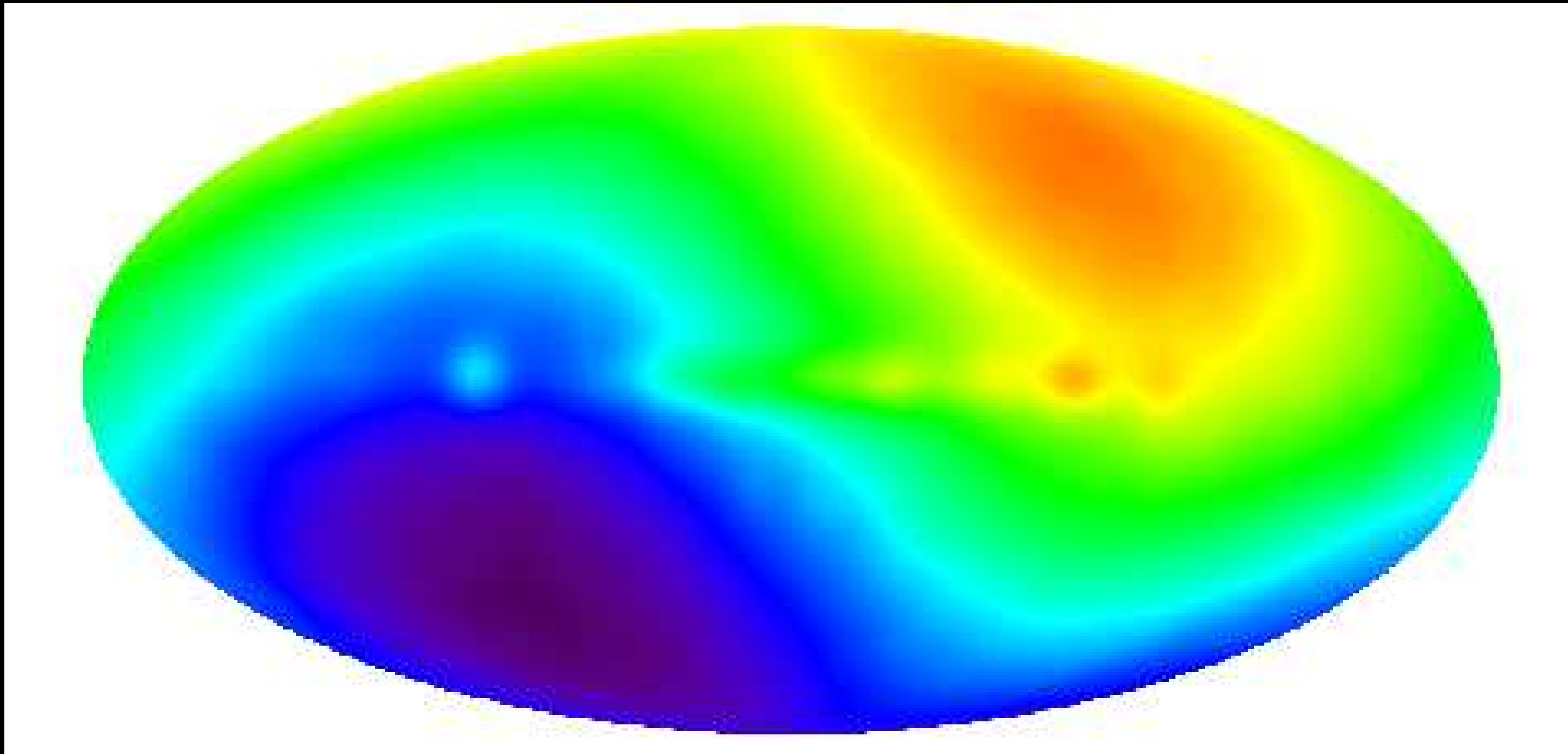
z_R: cosmological redshift

Older galaxy catalogues often attempt to correct the measured values of z to produce "corrected redshifts", e.g., by setting $v_\mathrm{G} = 0$ and

$$1 + z = (1 + z_\mathrm{R}) \left(1 + \frac{v_0}{c}\right) \sim 1 + z_\mathrm{R} - \frac{v_0}{c} \quad \Longrightarrow \quad z_\mathrm{R} \sim z + \frac{v_0}{c} \tag{5.44}$$

since v_0 was not well known before COBE $\Longrightarrow$ introduces unnecessary problems

$\Longrightarrow$ correction not used in recent redshift surveys! (see Harrison & Noonan, 1979, for details)



(COBE DMR; Bennett et al., 1996)

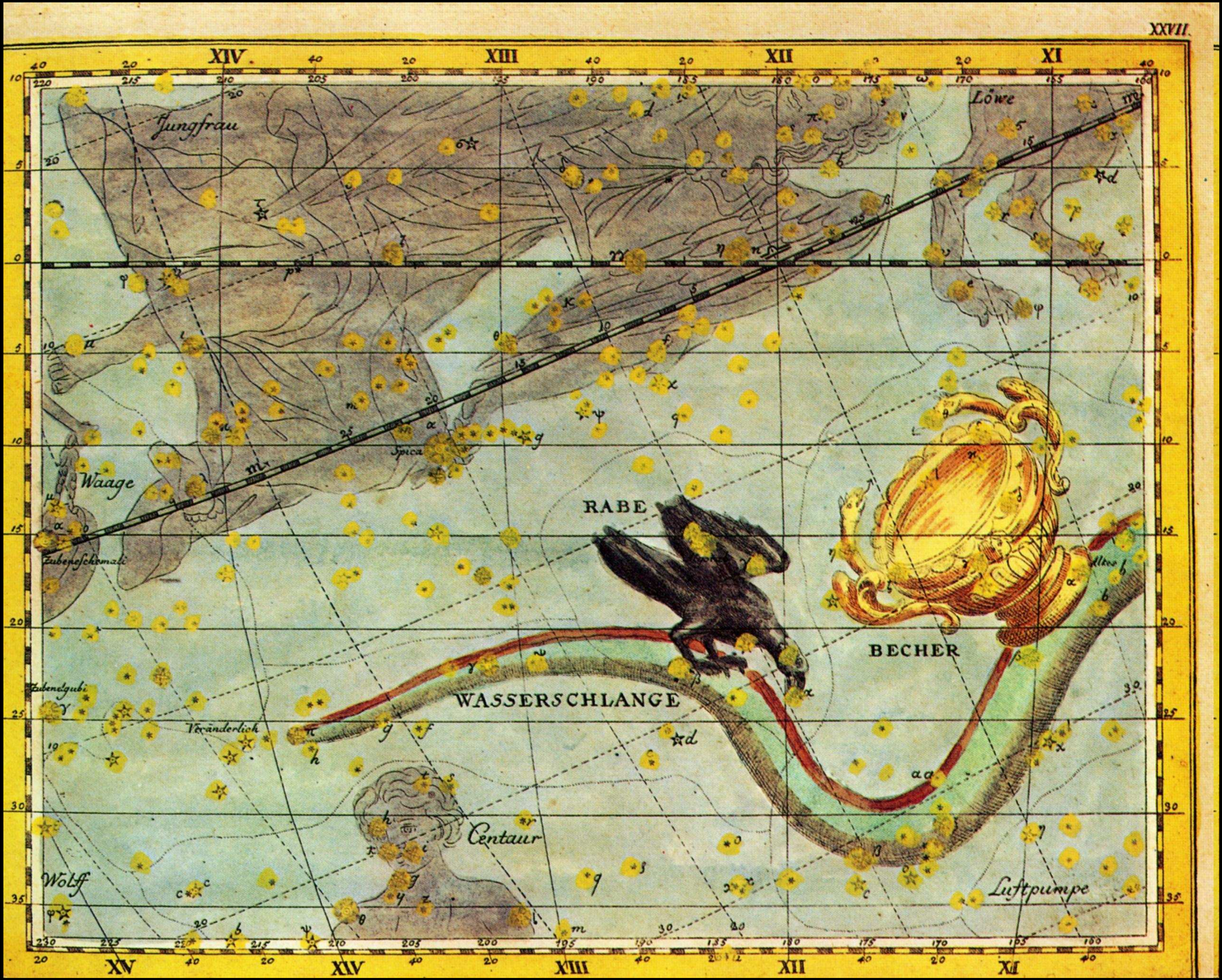
v_0 is easy to find $\Longrightarrow$ Measure velocity of Earth with respect to 3 K radiation. COBE finds $\Delta T = 3.353 \pm 0.024$ mK of 3K black-body spectrum of $T = 2.725 \pm 0.020$ K, using $\Delta T/T = v/c$.

$$v_0 = (369.1 \pm 2.6)\,\mathrm{km\,s^{-1}} \cdot \cos\theta_{\mathrm{CMB}} \tag{5.45}$$

where $\theta_{\mathrm{CMB}} = \angle(\boldsymbol{v}, \boldsymbol{v}_{\mathrm{CMB}})$, and $\boldsymbol{v}_{\mathrm{CMR}}$ points towards

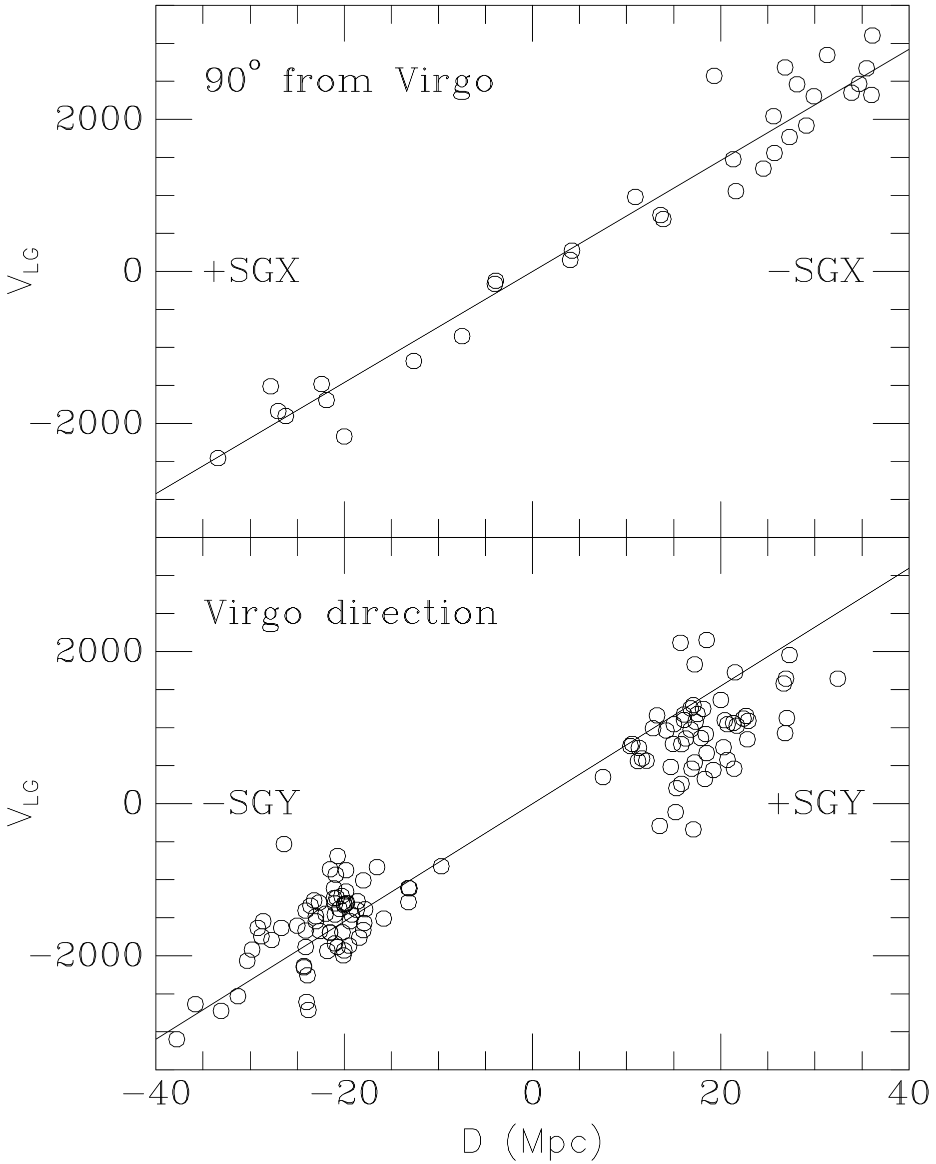
$$(l, b) = (264\overset{\circ}{.}26 \pm 0\overset{\circ}{.}33, 48\overset{\circ}{.}22 \pm 0\overset{\circ}{.}13)$$
$$(\alpha, \delta)_{\mathrm{J2000.0}} = (11^{\mathrm{h}}12\overset{\mathrm{m}}{.}2 \pm 0\overset{\mathrm{m}}{.}8, -7\overset{\circ}{.}06 \pm 0\overset{\circ}{.}16)$$

in constellation Crater.



The constellation Crater ("Becher") in Johan Elert Bode's Sternatlas
(after Slawik/Reichert, Atlas der Sternbilder, Spektrum, 2004)

Velocity Field, IV



To get feeling for v_G out to Virgo, need to study local velocity field surrounding local group and beyond.

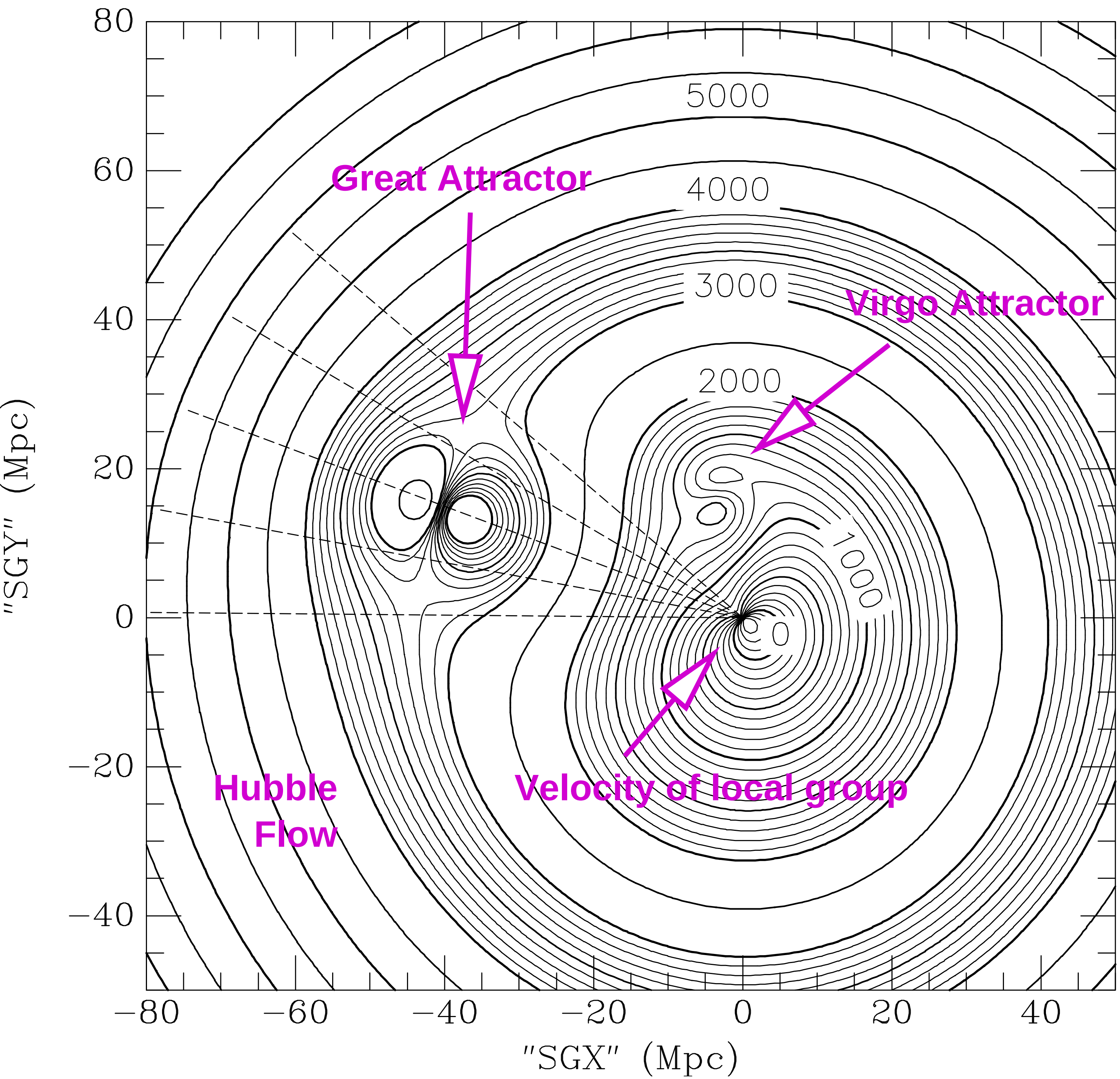
Two major velocity components:

1. Virgocentric infall (known since mid-1970s)
2. Motion towards great attractor ("Seven Samurai", 1980)

plus virialized galaxy motions within clusters.

General analysis: build maximum likelihood model of velocity field including above components *plus* Hubble flow. See Tonry et al. (2000) for details.

Galaxy moves within local group with $v \sim 630\,\mathrm{km\,s^{-1}}$



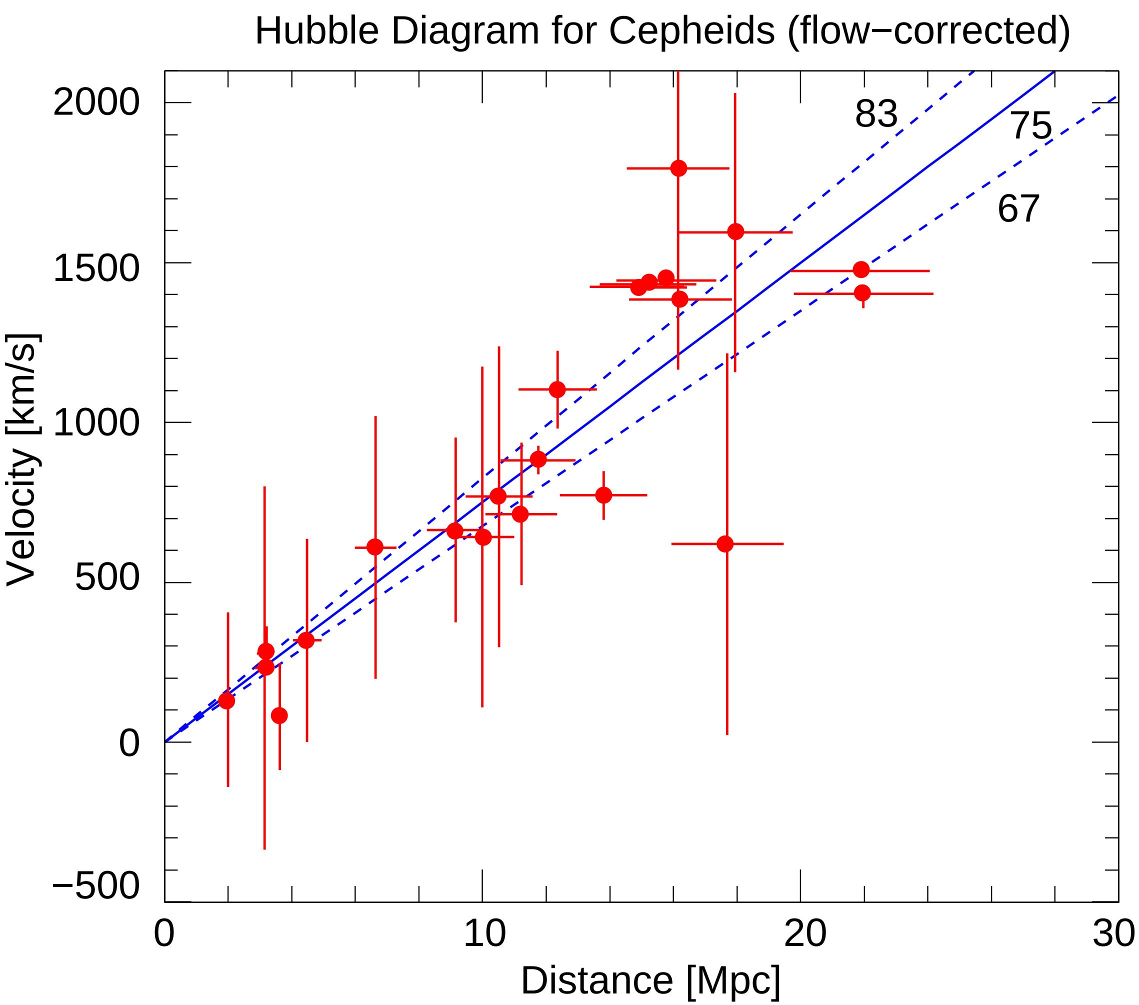
Decomposition of velocity field: (Mould et al., 2000, Tab. A1, note that Tonry et al. 2000 find slightly different values):

	$\alpha_{1950.0}$	$\delta_{1950.0}$	v (km s
Virgo	$12^{h}28^{m}$	$+12^{\circ}40'$	
GA	$13^{h}20^{m}$	$+44^{\circ}00'$	4
Shapley	$13^{h}30^{m}$	$+31^{\circ}00'$	13

(v wrt. center of local group; *not* taking Hubble flow into account!).

(Tonry et al., 2000, Fig. 20)

H from HST



Freedman et al. (2001, Fig. 1)

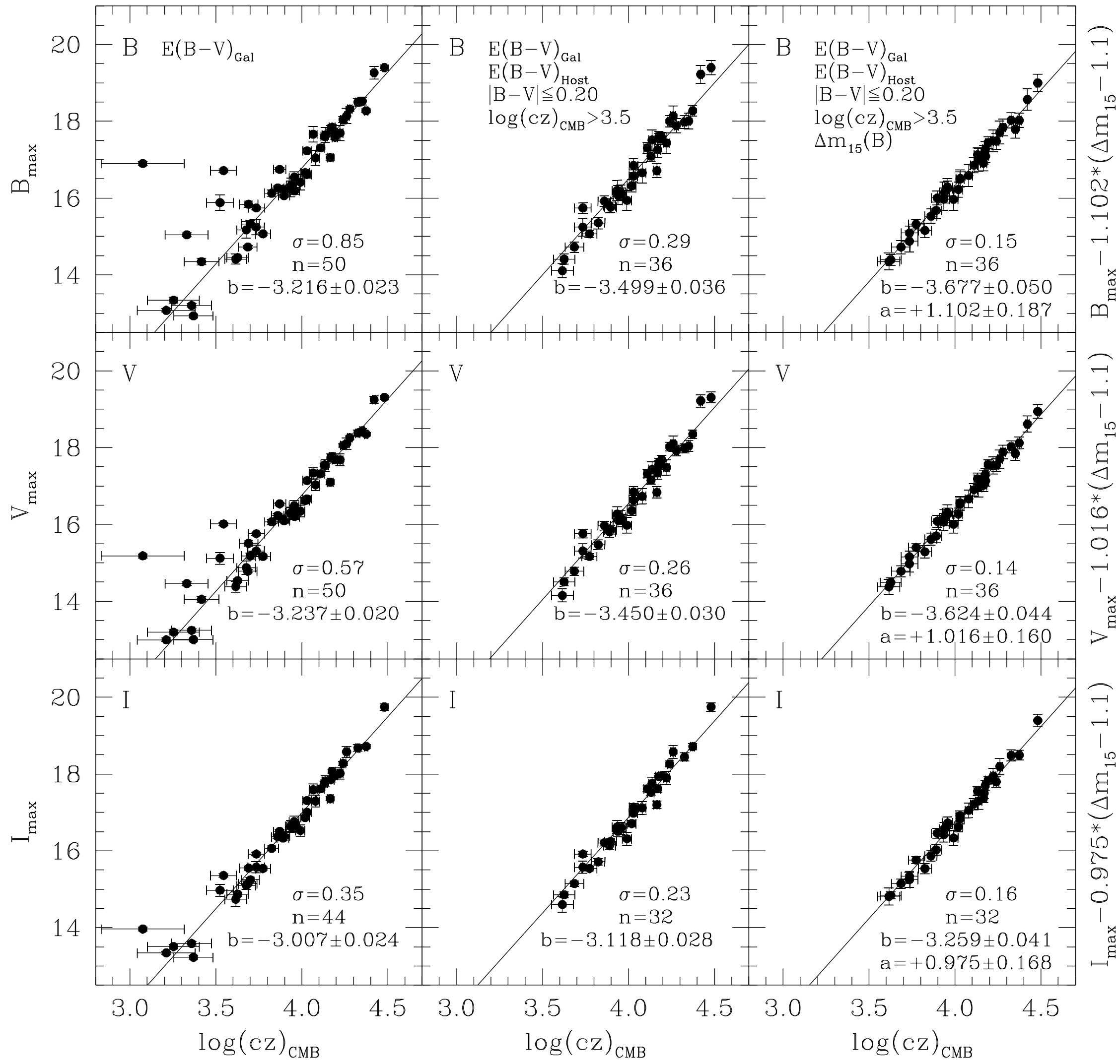
To obtain H_0:

1. Determine d with Cepheids and HST
2. Determine "v", corrected for local velocity fi eld
3. Draw Hubble-diagram
4. Regression Analysis $\Longrightarrow$ H_0

Value from HST Key Project:

$$H_0 = 75 \pm 10\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \quad (5.46)$$

H from HST



Cepheids alone: nearby

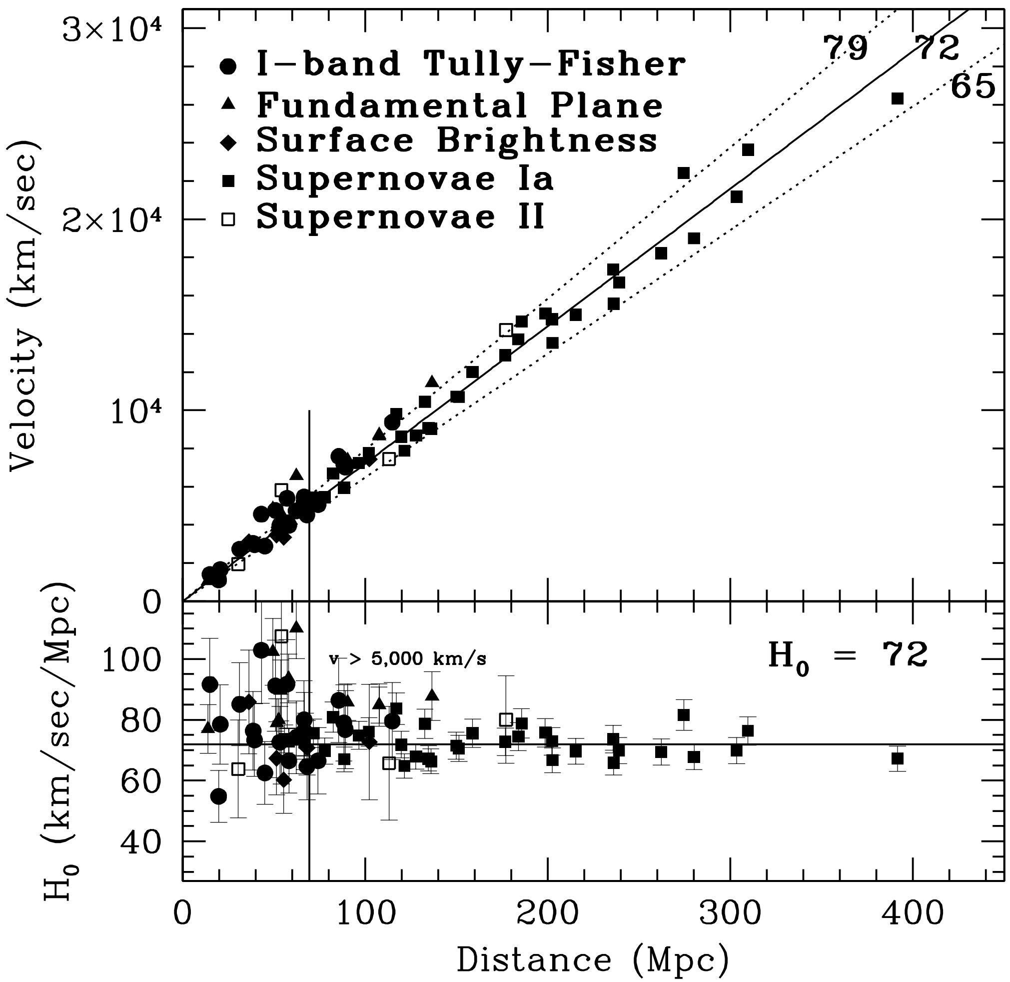
$\Longrightarrow$ systematic uncertainties due to local flow correction and small overall v

$\Longrightarrow$ use secondary candles to get to larger distances.

Example: magnitude-redshift diagram, analoguous to Hubble diagram ($m \propto -5 \log I$, and $I \propto 1/r^2 \propto 1/z^2$ because of Hubble $\Longrightarrow m \propto \log cz$).

(SN Ia Hubble relations; left: full sample, middle: excluding strongly reddened SN Iae, right: same as middle, correcting for light-curve shape Freedman et al., 2001, Fig. 2)

H from HST

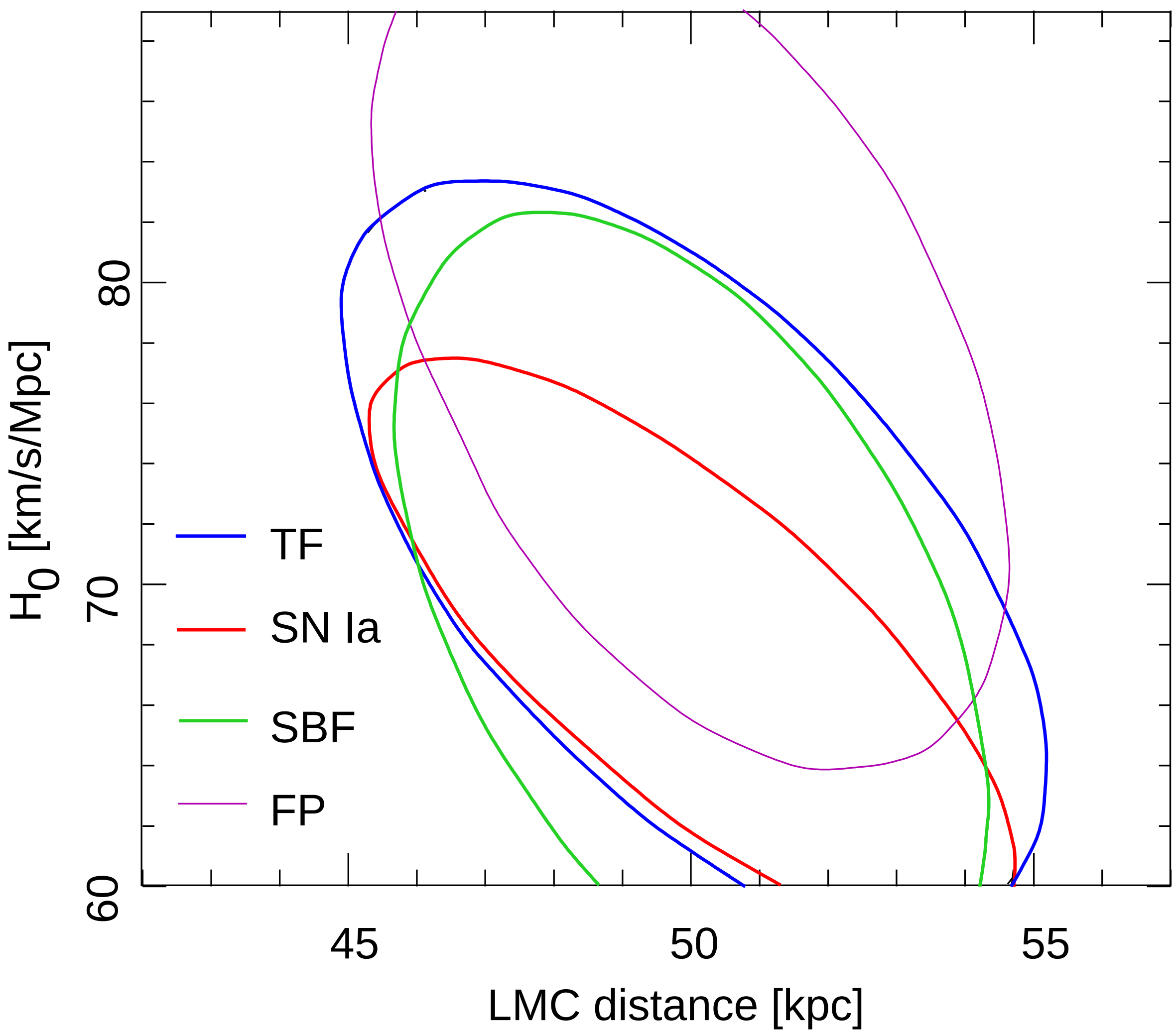


Freedman et al. (2001, Fig. 4)

Combining all secondary methods, best value found:

$$H_0 = 72 \pm 8\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \tag{5.47}$$

H from HST



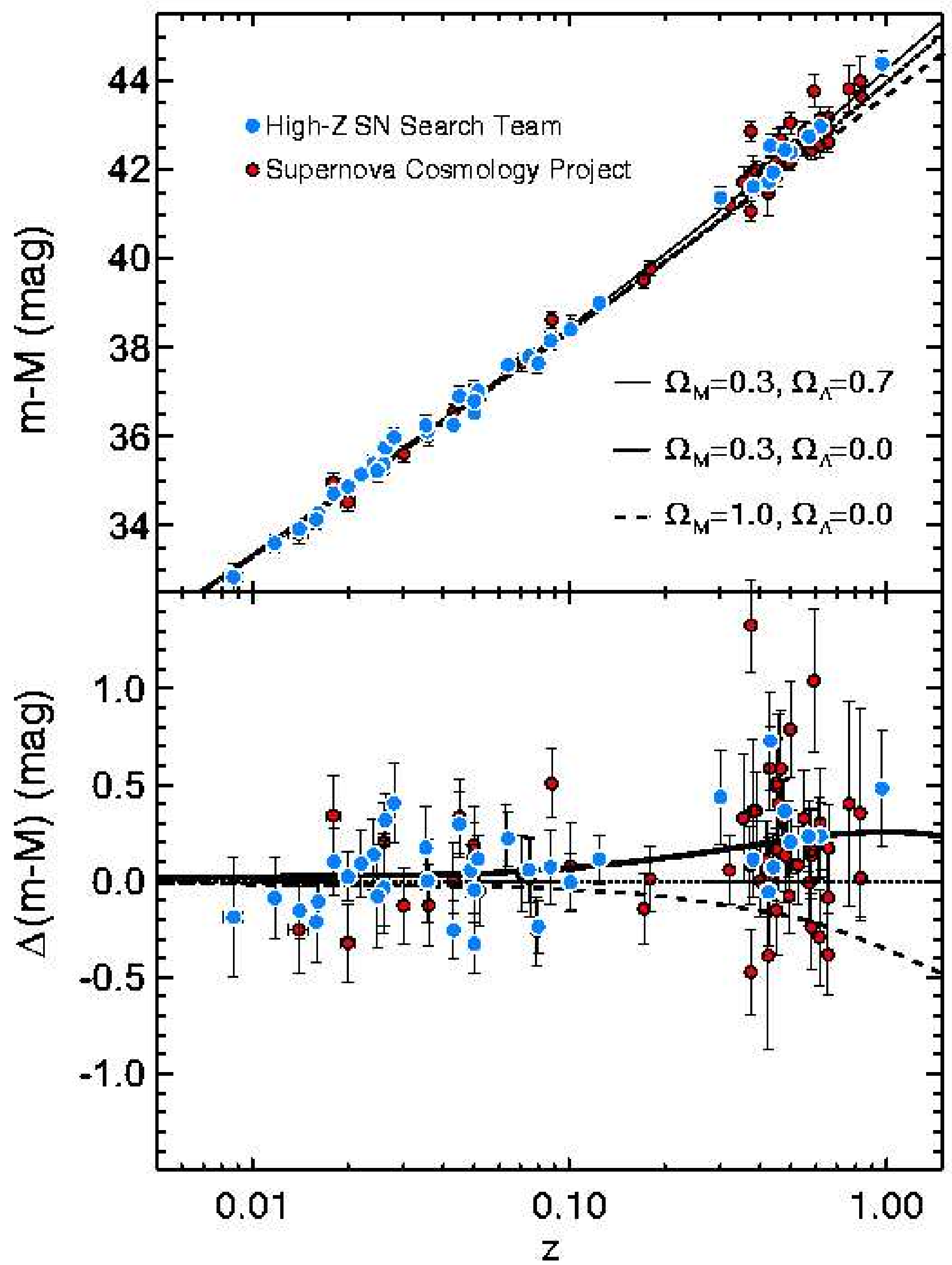
(after Mould et al., 2000, Fig. 5)

Major systematic uncertainty in current H_0 value: zero-point of Cepheid scale, i.e., distance to Large Magellanic Cloud.

Despite these problems:

$\Longrightarrow$ All current values approach $\sim$70 km s^{-1} Mpc^{-1}, with uncertainty $\sim$10%

H_0 controversy is over



For larger distances: There are deviations from Hubble-Relation!

Before we understand why: Need to understand the Big-Bang itself!

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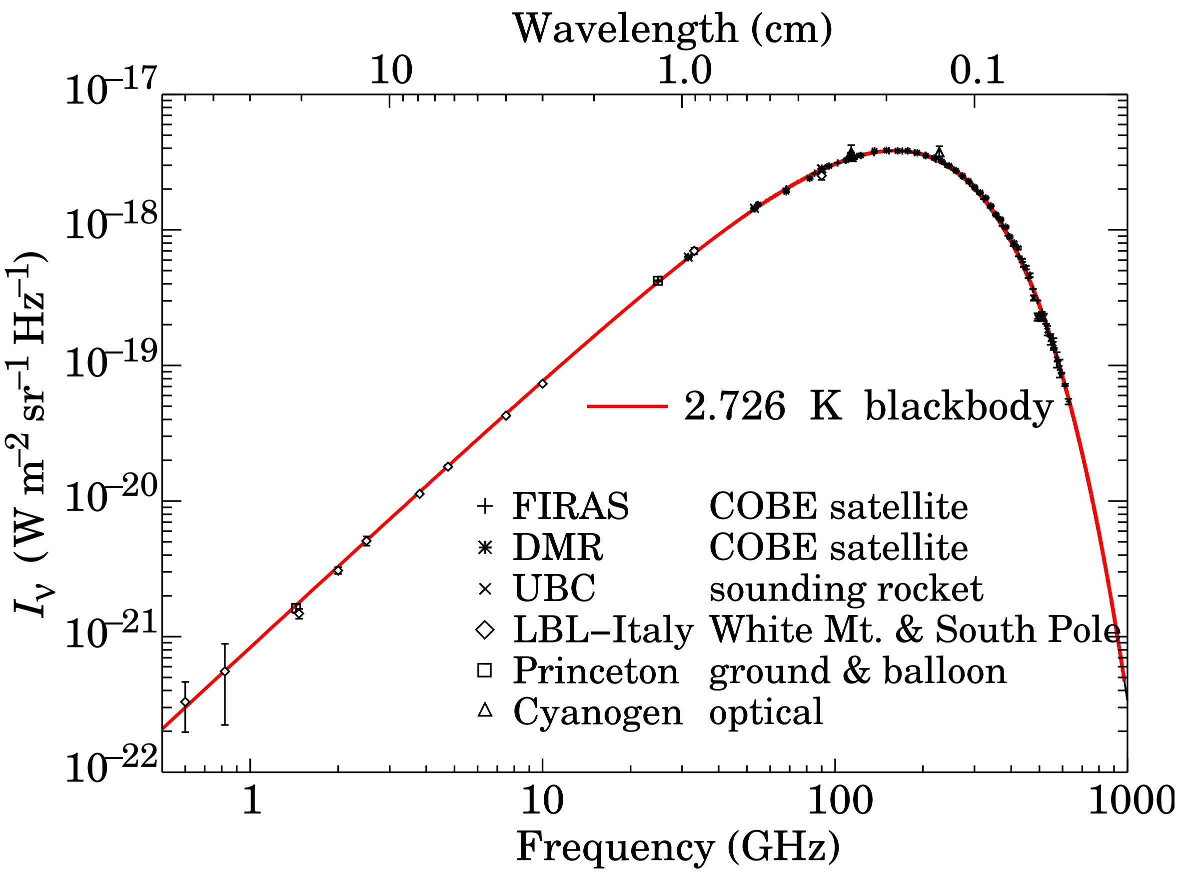
The Hot Big Bang

CMBR



(after Smoot, 1997, Fig. 1)

Penzias & Wilson (1965): “Measurement of Excess Antenna Temperature at 4080 Mc/s”
$\Longrightarrow$ Cosmic Microwave Background Radiation (CMBR)

The CMBR spectrum is fully consistent with a pure Planckian with temperature $T_{\text{CMBR}} = 2.728 \pm 0.004$ K: a relict of the hot big bang.

CMBR

Assumption: Early universe was hot and dense

$\Longrightarrow$ Equilibrium between matter and radiation.

Generation of radiation, e.g., in pair equilibrium,

$$\gamma + \gamma \longleftrightarrow \mathrm{e}^{-} + \mathrm{e}^{+} \tag{6.1}$$

Equilibrium with electrons, e.g., via Compton scattering:

$$\mathrm{e}^{-} + \gamma \longrightarrow \mathrm{e}^{-} + \gamma \tag{6.2}$$

where the electrons are linked to protons via Coulomb interaction.

Once density low and temperature below photoionization for Hydrogen,

$$\mathrm{H} + \gamma \longleftrightarrow \mathrm{p} + \mathrm{e}^{-} \tag{6.3}$$

Decoupling of radiation and matter $\Longrightarrow$ Adiabatic cooling of photon fi eld.

Proof for these assumptions, and lots of gory details: this and the next few lectures!

CMBR

Reminder: Planck formula for energy density of photons:

$$B_\lambda = \frac{\mathrm{d}u}{\mathrm{d}\lambda} = \frac{8\pi hc}{\lambda^5} \frac{1}{\exp(hc/k_\mathrm{B}T\lambda) - 1} \tag{6.4}$$

(units: $\mathrm{erg\,cm^{-3}\,Å^{-1}}$), where

$$k_\mathrm{B} = 1.38\times10^{-16}\,\mathrm{erg\,K^{-1}} \quad \text{(Boltzmann)} \quad \text{and} \quad h = 6.625\times10^{-27}\,\mathrm{erg\,s} \quad \text{(Planck)} \tag{6.5}$$

For $\lambda \gg hc/k_\mathrm{B}T$: Rayleigh-Jeans formula:

$$B_\lambda \sim \frac{8\pi k_\mathrm{B}T}{\lambda^4} \tag{6.6}$$

(classical case, diverges for $\lambda \longrightarrow 0$, “Jeans catastrophe”).

The wavelength of maximum emission is given by Wien’s displacement law:

$$\lambda_\mathrm{max} = 0.201\frac{hc}{k_\mathrm{B}T} \tag{6.7}$$

CMBR

The total energy density of the CMB is obtained by integration:

$$u = \int_0^\infty B_\lambda \, \mathrm{d}\lambda = \frac{8\pi^5 (kT)^4}{15 h^3 c^3} = \frac{4\sigma_{\mathrm{SB}}}{c} T^4 = a_{\mathrm{rad}} T^4 \qquad (6.8)$$

where

$$\sigma_{\mathrm{SB}} = 5.670 \times 10^{-5}\,\mathrm{erg\,cm^{-3}\,K^{-4}} \quad \text{Stefan-Boltzmann} \qquad (6.9)$$

$$a_{\mathrm{rad}} = 7.566 \times 10^{-15}\,\mathrm{erg\,cm^{-2}\,K^{-4}\,s^{-1}} \quad \text{radiation density constant} \qquad (6.10)$$

Since the energy of a photon is $E_\gamma = h\nu = hc/\lambda$, the total number density of photons is

$$n = \int_0^\infty \frac{B_\lambda \, \mathrm{d}\lambda}{hc/\lambda} = 20.28\, T^3\,\mathrm{photons\,cm^{-3}} \qquad (6.11)$$

Thus, for today's CMBR:

$$n_{\mathrm{CMBR}} = 400\,\mathrm{photons\,cm^{-3}} \qquad (6.12)$$

CMBR

For the CMBR today:

$$n_{\text{CMBR}} = 400\,\text{photons}\,\text{cm}^{-3} \tag{6.12}$$

Compare that to gravitating matter (protons for now).

$\Longrightarrow$ critical density:

$$\rho_{\text{c}} = \frac{3H^2}{8\pi G} = 1.88 \times 10^{-29} h^2\ \text{g}\,\text{cm}^{-3} = 1.13 \times 10^{-5}\, h^2\,\text{protons}\,\text{cm}^{-3} \tag{4.58}$$

since $m_{\text{p}} = 1.67 \times 10^{-24}$ g.

$\Longrightarrow$ photons dominate the particle number:

$$\frac{n_{\text{CMBR}}}{n_{\text{baryons}}} = \frac{3.54 \times 10^7}{\Omega h^2} \tag{6.13}$$

$\Longrightarrow$ baryons dominate the energy density:

$$\frac{u_{\text{CMBR}}}{u_{\text{baryons}}} = \frac{a_{\text{rad}} T^4}{\Omega \rho_{\text{c}} c^2} = \frac{4.20 \times 10^{-13}}{1.69 \times 10^{-8} \Omega h^2} = \frac{1}{40260 \Omega h^2} \tag{6.14}$$

That's why we talk about the matter dominated universe.

CMBR

The Universe was not always matter dominated:

Remember the scaling laws for the (energy) density of matter and radiation:

$$\begin{aligned}\rho_\mathrm{m} &\propto R^{-3} \\ \rho_\mathrm{r} &\propto R^{-4}\end{aligned} \quad \Longrightarrow \quad \frac{\rho_\mathrm{r}}{\rho_\mathrm{m}} \propto \frac{1}{R} \tag{4.63, 4.64}$$

$\Longrightarrow$ Photons dominate for large z, i.e., early in the universe!

Since $1+z = R_0/R$ (Eq. 4.40), matter-radiation equality was at

$$1 + z_\mathrm{eq} = 40260\,\Omega h^2 \tag{6.15}$$

(for $h = 0.75$, $1 + z_\mathrm{eq} = 22650$)

The above defi nition of z_eq is not entirely correct: neutrino background, which increases the background energy density, is ignored ($u_\nu \sim 68\% u_\gamma$, see later).

Formally, matter-radiation equality defi ned from $n_\text{baryons} = n_\text{relativistic particles}$, giving

$$1 + z_\mathrm{eq} = 23900\,\Omega h^2 \tag{6.16}$$

(for $h = 0.75$, $1 + z_\mathrm{eq} = 13440$).

CMBR

What happened to the temperature of the CMBR?

Compare CMBR spectrum today with earlier times.

(Differential) Energy density in $[\lambda, \lambda + \mathrm{d}\lambda]$:

$$\mathrm{d}u = B_\lambda \mathrm{d}\lambda \tag{6.17}$$

Cosmological redshift:

$$\frac{\lambda'}{\lambda} = \frac{R'}{R} = \frac{1}{1+z} = a \tag{4.47}$$

Taking the expansion into account:

$$\mathrm{d}u' = \frac{\mathrm{d}u}{a^4} = \frac{8\pi hc}{a^4\lambda^5}\frac{\mathrm{d}\lambda}{\exp(hc/kT\lambda)-1} = \frac{8\pi hc}{a^5\lambda^5}\frac{a\mathrm{d}\lambda}{\exp(hc/kT\lambda)-1}$$
$$= \frac{8\pi hc}{\lambda'^5}\frac{\mathrm{d}\lambda'}{\exp(hca/kT\lambda')-1} = B_{\lambda'}(T/a) \tag{6.18}$$

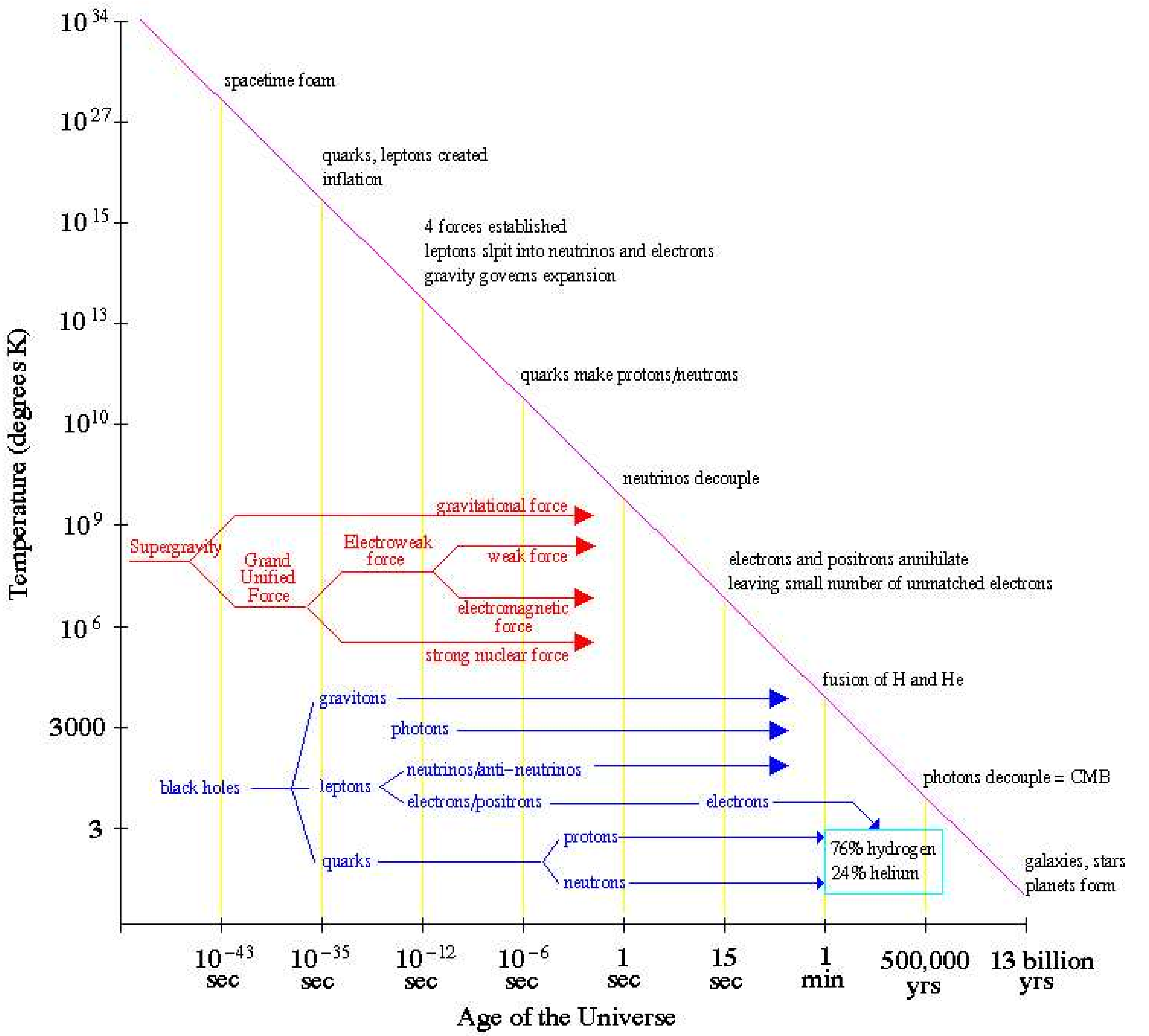
Therefore, the Planckian remains a Planckian, and the temperature of the CMBR scales as

$$T(z) = (1+z)T_0 \tag{6.19}$$

The early universe was hot $\Longrightarrow$ Hot Big Bang Model!

Overview

$a(t)$	t since BB	T[K] [K]	ρ_{matter} [g cm^{-3}]	Major Events
	10^{-42}	10^{30}		Planck era, "begin of physics"
	$10^{-40\ldots-30}$	10^{25}		Inflation?
10^{-13}	$\sim 10^{-5}$ s	$\sim 10^{13}$	$\sim 10^{9}$	generation of p-p^{-}, and baryon anti-baryon pairs from radiation background
3×10^{-9}	1 min	10^{10}	0.03	generation of e^{+}-e^{-} pairs out of radiation background
10^{-9}	10 min	3×10^{9}	10^{-3}	nucleosynthesis
$10^{-4} \ldots 10^{-3}$	$10^{6\ldots7}$ yr	$10^{3\ldots4}$	$10^{-21\ldots-18}$	End of radiation dominated epoch
7×10^{-4}	10^{7} yr	4000	10^{-20}	Hydrogen recombines, decoupling of matter and radiation
1	15×10^{9} yr	3	10^{-30}	now



Temperature (degrees K)
10^34
10^27
10^15
10^13
10^10
10^9
10^6
3000
3
spacetime foam
quarks, leptons created
inflation
4 forces established
leptons slpit into neutrinos and electrons
gravity governs expansion
quarks make protons/neutrons
neutrinos decouple
electrons and positrons annihilate
leaving small number of unmatched electrons
fusion of H and He
photons decouple = CMB
galaxies, stars
planets form
Supergravity
Grand Unified Force
Electroweak force
gravitational force
weak force
electromagnetic force
strong nuclear force
black holes
gravitons
photons
leptons
neutrinos/anti-neutrinos
electrons/positrons
electrons
quarks
protons
neutrons
76% hydrogen
24% helium
10^-43 sec
10^-35 sec
10^-12 sec
10^-6 sec
1 sec
15 sec
1 min
500,000 yrs
13 billion yrs
Age of the Universe

Thermodynamics, I

Density in early universe is very high.

Physical processes (e.g., photon-photon pair creation, electron-positron annihilation etc.) all have reaction rates

$$\Gamma \propto n\sigma v \tag{6.20}$$

where

n**:** number density (cm^{-3})
σ**:** interaction cross-section (cm^{2})
v**:** velocity (cm s^{-1})

Thermodynamic equilibrium reached if reaction rate much faster than "changes" in the system,

$$\Gamma \gg H \tag{6.21}$$

Where the Hubble parameter, H, is a good measure for $\left(\text{typical timescale of the Universe}\right)^{1}$.

If thermodynamic equilibrium holds, then we can assume evolution of universe as sequence of states of local thermodynamic equilibrium, and use standard thermodynamics.

Before looking at real universe, first need to derive certain useful formulae from relativistic thermodynamics.

Thermodynamics, II

For ideal gases, thermodynamics shows that number density $f(\boldsymbol{p})\,\mathrm{d}p$ of particles with momentum in $[p, p + \mathrm{d}p]$ is given by

$$f(\boldsymbol{p}) = \frac{1}{\exp\left((E-\mu)/k_\mathrm{B}T\right) + a} \tag{6.22}$$

where

$$a = \begin{cases} +1 & : \text{Fermions (spin=1/2, 3/2,...)} \\ -1 & : \text{Bosons (spin=1, 2,...)} \\ 0 & : \text{Maxwell-Boltzmann} \end{cases}$$

and where the energy includes the rest-mass:

$$E^2 = |\boldsymbol{p}|^2 c^2 + m^2 c^4 \tag{6.23}$$

μ is called the "chemical potential". It is preserved in chemical equilibrium:

$$i + j \leftrightarrow k + l \quad \Longrightarrow \quad \mu_i + \mu_j = \mu_k + \mu_l \tag{6.24}$$

photons: multi-photon processes exist $\Longrightarrow \mu_\gamma = 0$.
particles in thermal equilibrium: $\mu = 0$ as well because of the fi rst law of thermodynamics,

$$\mathrm{d}E = T\,\mathrm{d}S - P\,\mathrm{d}V + \mu\,\mathrm{d}N \tag{6.25}$$

and in equilibrium system stationary with respect to changes in particle number N.

Thermodynamics, III

In addition to number density: different particles have internal degrees of freedom, g.

Examples:

photons: two polarization states $\Longrightarrow g = 2$
neutrinos: one polarization state $\Longrightarrow g = 1$
electrons, positrons: spin=$1/2 \Longrightarrow g = 2$

Knowing g and $f(p)$, it is possible to calculate interesting quantities:

particle number density: $$n = \frac{g}{(2\pi\hbar)^3} \int f(\boldsymbol{p})\, \mathrm{d}^3p \quad (6.26)$$

energy density: $$u = \rho c^2 = \frac{g}{(2\pi\hbar)^3} \int E(\boldsymbol{p})\, f(\boldsymbol{p})\, \mathrm{d}^3p \quad (6.27)$$

To calculate the pressure, remember that kinetic theory shows:

$$P = \frac{n}{3}\langle pv\rangle = \frac{n}{3}\left\langle \frac{p^2c^2}{E}\right\rangle \quad (6.28)$$

such that

$$P = \frac{g}{(2\pi\hbar)^3} \int \frac{p^2c^2}{3E}\, f(\boldsymbol{p})\, \mathrm{d}^3p \quad (6.29)$$

Thermodynamics, IV

Generally, we are interested in knowing n, u, and P in two limiting cases:

1. the ultra-relativistic limit, where $k_\mathrm{B}T \gg mc^2$, i.e., kinetic energy dominates the rest-mass
2. the non-relativistic limit, where $k_\mathrm{B}T \ll mc^2$

Transitions between these limits (i.e., what happens during “cooling”) are usually much more complicated $\Longrightarrow$ ignore...

To derive the number density, the energy density, and the equation of state, note that Eq. (6.23) shows

$$E = \sqrt{p^2c^2 + m^2c^4} \tag{6.23}$$

such that

$$p = \sqrt{E^2 - m^2c^4}/c \tag{6.30}$$

Therefore

$$\frac{\mathrm{d}E}{\mathrm{d}p} = \frac{pc^2}{\sqrt{p^2c^2 + m^2c^4}} \tag{6.31}$$

from which it follows that

$$E\ \mathrm{d}E = pc^2\ \mathrm{d}p \tag{6.32}$$

Thus the following holds

$$\iiint_{-\infty}^{+\infty} \mathrm{d}^3p = \int_0^\infty 4\pi p^2\ \mathrm{d}p = \int_{mc^2}^\infty \frac{4\pi}{c^3}\left(E^2 - m^2c^4\right)^{1/2} E\ \mathrm{d}E \tag{6.33}$$

Going to a system of units where

$$c = k_\mathrm{B} = \hbar = 1 \tag{6.34}$$

to save me some typing, substitute these equations into Eqs. (6.26)–(6.29) to fi nd

$$n = \frac{g}{2\pi^2}\int_m^\infty \frac{\left(E^2 - m^2\right)^{1/2} E\ \mathrm{d}E}{\exp\left((E-\mu)/T\right) \pm 1} \tag{6.35}$$

$$\rho = \frac{g}{2\pi^2}\int_m^\infty \frac{\left(E^2 - m^2\right)^{1/2} E^2\ \mathrm{d}E}{\exp\left((E-\mu)/T\right) \pm 1} \tag{6.36}$$

$$P = \frac{g}{6\pi^2}\int_m^\infty \frac{\left(E^2 - m^2\right)^{3/2}\ \mathrm{d}E}{\exp\left((E-\mu)/T\right) \pm 1} \tag{6.37}$$

which can in some limiting cases be expressed in a closed form (Kolb & Turner, 1990, eq. 3.52 ff.) (see following viewgraphs).

Thermodynamics, V

In the ultra-relativistic limit, $k_\mathrm{B}T \gg mc^2$, and assuming $\mu = 0$,

$$n = \begin{cases} \frac{\zeta(3)}{\pi^2}\, g \left(\frac{k_\mathrm{B}T}{\hbar c}\right)^3 & \text{Bosons} \\ \frac{3}{4}\,\frac{\zeta(3)}{\pi^2}\, g \left(\frac{k_\mathrm{B}T}{\hbar c}\right)^3 & \text{Fermions} \end{cases} \tag{6.38}$$

$$u = \begin{cases} \frac{\pi^2}{30}\, g\, k_\mathrm{B}T \left(\frac{k_\mathrm{B}T}{\hbar c}\right)^3 & \text{Bosons} \\ \frac{7}{8}\,\frac{\pi^2}{30}\, g\, k_\mathrm{B}T \left(\frac{k_\mathrm{B}T}{\hbar c}\right)^3 & \text{Fermions} \end{cases} \tag{6.39}$$

$$P = \rho c^2/3 = u/3 \tag{6.40}$$

where $\zeta(3) = 1.202\ldots$, and $\zeta(s)$ is Riemann's zeta-function (see handout, Eq. 6.48).

Eq. (6.40) is a simple result of the fact that in the relativistic limit, $E \sim pc$. Inserting this and $v = c$ into Eq. (6.28) gives the desired result.

As expected, we find the T^4 proportionality from the Stefan Boltzmann law!

Obtaining the previous formulae is an exercise in special functions. For example, the $T \gg m$, $T \gg \mu$ case for ρ for Bosons (Eq. 6.39) is obtained as follows (setting $c = k_\mathrm{B} = \hbar = 1$):

$$\rho_\mathrm{Boson} = \frac{g}{2\pi^2} \int_m^\infty \frac{\left(E^2 - m^2\right)^{1/2} E^2 \, \mathrm{d}E}{\exp\left((E-\mu)/T\right) \pm 1} \tag{6.41}$$

because of $T \gg \mu$

$$\approx \frac{g}{2\pi^2} \int_m^\infty \frac{\left(E^2 - m^2\right)^{1/2} E^2 \, \mathrm{d}E}{\exp(E/T) \pm 1} \tag{6.42}$$

for *Bosons*, choose -1, and substitute $x = E/T$:

$$= \frac{g}{2\pi^2} \int_{m/T}^\infty \frac{\left(x^2T^2 - m^2\right)^{1/2} x^2 T^3 \, \mathrm{d}x}{\exp(x) - 1} \tag{6.43}$$

Since $T \gg m$,

$$\approx \frac{g}{2\pi^2} \int_0^\infty \frac{x^3 T^4 \, \mathrm{d}x}{\exp(x) - 1} \tag{6.44}$$

$$= \frac{gT^4}{2\pi^2} \int_0^\infty \frac{x^3 \, \mathrm{d}x}{\exp(x) - 1} \tag{6.45}$$

$$= \frac{gT^4}{2\pi^2} \cdot 6\zeta(4) \tag{6.46}$$

$$= \frac{\pi^2}{30} gT^4 \tag{6.47}$$

where $\zeta(s)$ is *Riemann's zeta-function*, which is defined by (Abramowitz & Stegun, 1964)

$$\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty \frac{x^{s-1}}{\exp(x) - 1} \, \mathrm{d}x \quad \text{for } \mathscr{R}e\, s > 1 \tag{6.48}$$

where $\Gamma(x)$ is the Gamma-function. Note that $\zeta(4) = \pi^4/90$.

For *Fermions*, everything is the same except for that we now have to choose the $+$ sign. The equivalent of Eq. (6.45) is then

$$\rho_{\mathrm{Fermi}} = \frac{gT^4}{2\pi^2}\int_0^\infty \frac{x^3\,\mathrm{d}x}{\exp(x)+1} \tag{6.49}$$

Now we can make use of formula 3.411.3 of Gradstein & Ryshik (1981),

$$\int_0^\infty \frac{x^{\nu-1}\,\mathrm{d}x}{\exp(\mu x)+1} = \frac{1}{\mu^\nu}(1-2^{1-\nu})\Gamma(\nu)\zeta(\nu) \quad \text{for } \mathscr{R}e\,\mu,\nu>1 \tag{6.50}$$

to see where the additional factor of $7/8$ in Eq. (6.39) comes from.

Thermodynamics, VI

In the non-relativistic limit: $k_\mathrm{B}T \ll mc^2$

$\Longrightarrow$ can ignore the ± 1 term in the denominator

$\Longrightarrow$ Same formulae for Bosons and Fermions!

$$n = \frac{2g}{(2\pi\hbar)^3}(2\pi m k_\mathrm{B} T)^{3/2} \mathrm{e}^{-mc^2/k_\mathrm{B}T} \tag{6.51}$$

$$u = nmc^2 \tag{6.52}$$

$$P = nk_\mathrm{B}T \tag{6.53}$$

Therefore:

- density dominated by rest-mass ($\rho = u/c^2 = mn$)
- $P \ll \rho c^2/3$, i.e., *much* smaller than for relativistic particles.
- Particle pressure only important if particles are relativistic.

Obviously, relativistic particles with $m = 0$ (or very close to 0) will never get nonrelativistic. Still, they can "decouple" from the rest of the universe when the interaction rates go to 0.

Equation of State

Pressure of ultra-relativistic particles $\gg$ Pressure of nonrelativistic particles
$\Longrightarrow$ Nonrelativistic particles unimportant for equation of state.

For relativistic particles:

$$u_{\text{bosons}} = \frac{\pi^2}{30} g\, k_{\text{B}} T \left(\frac{k_{\text{B}}T}{\hbar c}\right)^3 \quad \text{and} \quad u_{\text{fermions}} = \frac{7}{8} u_{\text{bosons}} \tag{6.39}$$

$\Longrightarrow$ Total energy density for mixture of particles:

$$u = g_* \cdot \frac{\pi^2}{30} k_{\text{B}} T \left(\frac{k_{\text{B}}T}{\hbar c}\right)^3 \tag{6.54}$$

where the effective degeneracy factor

$$g_* = \sum_{\text{bosons}} g_{\text{B}} \left(\frac{T_{\text{B}}}{T}\right)^4 + \frac{7}{8} \sum_{\text{fermions}} g_{\text{F}} \left(\frac{T_{\text{F}}}{T}\right)^4 \tag{6.55}$$

g_* counts total number of internal degrees of freedom of *all* relativistic bosonic and fermionic species, i.e., all relativistic particles which are in thermodynamic equilibrium

The pressure is obtained from Eq. (6.54) via $P = u/3$.

Early Expansion, I

Knowing the equation of state, we can now use Friedmann equations to determine the early evolution of the universe.

Friedmann:

$$\dot{R}^2 = \frac{8\pi G}{3}\rho R^2 - kc^2 \tag{4.55}$$

or, dividing by R^2

$$\frac{\dot{R}^2}{R^2} = H(t)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2} \tag{4.56}$$

But: The early universe is dominated by relativistic particles

$\Longrightarrow \rho \propto R^{-4}$

$\Longrightarrow$ Density-term dominates

$\Longrightarrow$ we can set $k = 0$.

Early universe is asymptotically flat!

This will prove to be one of the most crucial problems of modern cosmology...

Early Expansion, II

To obtain the evolution of the early universe, insert the Equation of State (Eq. 6.54) into Eq. (4.56):

$$H(t)^2 = \frac{8\pi G}{3}\, g_*\, \frac{\pi^2}{30}\frac{(k_\mathrm{B}T)^4}{(\hbar c)^3} = \frac{4\pi^3 G}{45(\hbar c)^3}\, g_*\, (k_\mathrm{B}T)^4 \quad (6.56)$$

such that

$$H(t) = \left(\frac{4\pi^3 G}{45(\hbar c)^3}\right)^{1/2} g_*^{1/2}\,(k_\mathrm{B}T)^2 \quad (6.57)$$

On the other hand, since $\rho \propto R^{-4}$ (relativistic background),

$$\rho = \rho_0 \left(\frac{R_0}{R}\right)^4 \quad (6.58)$$

Friedmann:

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \sqrt{\frac{8\pi G\rho_0}{3}\frac{R_0^2}{R}} \quad (6.59)$$

Introducing the dimensionless scale factor, $a = R/R_0$ (Eq. 4.29), gives

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \sqrt{\frac{8\pi G\rho_0}{3}\frac{1}{a}} =: \xi a^{-1} \quad (6.60)$$

Early Expansion, III

And using separation of variables gives

$$\int_0^{a(t)} a\,\mathrm{d}a = \int_0^t \xi\,\mathrm{d}t \quad \Longrightarrow \quad a(t) = \xi^{1/2} \cdot t^{1/2} \tag{6.61}$$

Therefore, the Hubble constant evolves as

$$H(t) = \frac{\dot{a}}{a} = \frac{1}{2t} \tag{6.62}$$

Equating Eqs. (6.57) and (6.62) gives the time-temperature relationship:

$$t = \left(\frac{45(\hbar c)^3}{16\pi^3 G}\right)^{1/2} \frac{1}{g_*^{1/2}} \frac{1}{(k_\mathrm{B}T)^2} \tag{6.63}$$

Inserting all constants and converting to more useful units gives

$$t = \frac{2.4\ \mathrm{sec}}{g_*^{1/2}} \cdot \left(\frac{k_\mathrm{B}T}{1\,\mathrm{MeV}}\right)^{-2} \tag{6.64}$$

. . . one of the most useful equations for the early universe.

Elementary Particles, I

Behavior of universe depends on g_* $\Longrightarrow$ Strong dependency on elementary particle physics.

Generally, particles present when energy in other particles allows generation of particle–antiparticle pairs, i.e., when $k_\mathrm{B}T \gtrsim mc^2$ (threshold temperature)

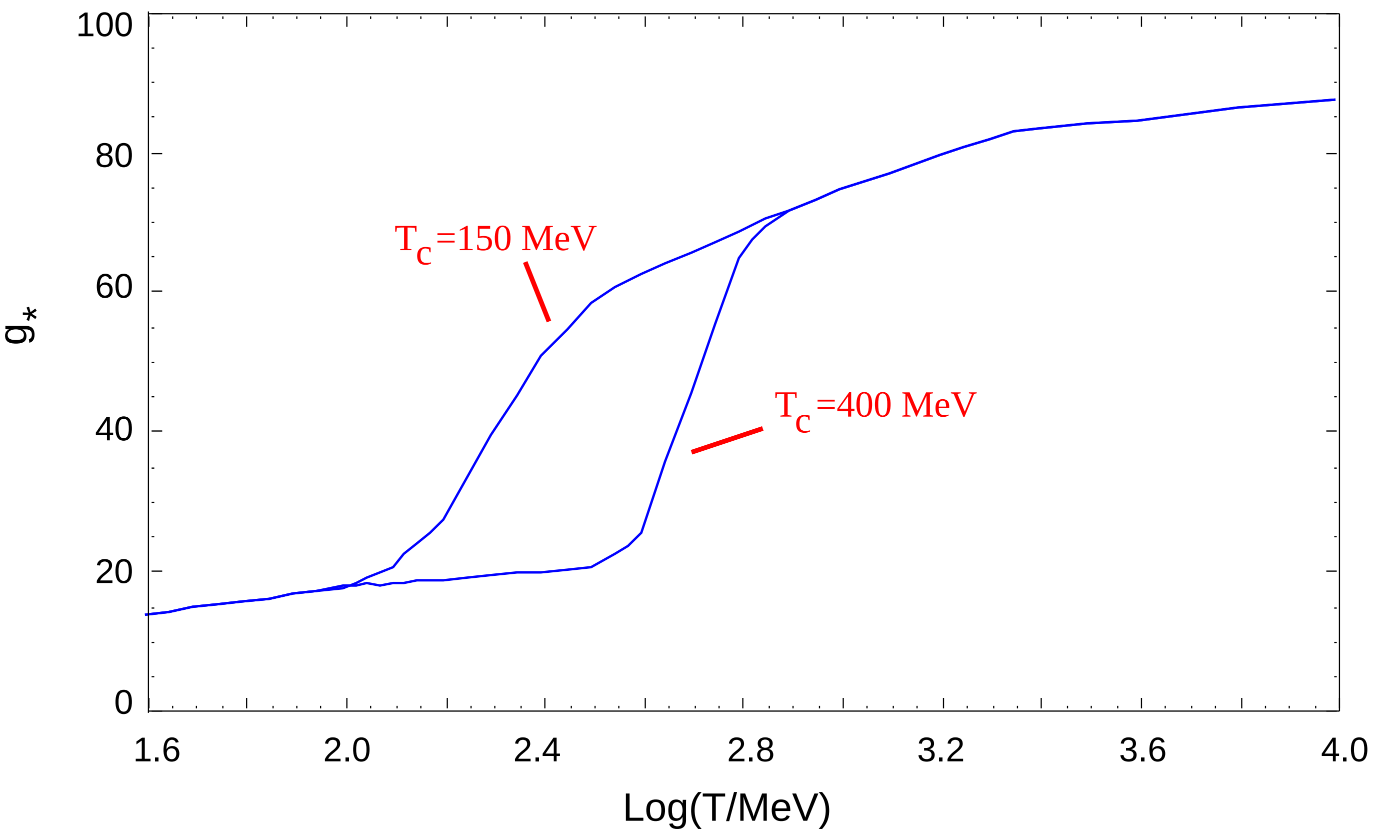
Current particle physics provides the following picture (Olive, 1999, Tab. 1):

Temp.	New Particles	$4g_*$
$k_\mathrm{B}T < m_\mathrm{e}c^2$	γ's and ν's	29
$m_\mathrm{e}c^2 < k_\mathrm{B}T < m_\mu c^2$	$\mathrm{e}^\pm$	43
$m_\mu c^2 < k_\mathrm{B}T < m_\pi c^2$	$\mu^\pm$	57
$m_\pi c^2 < k_\mathrm{B}T < k_\mathrm{B}T_\mathrm{c}$	π's	69
$k_\mathrm{B}T_\mathrm{c} < k_\mathrm{B}T < m_\mathrm{strange}c^2$	$-\pi$'s$+$u, $\bar{\mathrm{u}}$, d, $\bar{\mathrm{d}}$, gluons	205
$m_\mathrm{s}c^2 < k_\mathrm{B}T < m_\mathrm{charm}c^2$	s, $\bar{\mathrm{s}}$	247
$m_\mathrm{c}c^2 < k_\mathrm{B}T < m_\tau c^2$	c, $\bar{\mathrm{c}}$	289
$m_\tau c^2 < k_\mathrm{B}T < m_\mathrm{bottom}c^2$	$\tau^\pm$	303
$m_\mathrm{b}c^2 < k_\mathrm{B}T < m_\mathrm{W,Z}c^2$	b, $\bar{\mathrm{b}}$	345
$m_\mathrm{W,Z}c^2 < k_\mathrm{B}T < m_\mathrm{top}c^2$	$\mathrm{W}^\pm$, Z	381
$m_\mathrm{t}c^2 < k_\mathrm{B}T < m_\mathrm{Higgs}c^2$	t, $\bar{\mathrm{t}}$	423
$m_\mathrm{H}c^2 < k_\mathrm{B}T$	H^0	427

T_c: energy of confinement-deconfinement for transitions quarks $\Longrightarrow$ hadrons, somewhere between 150 MeV and 400 MeV.

Example: photons (2 polarization states, i.e., $g = 2$) and three species of neutrinos ($g = 1$, but with distinguishable anti-particles) $\Longrightarrow$ $g_* = 2 + (7/8) \cdot 2 \cdot 3 = 58/8 = 29/4$.

Elementary Particles, II



(Olive, 1999, Fig. 1)

Will now consider times when only Neutrinos and Electron/Positrons present (after baryogenesis, see next lecture for that).

Interlude

Previous (abstract) formulae allow to estimate quantities like

1. The existence and energy of primordial neutrinos,
2. The formation of neutrons,
3. The formation of heavier elements.

Detailed computations require solving nonlinear differential equations
$\Longrightarrow$ diffi cult, only numerically possible.

Essentially, need to self-consistently solve Boltzmann equation in expanding universe for evolution of phase space density with time, using the correct QCD/QED reaction rates $\Longrightarrow$ too complicated (at least for me. . .).

Will use approximate analytical way here, which gives surprisingly exact answers.

Neutrinos, I

Neutrino equilibrium caused by weak interactions such as

$$\mathrm{e}^- + \mathrm{e}^+ \longleftrightarrow \nu + \bar{\nu} \quad \text{or} \quad \mathrm{e}^- + \nu \longleftrightarrow \mathrm{e}^- + \nu \quad etc. \tag{6.65}$$

Reaction rate for these processes:

$$\Gamma = n \langle \sigma v \rangle \tag{6.66}$$

where the thermally averaged interaction cross-section is

$$\langle \sigma v \rangle \approx \left\langle \frac{\alpha^2 p}{m_\mathrm{W}^4} \cdot p \right\rangle \sim 10^{-2} \frac{(k_\mathrm{B} T)^2}{m_\mathrm{W}^4} \tag{6.67}$$

m_W: mass of W-boson (exchange particle of weak interaction), $\alpha \approx 1/137$: fine structure constant.

But in the ultra-relativistic limit, $n \propto T^3$ (Eq. 6.38), such that

$$\Gamma_\mathrm{weak} \propto \frac{\alpha^2 T^5}{m_\mathrm{W}^4} \tag{6.68}$$

Neutrinos, II

Because of Eqs. (6.62) and (6.63), the temperature dependence of the Hubble constant is

$$H(T) = 1.66 g_*^{1/2} \cdot \frac{T^2}{m_\mathrm{P}} \tag{6.69}$$

where m_P is the Planck mass, $m_\mathrm{P}c^2 = 1.22 \times 10^{19}$ GeV (see later, Eq. 7.24).

Neutrino equilibrium possible as long as $\Gamma_\mathrm{weak} > H$, i.e., (inserting exact numbers)

$$k_\mathrm{B} T_\mathrm{dec} \gtrsim \left(\frac{500\, c^6\, m_\mathrm{W}^4}{m_\mathrm{P}} \right)^{1/3} \sim 1\,\mathrm{MeV} \tag{6.70}$$

Neutrinos decouple $\sim 1\,\mathrm{s}$ after the big bang.

This follows from Eq. (6.64), remembering that for this phase, $g_* \sim 10$.

Since decoupling, primordial neutrinos just follow expansion of universe, virtually no interaction with “us” anymore.

Entropy, I

The entropy of particles is defined through

$$S = \frac{E + PV}{T} \tag{6.71}$$

Important for cosmology: relativistic limit. Define the entropy density,

$$s = \frac{S}{V} = \frac{E/V + P}{T} = \frac{u + P}{T} \approx \frac{4}{3}\frac{u}{T} \tag{6.72}$$

(last step for relativistic limit; Eq. 6.40)

Inserting Eq. (6.39) ($u \propto (7/8)T^4$; 7/8 for Fermions only) gives

$$s = \frac{7}{8}\frac{2\pi^2}{45} g k_\mathrm{B} \left(\frac{k_\mathrm{B}T}{\hbar c}\right)^3 = \frac{7}{8}\frac{2\pi^4}{45\,\zeta(3)} k_\mathrm{B}\, n \tag{6.73}$$

Since $s \propto n$ for backgrounds, $\eta = n_\mathrm{CMBR}/n_\mathrm{baryons}$ is often called "entropy per baryon".

Entropy, II

For a mixture of backgrounds, Eq. (6.73) gives

$$\frac{s}{k_B} = g_{*,S} \cdot \frac{2\pi^2}{45} \left(\frac{k_B T}{\hbar c}\right)^3 \tag{6.74}$$

where $g_{*,S}$ is the analogue to g_* (Eq. 6.55),

$$g_{*,S} = \sum_{\text{bosons}} g_B \left(\frac{T_B}{T}\right)^3 + \frac{7}{8} \sum_{\text{fermions}} g_F \left(\frac{T_F}{T}\right)^3 \tag{6.75}$$

Note that if the species are not at the same temperature, $g_* \neq g_{*,S}$.

Entropy per mass today:

$$\frac{S}{M} = \frac{10^{16}}{\Omega h^2}\,\text{erg}\,\text{K}^{-1}\,\text{g}^{-1} \tag{6.76}$$

while the entropy gain of heating water at 300 K by 1 K is $\sim 1.4 \times 10^5\,\text{erg}\,\text{K}^{-1}\,\text{g}^{-1}$.

$\Longrightarrow$ "Human attempts to obey 2nd law ... are swamped by ... microwave background" (Peacock, 1999, p. 277).

$\Longrightarrow$ $S =$ const. for universe to very good approximation.

$\Longrightarrow$ Universe expansion is adiabatic!

Reheating

After decoupling of neutrinos, neutrino distribution just gets redshifted (similar to CMBR, Eq. 6.19):

$$\frac{T_\nu}{T_{\text{dec}}} = \frac{R_{\text{dec}}}{R(t)} \implies T_\nu \propto R^{-1} \tag{6.77}$$

On the other hand, the temperature of the universe is

$$T \propto g_{*,S}^{1/3} \, R^{-1} \tag{6.78}$$

This follows from $S/V \propto T^3$ (Eq. 6.74), $V \propto R^3$, and $S = \text{const.}$ (adiabatic expansion of the universe).

$\Longrightarrow$ as long as $g_{*,S} = \text{const.}$ we have $T_\nu = T$

$\Longrightarrow$ Immediately after decoupling, neutrino background appears as if it is still in equilibrium.

However: Temperature for neutrino decoupling $\sim 2m_e c^2$. But, for $kT_{\text{BB}} < 2\,m_e c^2$, pair creation,

$$\gamma + \gamma \longleftrightarrow e^- + e^+ \tag{6.79}$$

is kinematically impossible.

$\Longrightarrow$ Shortly after neutrino decoupling: $e^\pm$ annihilation

$\Longrightarrow$ $g_{*,S}$ changes!

$\Longrightarrow$ We expect that $T_{\text{CMBR}} \neq T_\nu$.

Reheating

Difference in $g_{*,S}$:

- before annihilation: e$^-$, e$^+$, $\gamma \Longrightarrow g_{*,S} = 2 + 2 \cdot 2 \cdot (7/8) = 11/2$.
- after annihilation: $\gamma \Longrightarrow g_{*,S} = 2$

But: the total entropy for particles in equilibrium conserved ("expansion is adiabatic"):

$$g_{*,S}(T_{\text{before}}) \cdot T_{\text{before}}^3 = g_{*,S}(T_{\text{after}}) \cdot T_{\text{after}}^3 \tag{6.80}$$

such that

$$T_{\text{after}} = \left(\frac{11}{4}\right)^{1/3} T_{\text{before}} \sim 1.4 \cdot T_{\text{before}} \tag{6.81}$$

Since $T_{\text{after}} > T_{\text{before}}$: "reheating".

Note that in reality the annihilation is not instantaneous and T decreases (albeit less rapidly) during "reheating"...

$\Longrightarrow$ Since neutrino-background does not "see" annihilation
$\Longrightarrow$ just continues to cool
$\Longrightarrow$ current temperature of neutrinos is

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_{\text{CMBR}} \sim 1.95\,\text{K} \tag{6.82}$$

History

After reheating: universe consists of p, n, γ (and e^- to preserve charge neutrality)

$\Longrightarrow$ Ingredients for Big Bang Nucleosynthesis (BBN).

Historical perspective: Cross section to make Deuterium:

$$\langle \sigma v \rangle (\mathrm{p} + \mathrm{n} \longrightarrow \mathrm{D} + \gamma) \sim 5 \times 10^{-20}\,\mathrm{cm}^3\,\mathrm{s}^{-1} \tag{6.83}$$

Furthermore, we need temperatures of $T_{\mathrm{BBN}} \sim 100\,\mathrm{keV}$, i.e., $t_{\mathrm{BBN}} \sim 200\,\mathrm{s}$ (Eq. 6.64).
By Eq. (6.20) this implies a particle density of

$$n \sim \frac{1}{\langle \sigma v \rangle \cdot t_{\mathrm{BBN}}} \sim 10^{17}\,\mathrm{cm}^{-3} \tag{6.84}$$

Today: Baryon density $n_{\mathrm{B}} \sim 10^{-7}\,\mathrm{cm}^{-3}$. Since $n \propto R^{-3}$,

$$T(\mathrm{today}) = \left(\frac{n_{\mathrm{B}}}{n}\right)^{1/3} \cdot T_{\mathrm{BBN}} \sim 10\,\mathrm{K} \tag{6.85}$$

pretty close to the truth...

The above discussion was first asserted by George Gamov and coworkers in 1948, and was the first prediction of the cosmic microwave background radiation!

Observations: BBN is required by observations, since no other production region for Deuterium known, and since He-abundance $\sim$ 25% by mass everywhere.

Proton/Neutron, I

Initial conditions for BBN: Set by Proton-Neutron-Ratio.

For $t \ll 1\,\mathrm{s}$, equilibrium via weak interactions:

$$\begin{aligned} \mathrm{n} &\longleftrightarrow p + \mathrm{e}^- + \bar{\nu}_\mathrm{e} \\ \nu_\mathrm{e} + \mathrm{n} &\longleftrightarrow p + \mathrm{e}^- \\ \mathrm{e}^+ + \mathrm{n} &\longleftrightarrow p + \bar{\nu}_\mathrm{e} \end{aligned} \tag{6.86}$$

Reactions fast as long as particles relativistic.

But once $T \sim 1\,\mathrm{MeV}$: n, p become non-relativistic

$\Longrightarrow$ Boltzmann statistics applies (or use Eq. 6.51):

$$\frac{n_\mathrm{n}}{n_\mathrm{p}} = \mathrm{e}^{-\Delta mc^2/k_\mathrm{B}T} = \mathrm{e}^{-1.3\,\mathrm{MeV}/k_\mathrm{B}T} \tag{6.87}$$

$\Longrightarrow$ Suppression of n with respect to p because of larger mass ($m_\mathrm{n}c^2 = 939.57\,\mathrm{MeV}$, $m_\mathrm{p}c^2 = 938.27\,\mathrm{MeV}$)

Proton/Neutron, II

As usual, the n, p abundance freezes out when $\Gamma \gg H$.

For the neutron, proton equilibrium, the reaction rate is

$$\Gamma(\nu_\mathrm{e} + \mathrm{n} \leftrightarrow \mathrm{p} + \mathrm{e}^-) \sim 2.1 \left(\frac{T}{1\,\mathrm{MeV}}\right)^5 \mathrm{s}^{-1} \tag{6.88}$$

The neutron abundance freezes out at $k_\mathrm{B}T \sim 0.8\,\mathrm{MeV}$ ($t = 1.7\,\mathrm{s}$), such that $n_\mathrm{n}/n_\mathrm{p} = 0.2$

After that: Neutron decay ($\tau_\mathrm{n} = 886.7 \pm 1.2\,\mathrm{s}$).

$\Longrightarrow$ Nucleosynthesis has to be over before neutrons are decayed away!

$\Longrightarrow$ Nucleosynthesis only takes a few minutes at most!

Deuterium

The first step in nucleosynthesis is the formation of deuterium (binding energy $E_\mathrm{B} = 2.225\,\mathrm{MeV}$, i.e., $1.7(m_\mathrm{n} - m_\mathrm{p})c^2$):

$$\mathrm{p} + \mathrm{n} \longleftrightarrow \mathrm{D} + \gamma \tag{6.89}$$

Note: Both fusion and photodisintegration are possible:

$$\Gamma_\mathrm{fusion} = n_\mathrm{B}\langle\sigma v\rangle \tag{6.90}$$

$$\Gamma_\mathrm{photo} = n_\gamma\langle\sigma v\rangle \mathrm{e}^{-E_\mathrm{B}/k_\mathrm{B}T} \tag{6.91}$$

At first: photodisintegration dominates since $\eta^{-1} = n_\gamma/n_\mathrm{B} \sim 10^{10}$ (see Eq. 6.73).

Build up of D is only possible once $\Gamma_\mathrm{fusion} > \Gamma_\mathrm{photo}$, i.e., when

$$\frac{n_\gamma}{n_\mathrm{B}}\mathrm{e}^{-E_\mathrm{B}/k_\mathrm{B}T} \sim 1 \tag{6.92}$$

Inserting numbers shows that

Deuterium production starts at $k_\mathrm{B}T \sim 100\,\mathrm{keV}$, or $t \sim 100\,\mathrm{s}$.

Heavier Elements, I

Once deuterium present:

nucleosynthesis of lighter elements:

$$
\begin{aligned}
\mathrm{D} + \mathrm{D} &\longrightarrow \mathrm{T} + \mathrm{p} \\
\mathrm{D} + \mathrm{n} &\longrightarrow \mathrm{T} + \gamma \\
\mathrm{D} + \mathrm{p} &\longrightarrow {}^{3}\mathrm{He} + \gamma \\
\mathrm{D} + \mathrm{D} &\longrightarrow {}^{3}\mathrm{He} + \mathrm{n} \\
{}^{3}\mathrm{He} + \mathrm{n} &\longrightarrow \mathrm{T} + \mathrm{p}
\end{aligned}
\tag{6.93}
$$

production of ^{4}He:

$$
\begin{aligned}
\mathrm{D} + \mathrm{D} &\longrightarrow {}^{4}\mathrm{He} + \gamma \\
\mathrm{D} + {}^{3}\mathrm{He} &\longrightarrow {}^{4}\mathrm{He} + \mathrm{p} \\
\mathrm{T} + \mathrm{D} &\longrightarrow {}^{4}\mathrm{He} + \mathrm{n} \\
{}^{3}\mathrm{He} + {}^{3}\mathrm{He} &\longrightarrow {}^{4}\mathrm{He} + 2\mathrm{p} \\
\mathrm{T} + \mathrm{p} &\longrightarrow {}^{4}\mathrm{He} + \gamma \\
{}^{3}\mathrm{He} + \mathrm{n} &\longrightarrow {}^{4}\mathrm{He} + \gamma
\end{aligned}
\tag{6.94}
$$

Heavier Elements, II

Element gap at $A = 5$ can be overcome to produce Lithium:

$$\begin{aligned} {}^3\text{He} + {}^4\text{He} &\longrightarrow {}^7\text{Be} + \gamma \\ {}^7\text{Be} &\longrightarrow {}^7\text{Li} + \text{e}^+ + \nu_\text{e} \\ \text{T} + {}^4\text{He} &\longrightarrow {}^7\text{Li} + \text{e}^+ + \nu_\text{e} \end{aligned} \tag{6.95}$$

Gap at $A = 8$ prohibits production of heavier isotopes.

$\Longrightarrow$ Major product of BBN: ^{4}He.

Mass fraction of ^{4}He can be estimated assuming all neutrons incorporated into ^{4}He
$\Longrightarrow$ number density of H=number of remaining protons, i.e., mass fraction

$$X = \frac{n_\text{p} - n_\text{n}}{n_\text{p} + n_\text{n}} \tag{6.96}$$

and

$$Y = 1 - \frac{n_\text{p} - n_\text{n}}{n_\text{p} + n_\text{n}} = 2\left(1 + \frac{n_\text{p}}{n_\text{n}}\right)^{-1} \tag{6.97}$$

Because of neutron decay, at $k_\text{B}T = 0.8\,\text{MeV}$: $n_\text{n}/n_\text{p} = 1/7$, such that

BBN predicts primordial He-abundance of $Y = 0.25$.

Remarkable Things

Note the following coincidences:

1. Freeze out of nucleons simultaneous to freeze out of neutrinos.
2. ... and parallel to electron-positron annihilation.
3. Expansion is slow enough that neutrons can be bound to nuclei.

$\Longrightarrow$ Long chain of coincidences makes our current universe possible!

Detailed Calculations, I

1. Generally, BBN operates as a function of the entropy per baryon, η.

   Remember that the entropy density for a baryon is

$$s = \frac{7}{8}\frac{2\pi^2}{45} g k_{\mathrm{B}} \left(\frac{k_{\mathrm{B}}T}{\hbar c}\right)^3 = \frac{7}{8}\frac{2\pi^4}{45\,\zeta(3)} k_{\mathrm{B}}\, n \qquad (6.73)$$

   and therefore the entropy per baryon is

$$\eta = \frac{n_{\mathrm{CMBR}}}{n_{\mathrm{baryons}}} \qquad (6.98)$$

   Note that η is related to Ω in baryons, Ω_{B}:

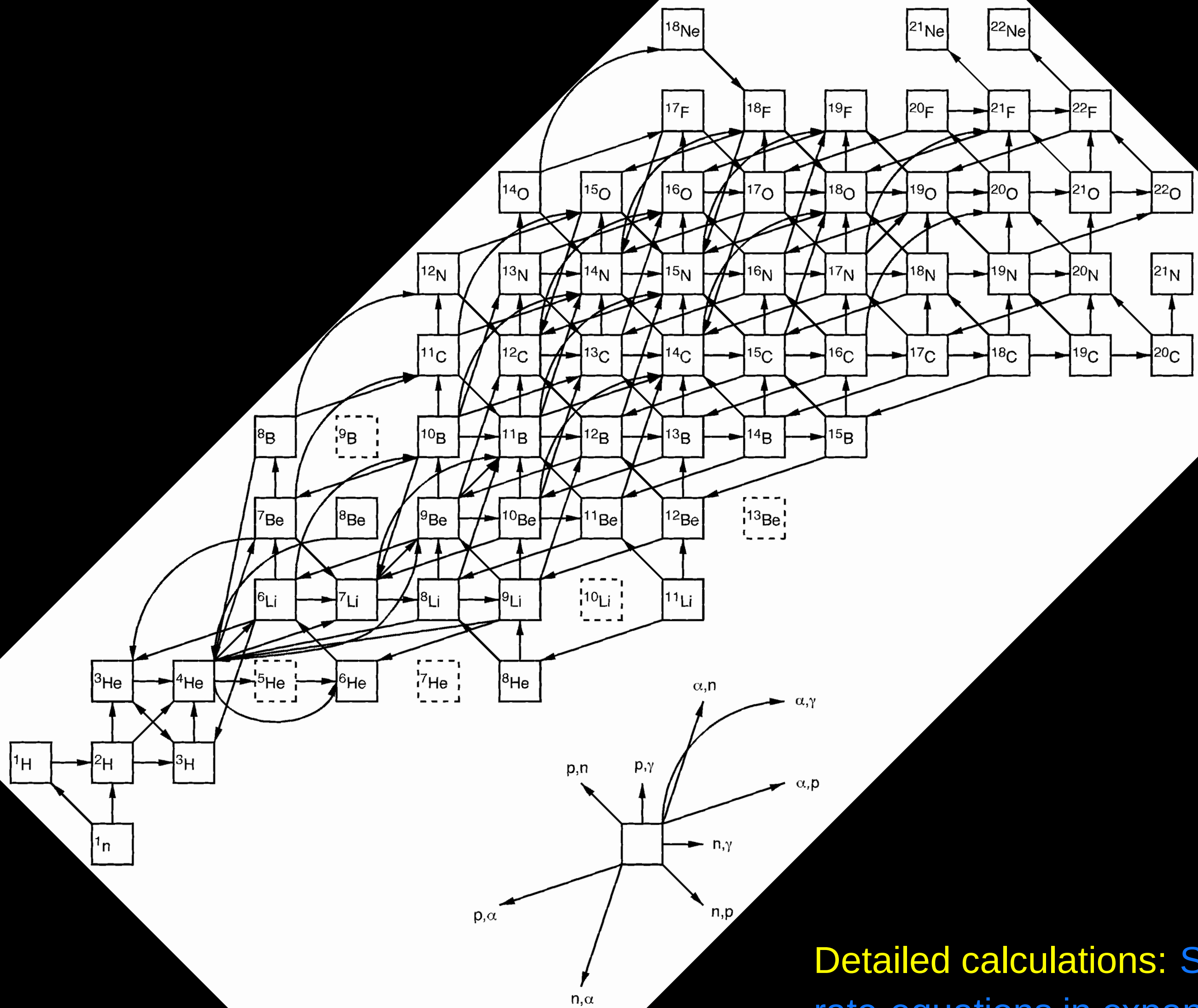
$$\Omega_{\mathrm{B}} = 3.67 \times 10^7 \cdot \eta \qquad (6.99)$$

   (since η, Ω determine expansion behavior)

   $\Longrightarrow$ Perform computations as function of η!

2. Since Y is set by $n_{\mathrm{p}}/n_{\mathrm{n}}$

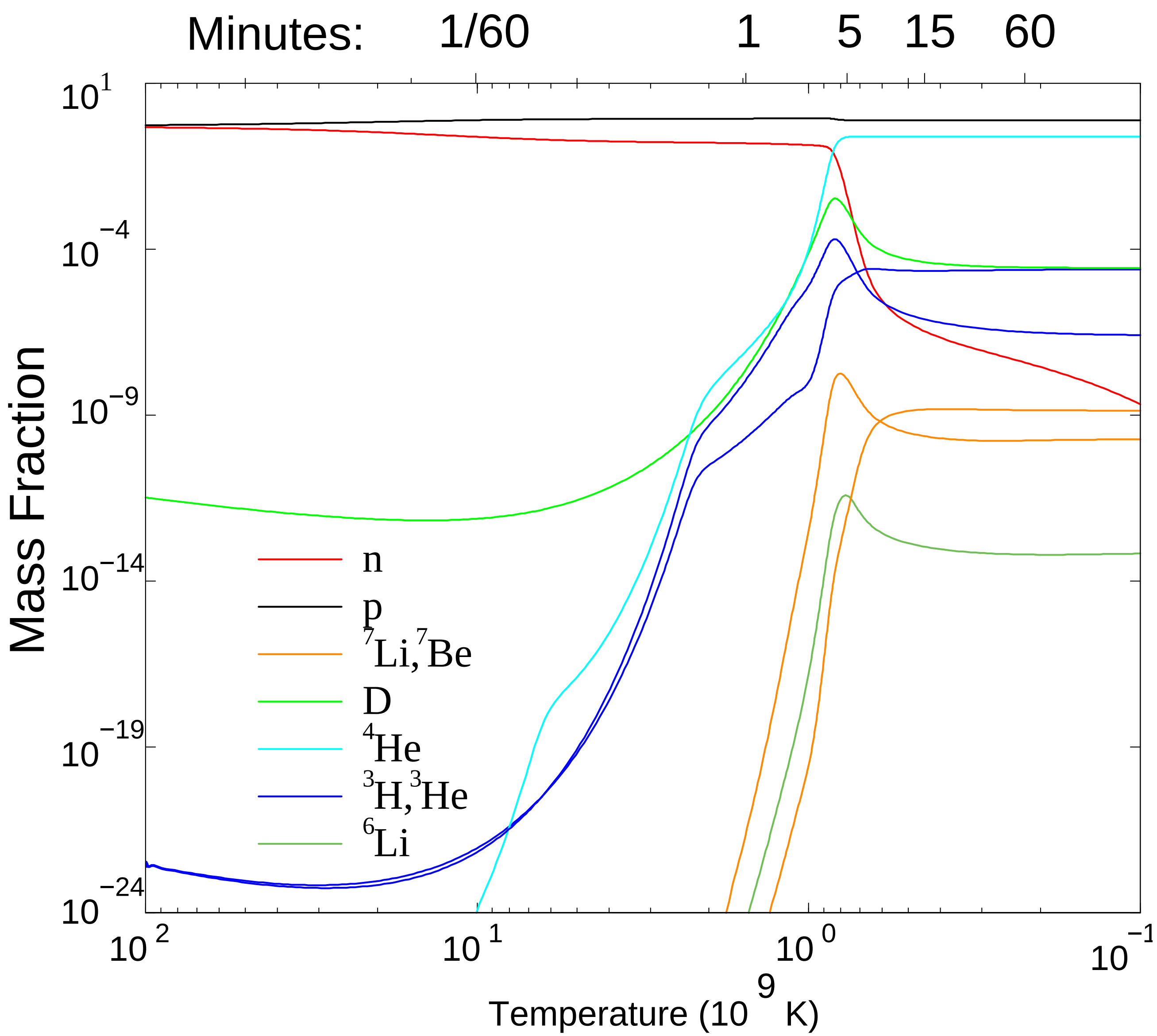
   $\Longrightarrow$ He abundance is relatively independent from η



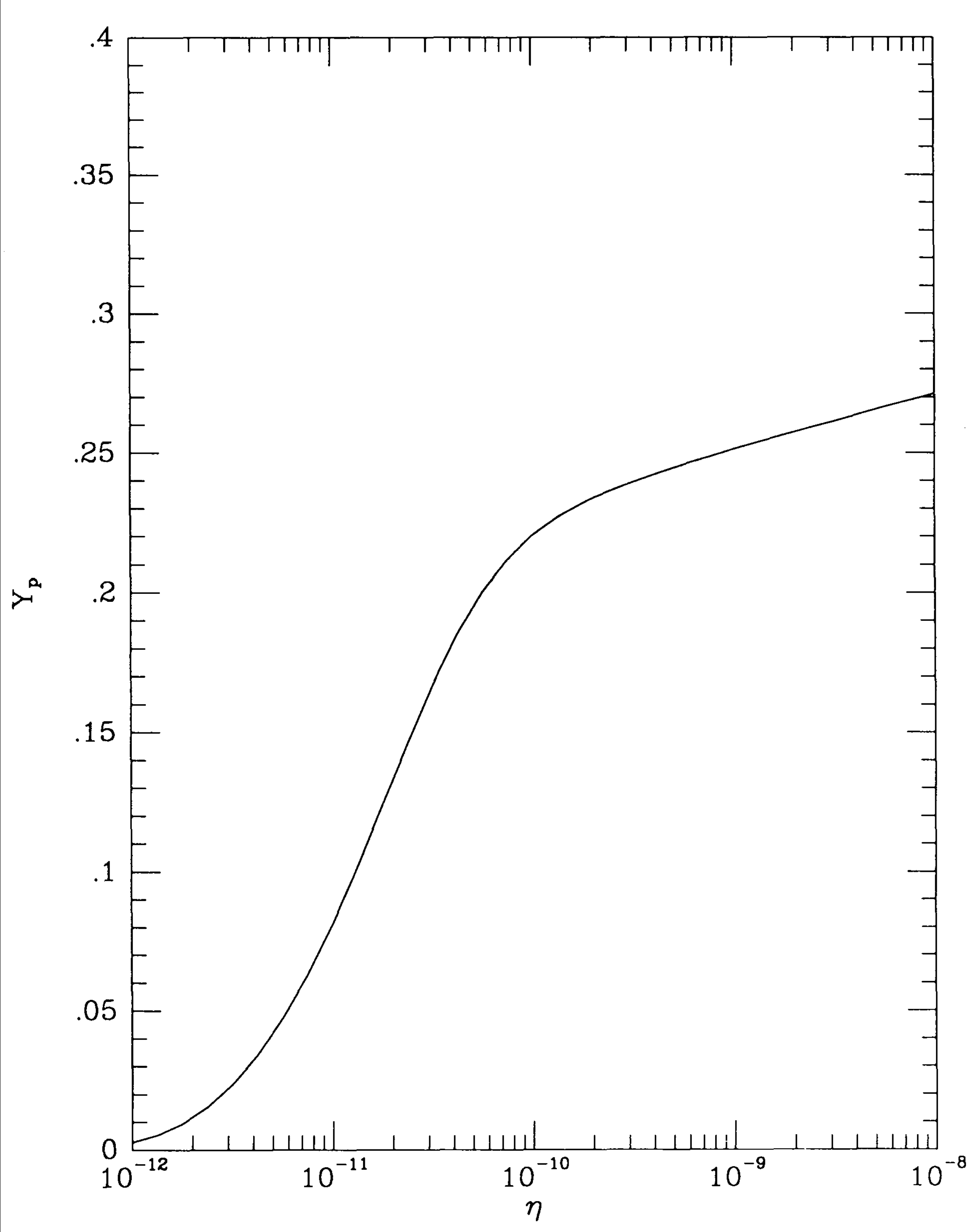
(Olive, 1999, Fig. 3)

Detailed calculations: Solution of rate-equations in expanding universe, see, e.g., Wagoner, Fowler & Hoyle (1967), Thomas et al. (1993), Olive (1999), Tytler et al. (2000), and (Kneller & Steigman, 2004).

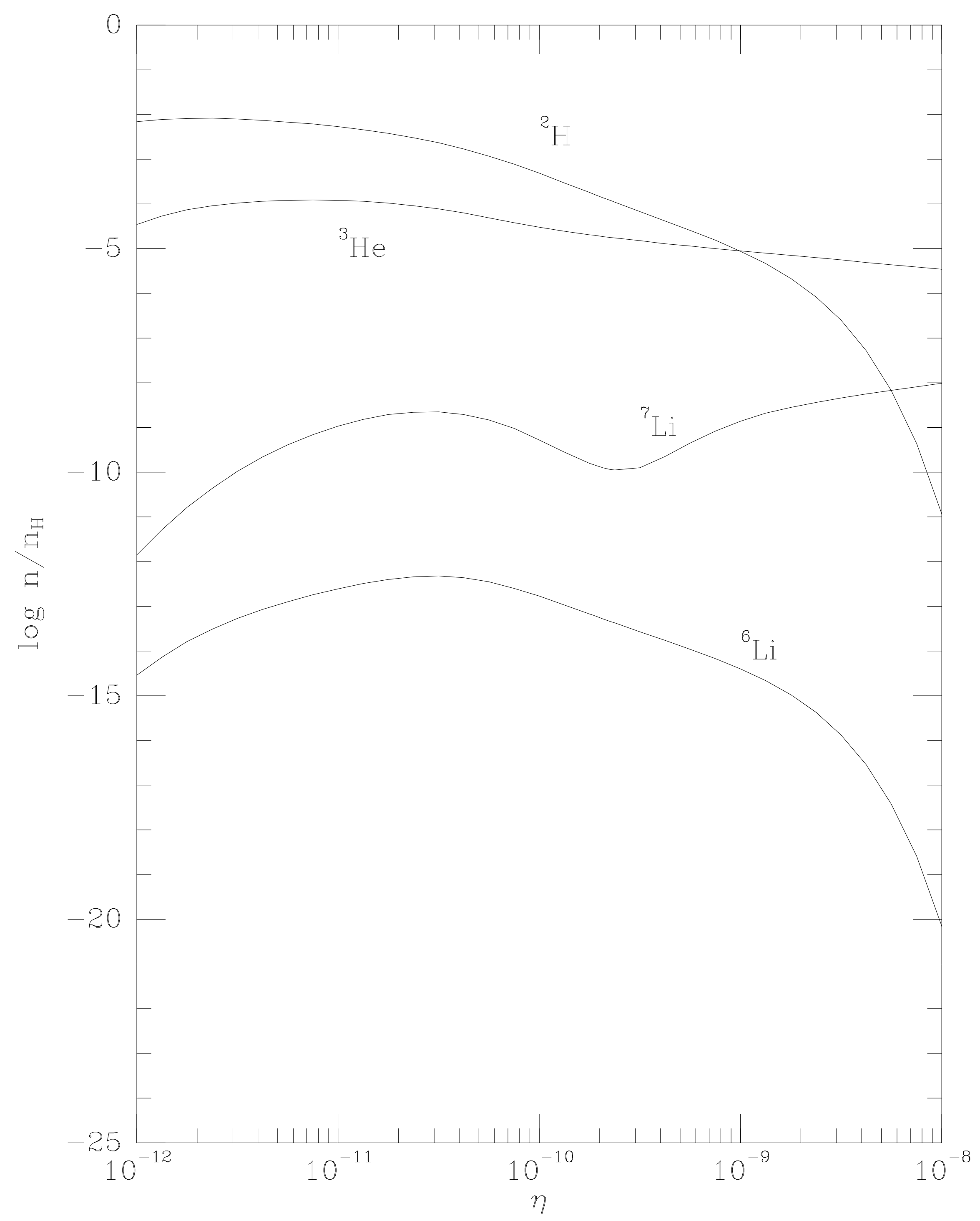
Detailed Calculations, III



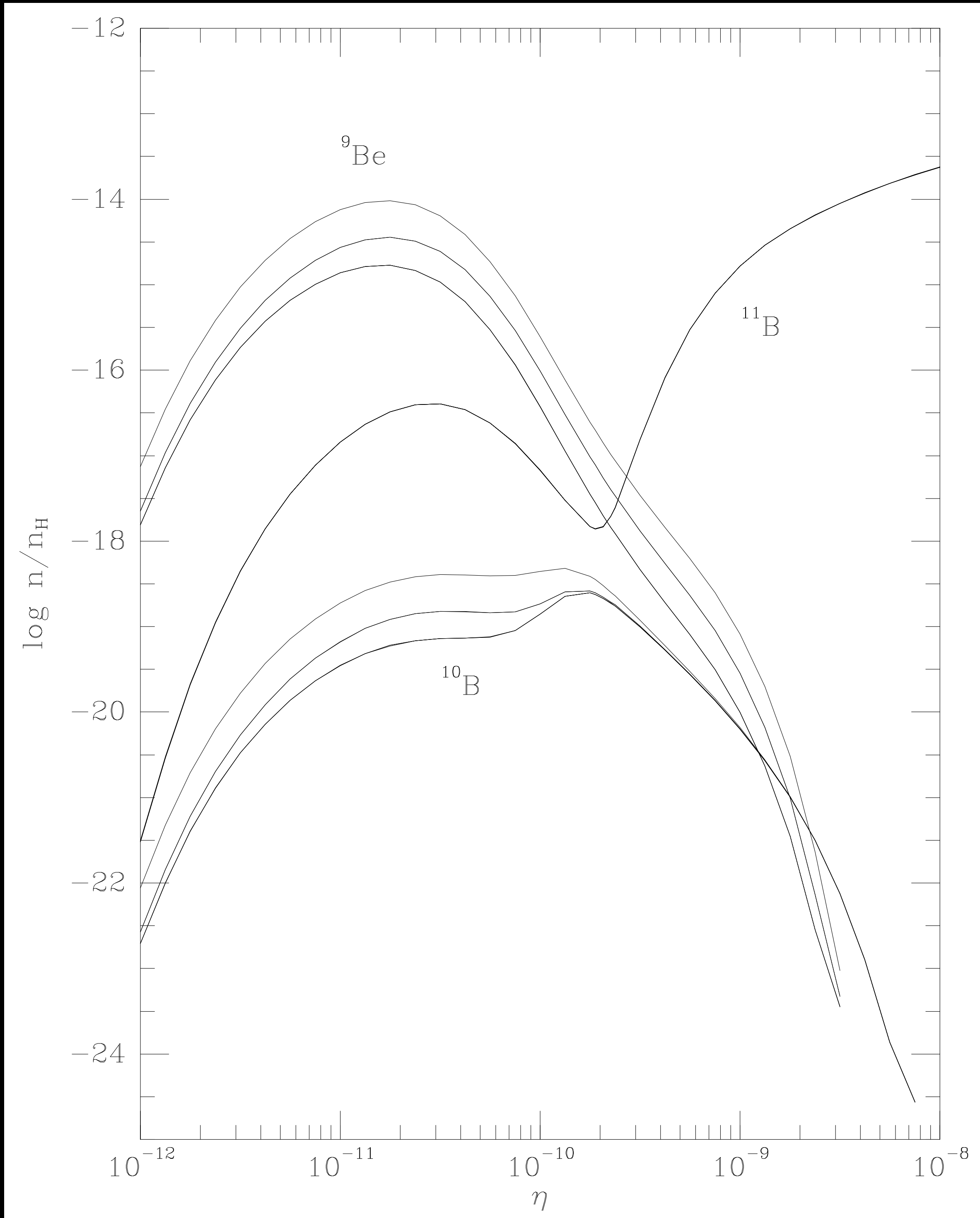
Build-up of abundances as function of time for $\eta = 5.1 \times 10^{-10}$ (Burles, Nollett & Turner, 1999, Fig. 3), remember: $\eta = n_{\mathrm{CMBR}}/n_{\mathrm{baryons}}$



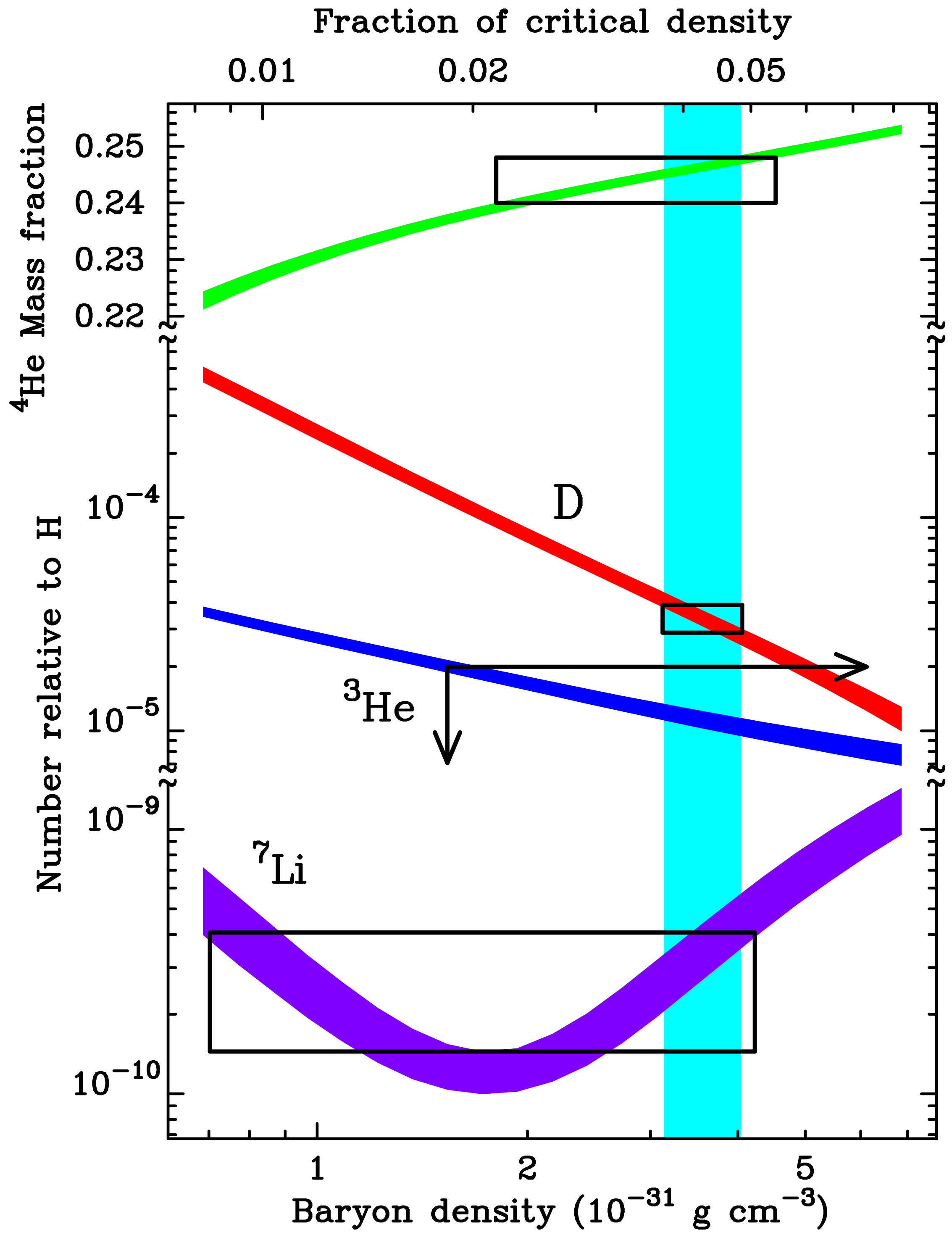
He abundance as function of η
(Thomas et al., 1993, Fig. 3a)



Light-element abundances as function of η (Olive, 1999, Fig. 4)



Intermediate mass abundances as function of η (Olive, 1999, Fig. 5)



BBN observations strongly constrain $\Omega_{Baryons}$.

(Burles, Nollett & Turner, 1999, Fig. 1)

Confrontation with WMAP

As we will see later: fluctuations in cosmic microwave background allow for a tight determination of cosmological parameters.

Best results so far from Wilkinson Microwave Anisotropy Probe (WMAP; see Spergel et al. 2007):

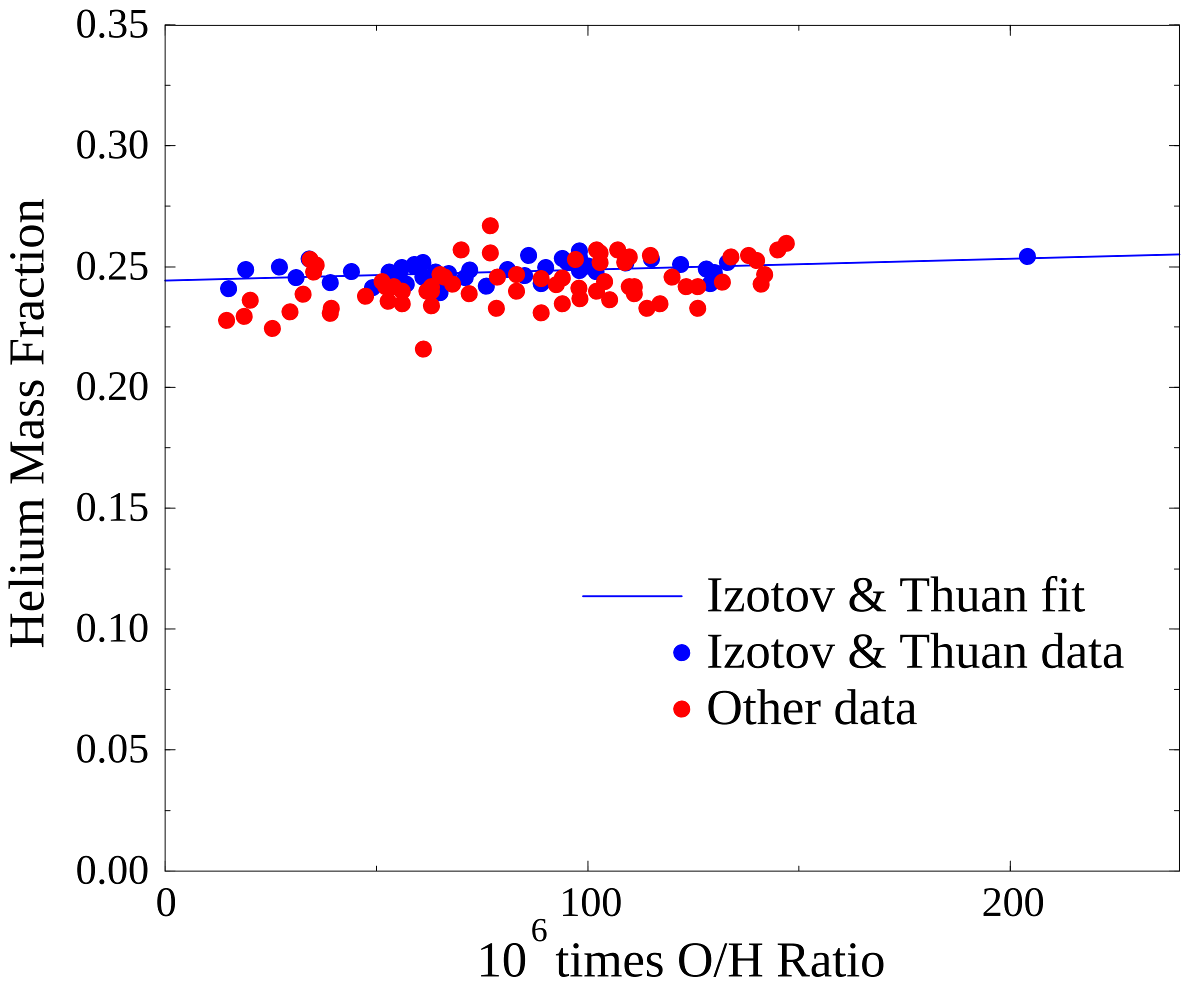
$$\Omega_\mathrm{b}h^2 = 0.02233^{+0.00072}_{-0.00091} \tag{6.100}$$

With the most modern BBN calculations (Kneller & Steigman, 2004), this gives (Molaro, 2007):

Element	SBBN+WMAP
Y_p	$0.2482^{+0.0004}_{-0.0003}$
$^3\mathrm{He}/\mathrm{H}$	$(10.5 \pm 0.6) \times 10^{-6}$
D/H	$(25.7^{+1.7}_{-1.3}) \times 10^{-6}$
Li/H	$(4.41^{+0.3}_{-0.4}) \times 10^{-10}$

$\Longrightarrow$ Can use WMAP parameters and BBN theory to compare BBN theory with measurements

^{4}He



(Burles, Nollett & Turner, 1999, Fig. 4)

^{4}He produced in stars

$\Longrightarrow$ extrapolate to zero metallicity in systems of low metallicity (i.e., minimize stellar processing).

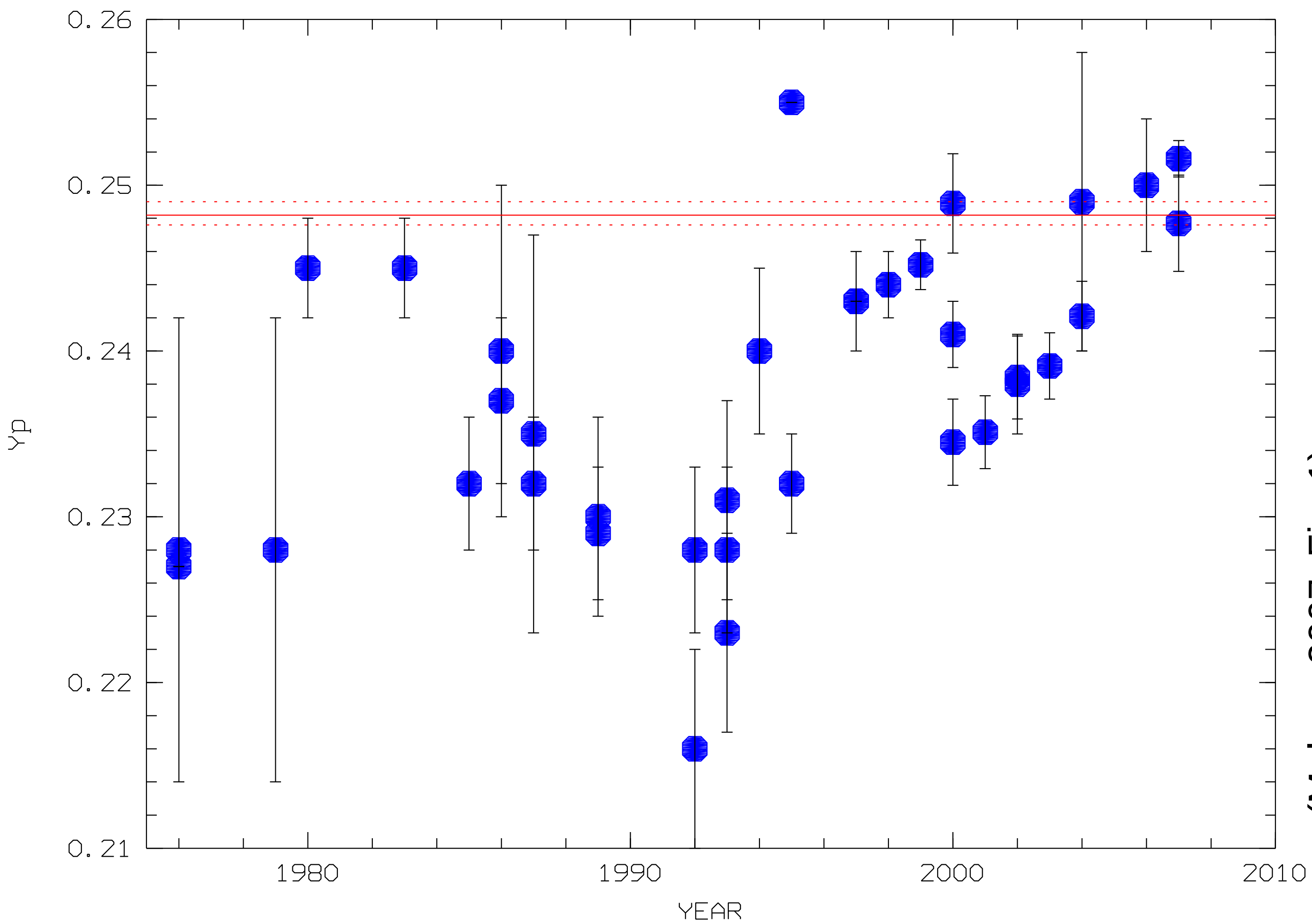
Best determination from He II$\longrightarrow$He I recombination lines in H II regions (metallicity $\sim$ 20% solar).

Result: Linear correlation He vs. O

$\Longrightarrow$ extrapolate to zero oxygen to obtain primordial abundances.

Result: $Y = 0.234 \pm 0.005$ (Olive, 1999).

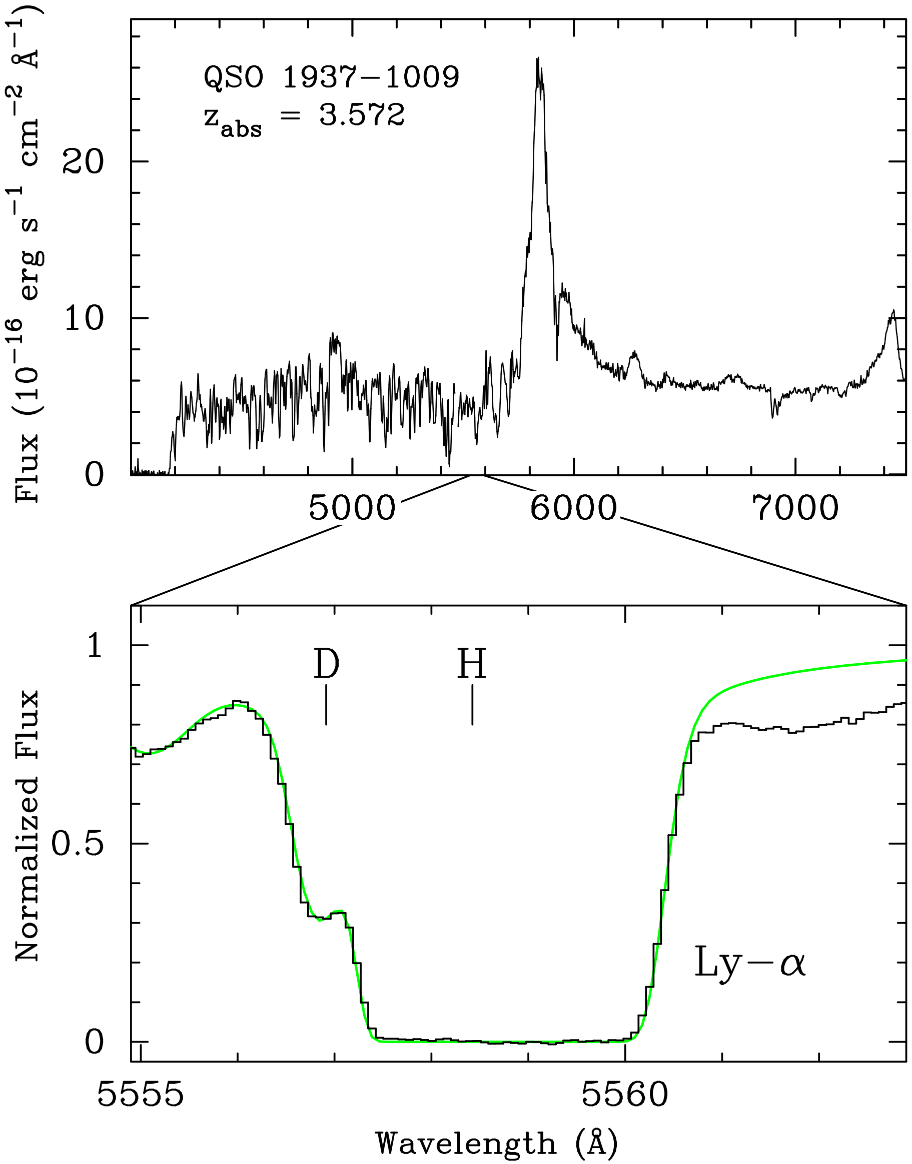
WMAP BBN and He



(Molaro, 2007, Fig. 1)

After improving He recombination physics and intrinsic absorption, He abundances are now in agreement with BBN prediction using Ω_B from WMAP.

Deuterium, I



Stars destroy D in fusion processes

$\Longrightarrow$ use as non-processed material as possible!

Lyα forest: absorption of quasar light by intervening material

$\Longrightarrow$ Some absorption lines in the Lyα forest show asymmetric line structure caused by primordial deuterium.

Remember the Balmer formula:

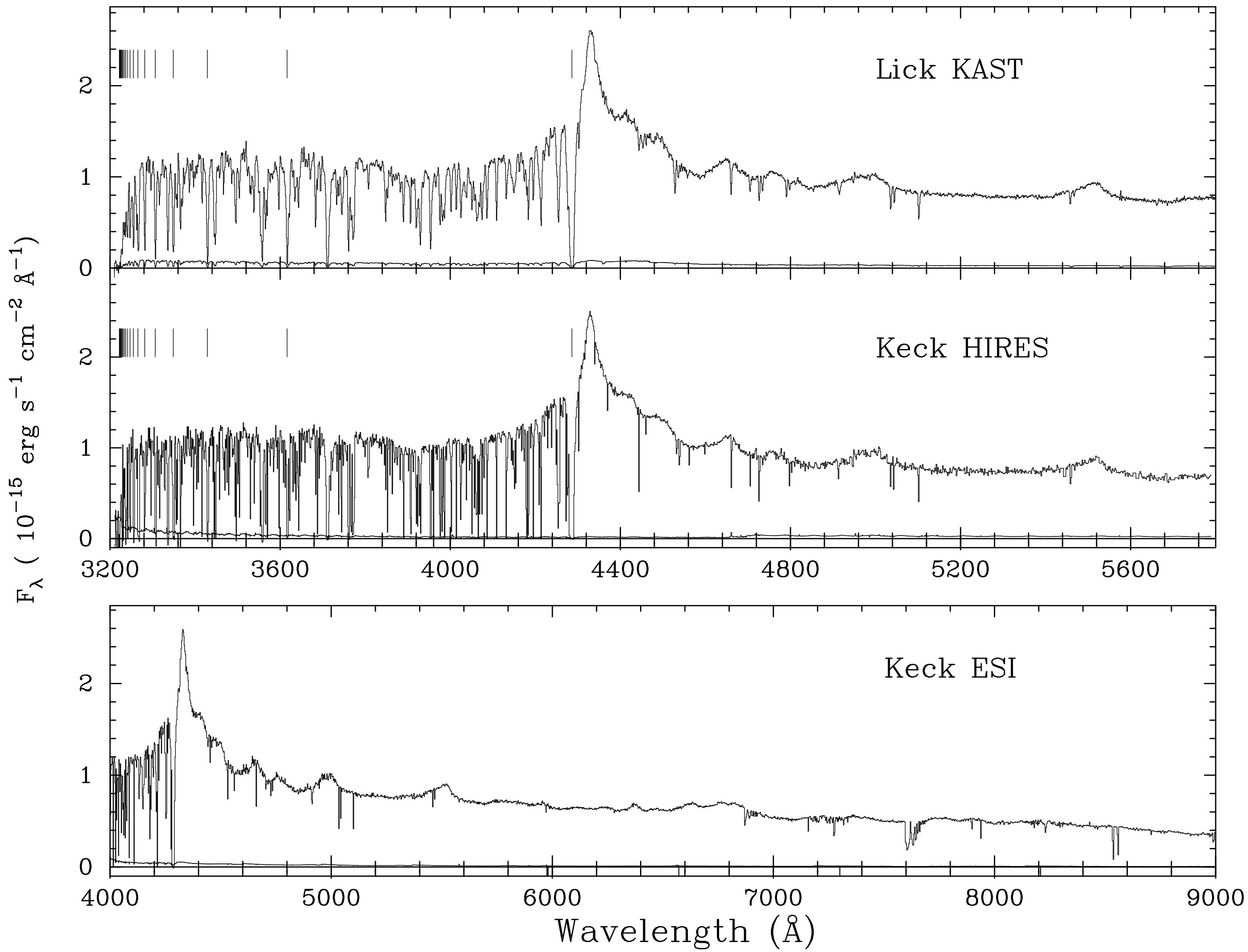
$$\frac{1}{\lambda_{n,m}} = R_{\mathrm{H}}\left(\frac{1}{m} - \frac{1}{n}\right) \tag{6.101}$$

with with Rydberg constant

$$R_{\mathrm{H}} = \frac{m_{\mathrm{e}} m_{\mathrm{p}}}{m_{\mathrm{e}} + m_{\mathrm{p}}} \frac{e^4}{8\pi\epsilon_0^2 h^3} \tag{6.102}$$

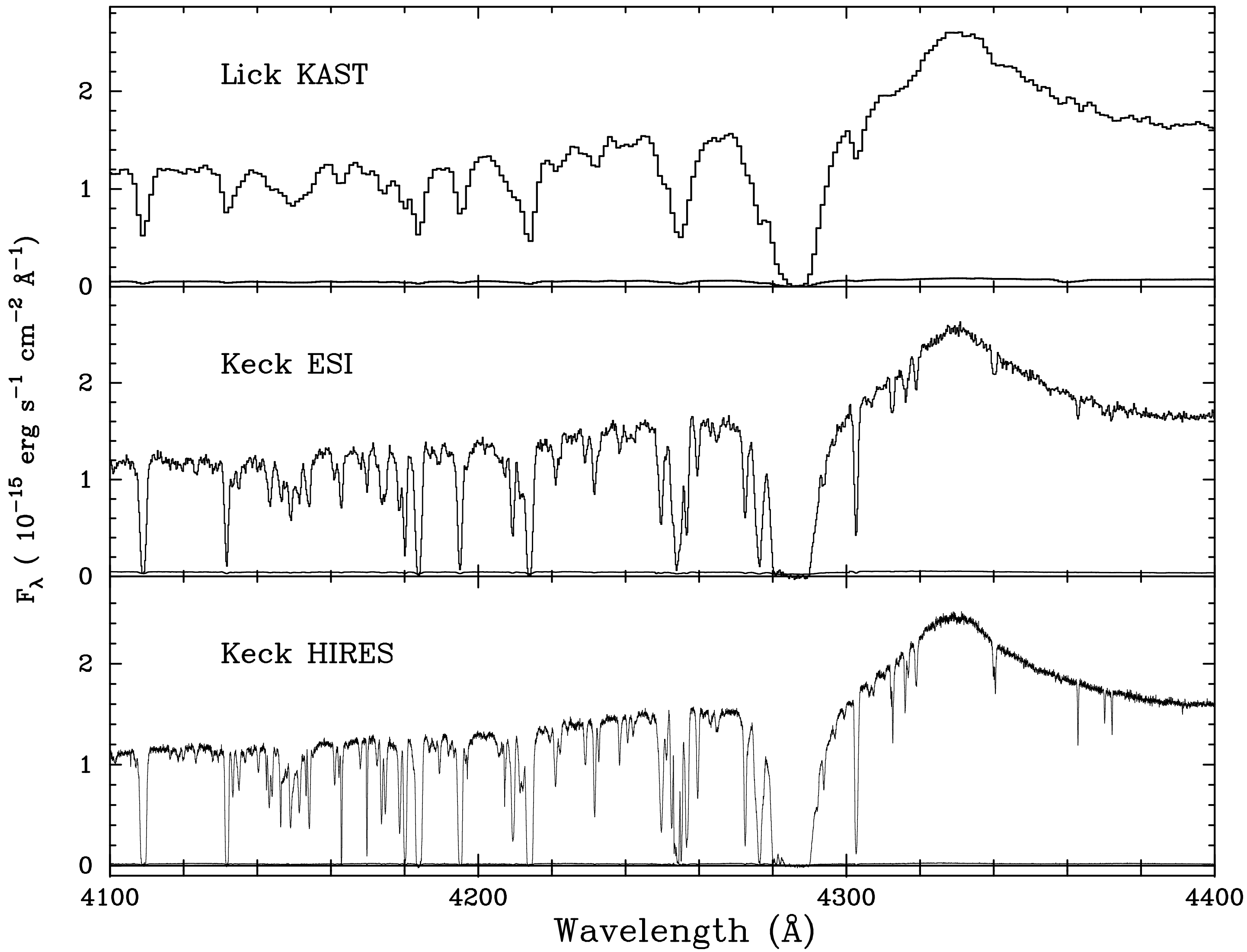
(QSO 1937−1009; top: 3 m Lick, bottom: Keck; Burles, Nollett & Turner, 1999, Fig. 2)

Deuterium, II



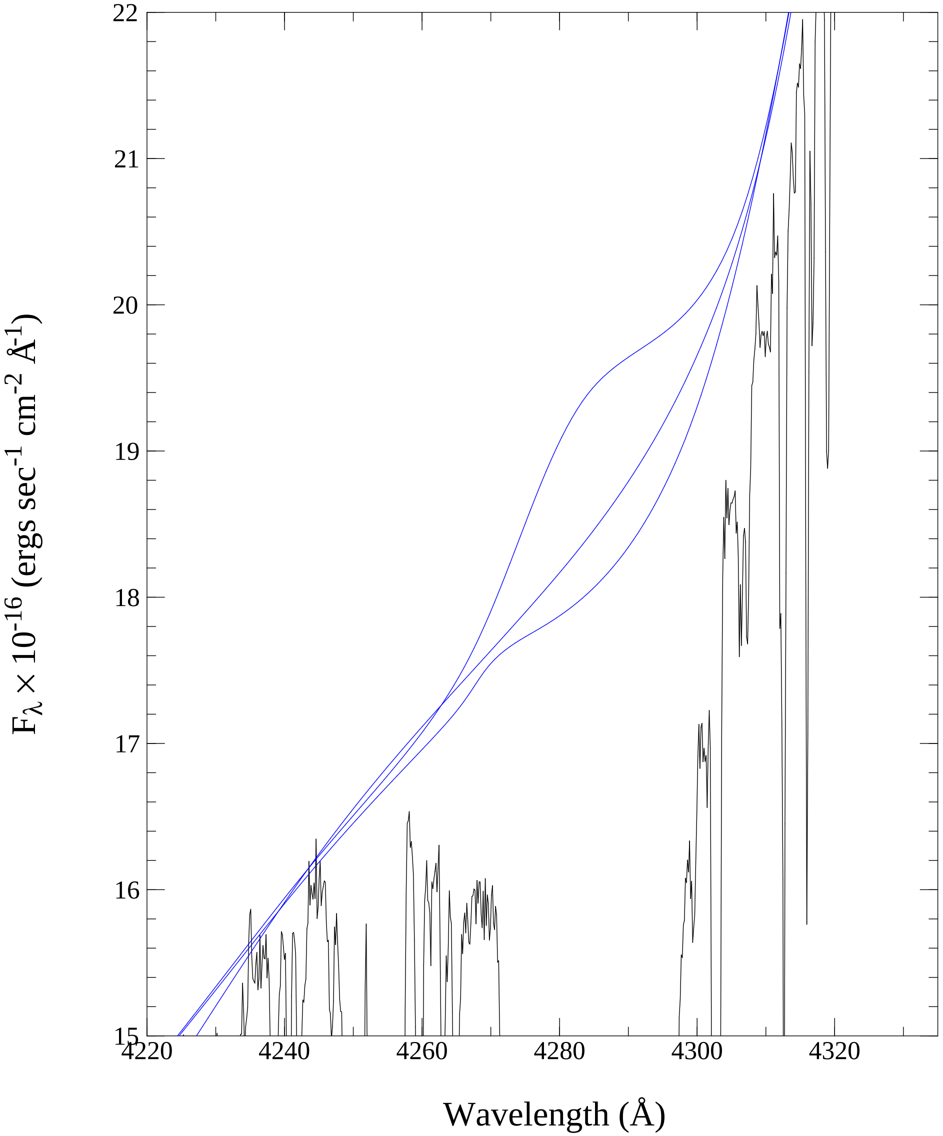
(Kirkman et al., 2003, Fig. 1): Lyman forest against three QSOs

Deuterium, III



(Kirkman et al., 2003, Fig. 2): use absorption close to 4285 Å to measure D/H

Deuterium, IV



To measure abundances, measure column from the optical depth:

$$\tau(\lambda) = n\sigma(\lambda)\ell = N\sigma(\lambda) \qquad (6.103)$$

where σ: absorption cross section of line, N: column density. This can be measured from

$$I_{\text{obs}}(\lambda) = I_{\text{cont}}(\lambda)\mathrm{e}^{-\tau(\lambda)} \qquad (6.104)$$

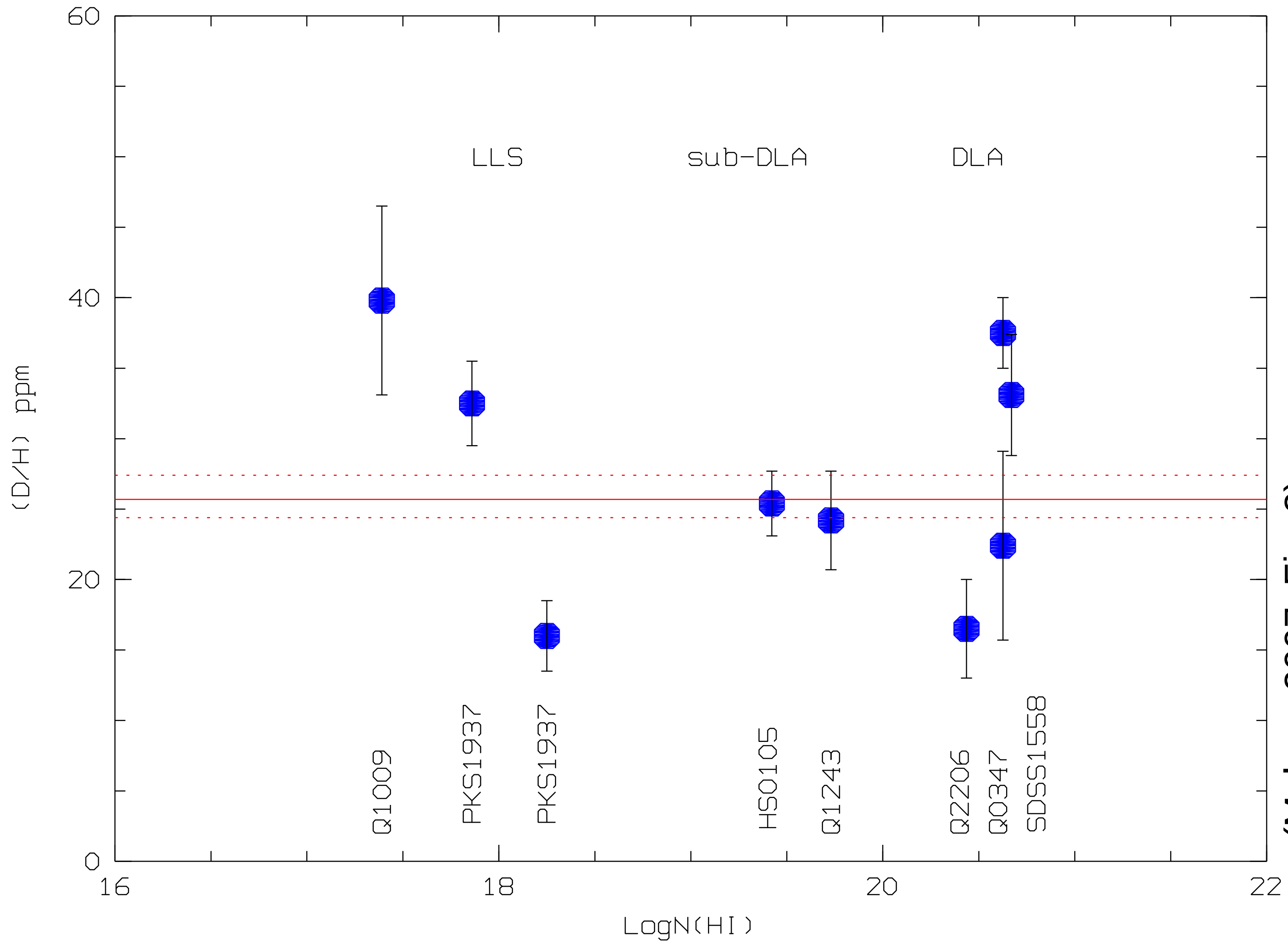
$\Longrightarrow$ Need to know the continuum, I_{cont}
Very difficult to do in Lyα forest (see Figure)

Currently best result for D/H (Kirkman et al., 2003):

$$\text{D/H} = 2.78^{+0.44}_{-0.38} \times 10^{-5}$$

Corresponding to $\eta = 5.9 \pm 0.5 \times 10^{-10}$ or $\Omega_{\text{B}}h^2 = 0.0214$ ($\pm 9.3\%$).

WMAP BBN and D

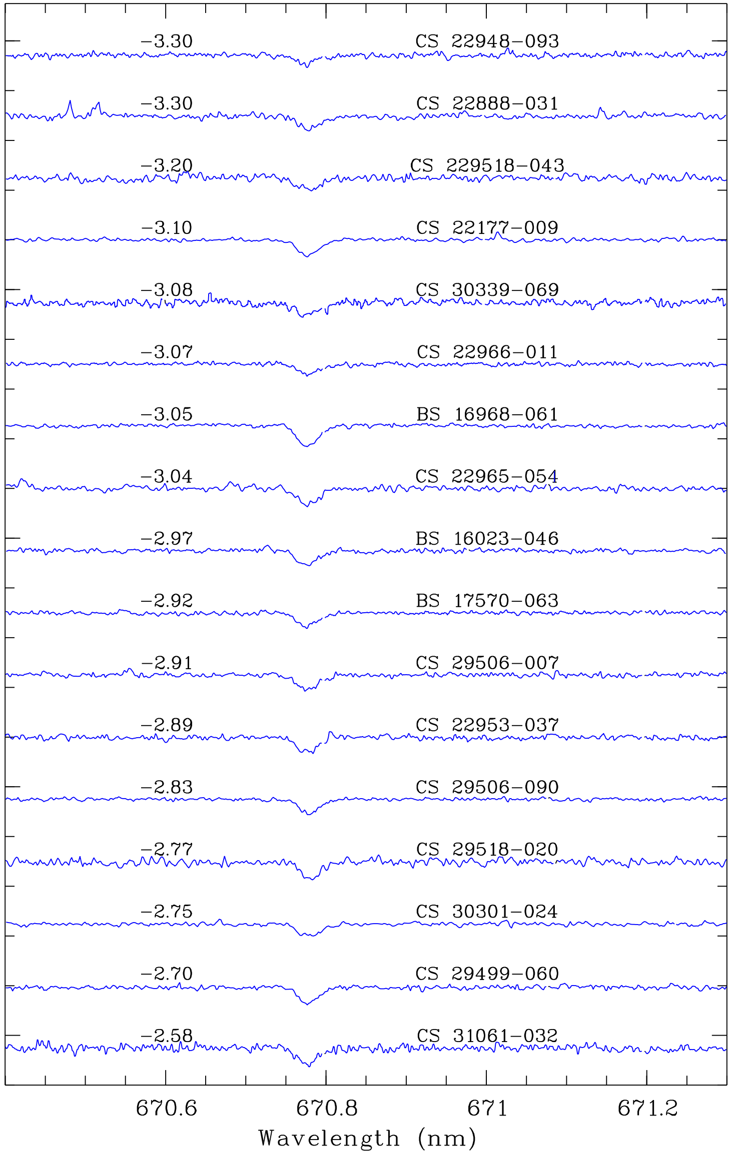


(Molaro, 2007, Fig. 2)

Measured deuterium abundances agree with WMAP predictions

Although there are issues with Milky Way deuterium abundances. . .

Lithium, I

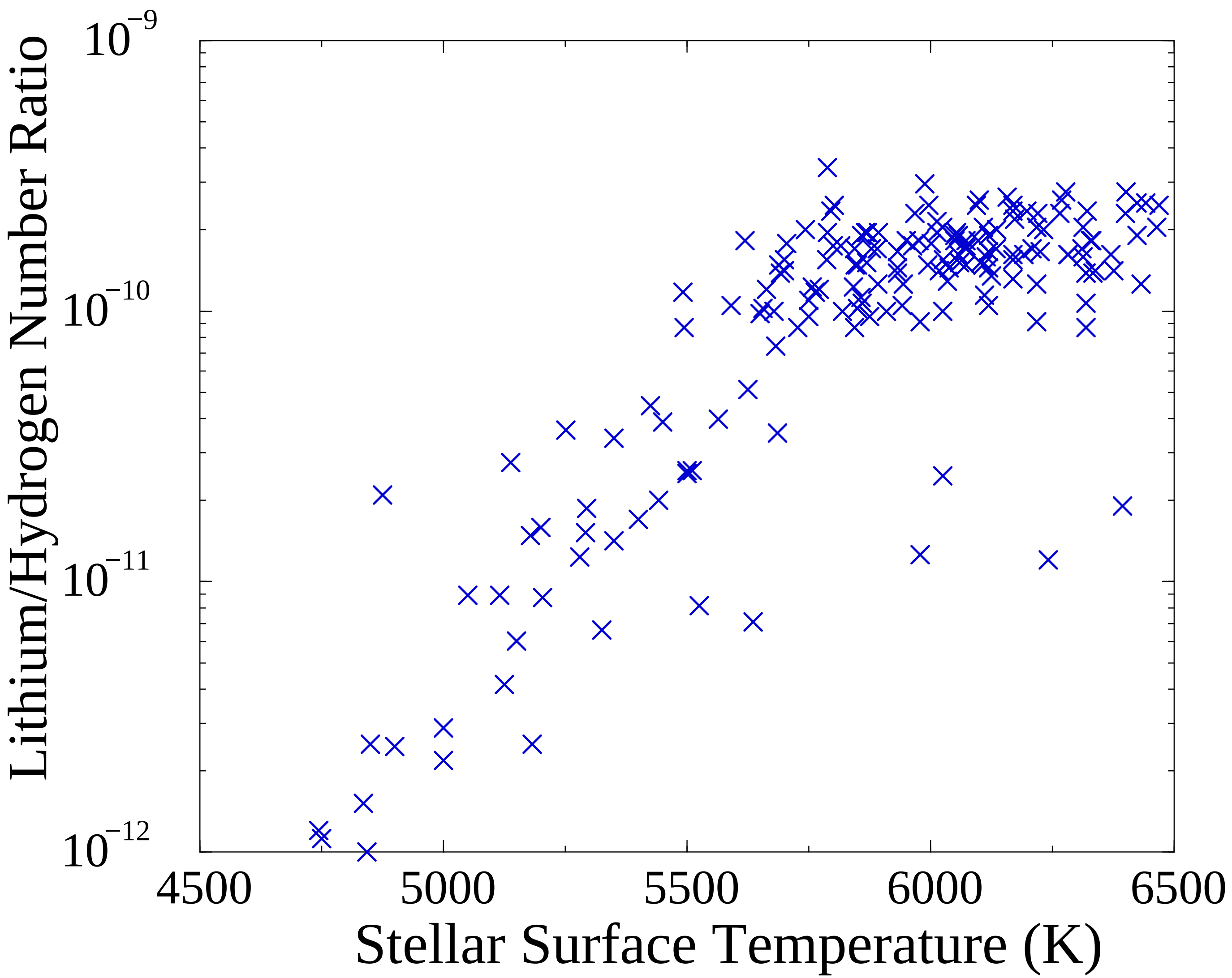


Lithium lines (Li doublet at 6707 Å) are visible in some stars

⟹ allow measurement of Li abundance

Li line as a function of [Fe/H]
(Bonifacio et al., 2007, Fig. 1)

Lithium, II



Spite & Spite (1982): Old halo stars with very low [Fe/H]) show primordial Lithium abundance,

$^7\mathrm{Li}/\mathrm{H} = 1.6 \times 10^{-10}$

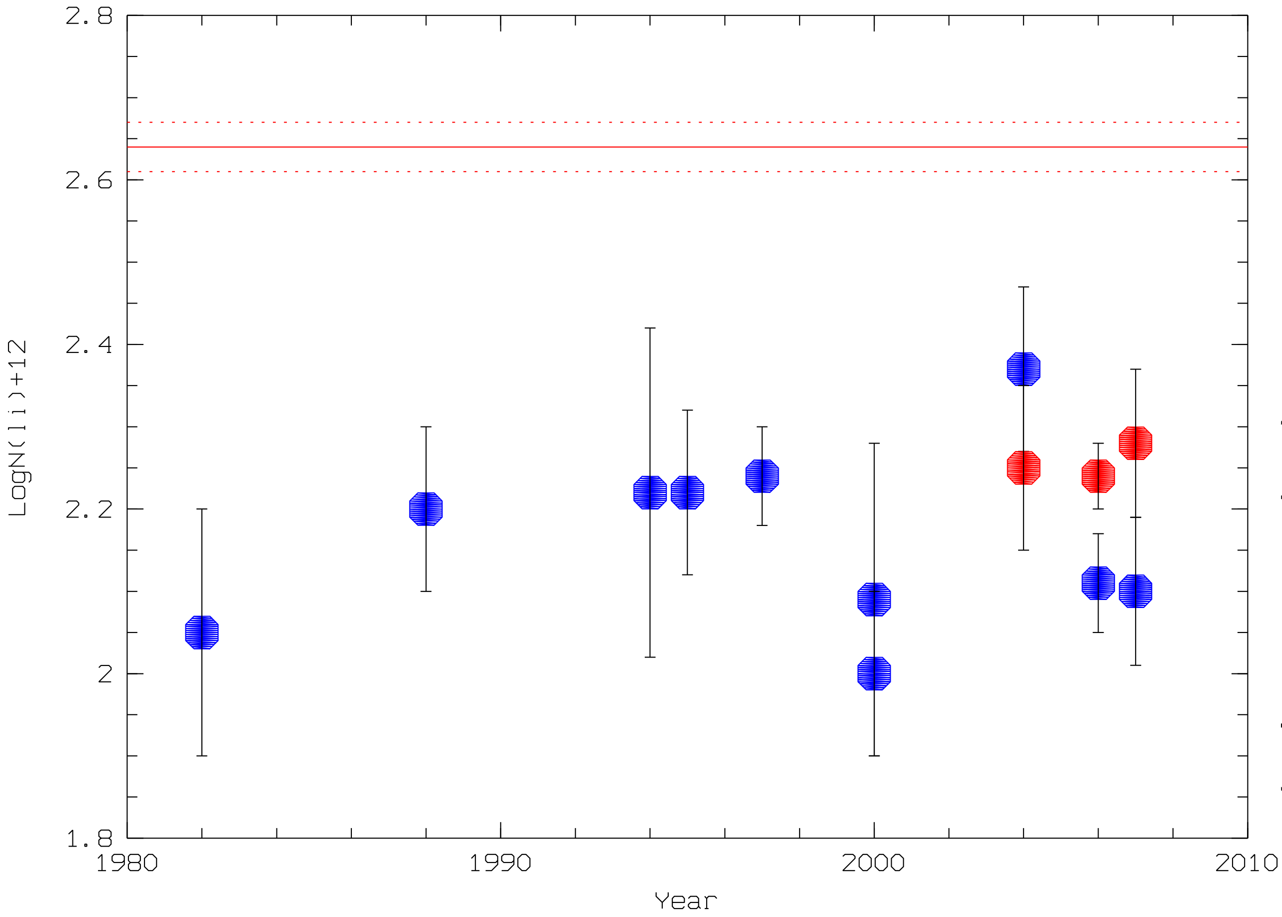
"Spite plateau"

Lower temperature stars: outer convection zone $\Longrightarrow$ Li burning destroys Li.

Cannot use galactic objects since spallation of heavier nuclei by cosmic rays produces Li (up to 10$\times$ primordial!).

(Burles, Nollett & Turner, 1999, Fig. 5)

WMAP BBN and Li

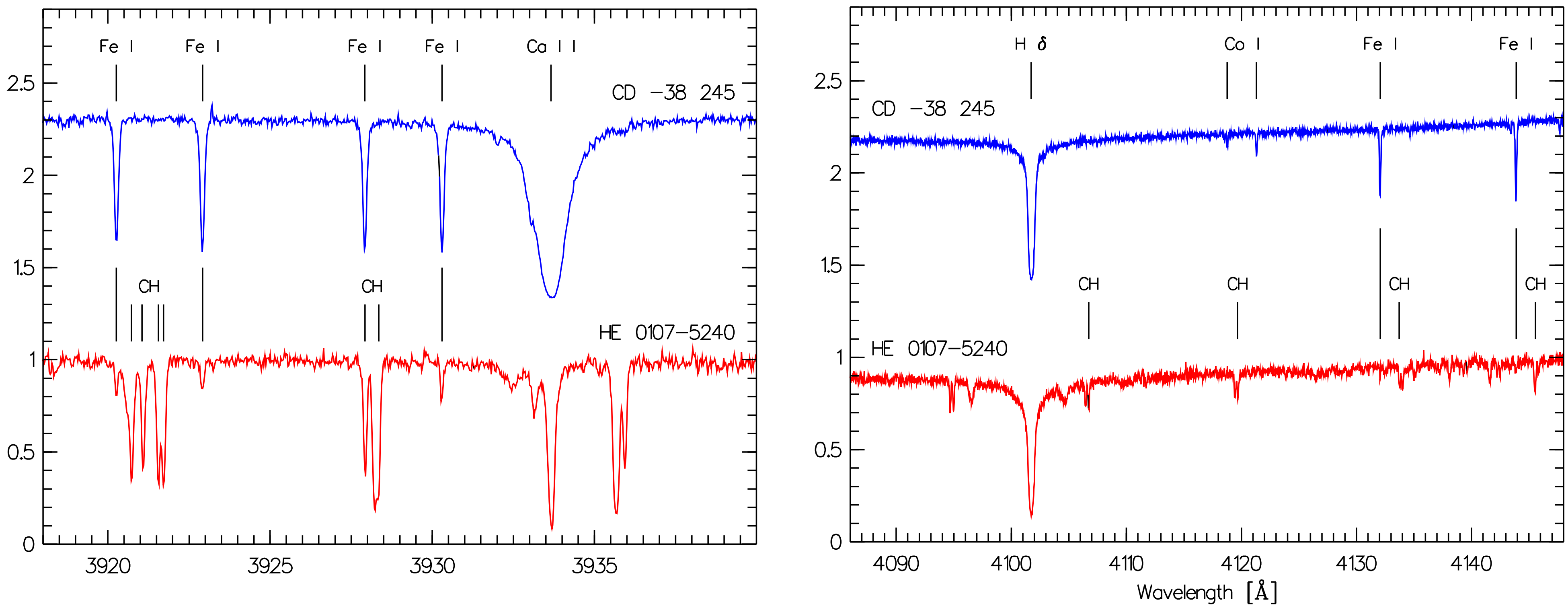


(Molaro, 2007, Fig. 3)

Lithium has a big problem!

Temperature sensitivity might have been underestimated, also rotational mixing, diffusion, and differences between 1D- and 3D-radiative transfer in stellar atmosphere models might play a role. However, no convincing solution has been proposed as of today.

Outlook: Population III

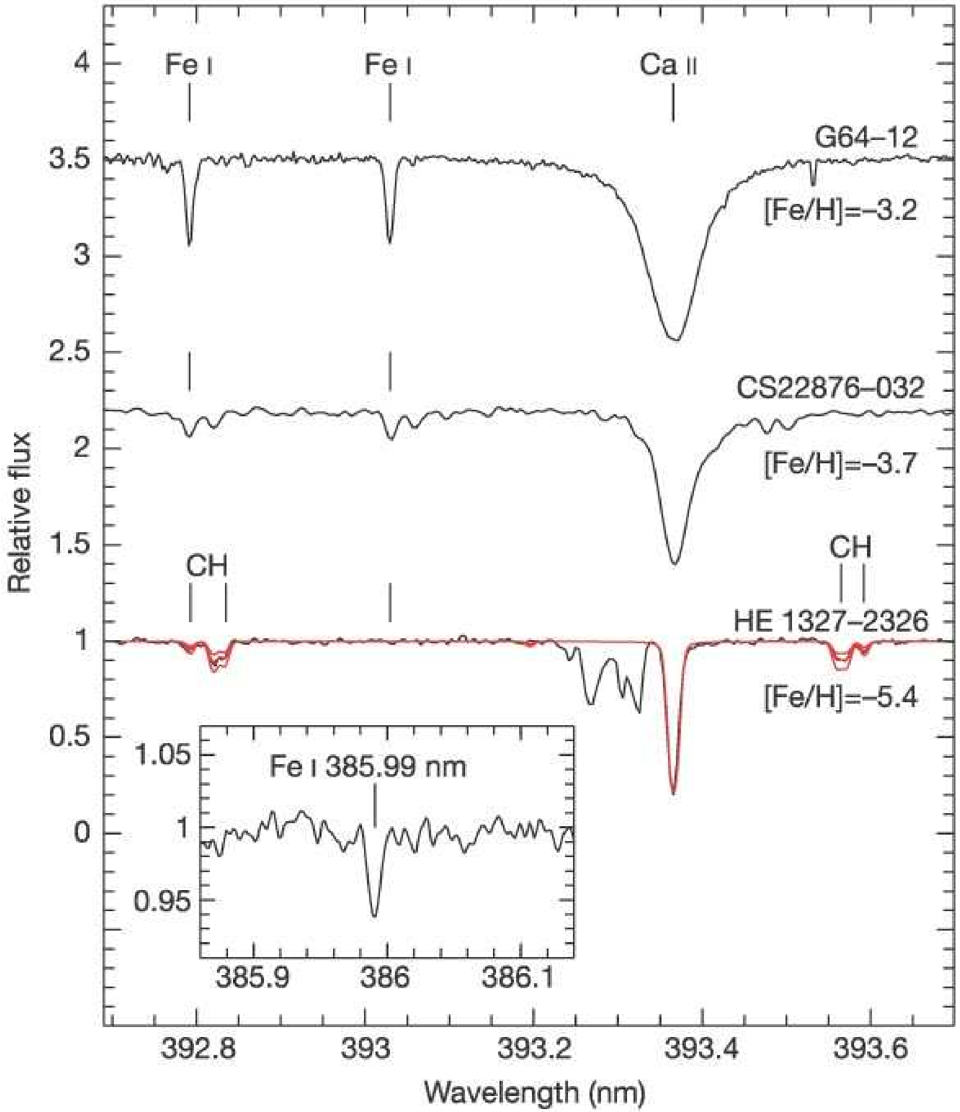


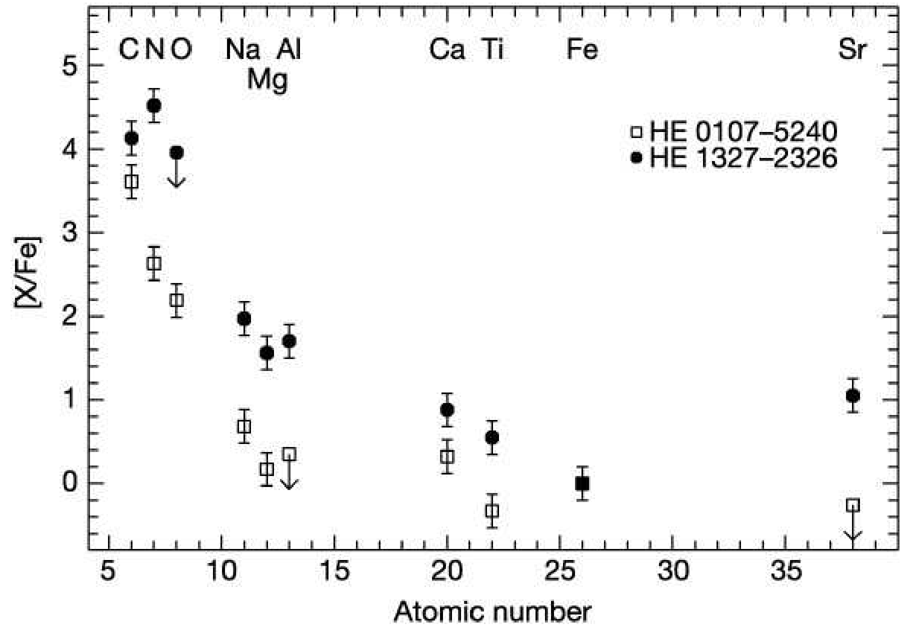
(HE0107−5240, metallicity 1/200000 solar; after Christlieb et al., 2002, Fig. 1)

Earliest stars should only have H, He, i.e., $Z = 0 \Longrightarrow$ detection of such stars would enable the *direct* measure of primordial abundances.

"population III star", formed either from primordial gas cloud (and got some elements later through accretion from ISM), or from debris from type II SN explosion.

Outlook: Population III





(Frebel et al., 2005, Fig. 2)

Lowest metallicity known:

HE1327−2326, with Fe-abundance of 1/250000 solar

(Frebel et al., 2005, Fig. 1)

Summary

Summary: History of the universe after its first 0.01 s (after Islam, 1992, Ch. 7, see also Weinberg, The first three minutes).

$t = 0.01\,\mathbf{s}$ $T = 10^{11}\,\mathbf{K}$ $\rho \sim 4 \times 10^{11}\,\mathbf{g\,cm^{-3}}$

Main constitutents: γ, ν, $\bar{\nu}$, e^--e^+ pairs.

No nuclei (instable). n and p in thermal balance.

$t = 0.1\,\mathbf{s}$ $T = 3 \times 10^{10}\,\mathbf{K}$ $\rho \sim 3 \times 10^{7}\,\mathbf{g\,cm^{-3}}$

Main constitutents: γ, ν, $\bar{\nu}$, e^--e^+ pairs. No nuclei.

$n + \nu \leftrightarrow p + e^-$: mass difference becomes important, 40% n, 60% p (by mass).

Summary

$t = 1.1\,\mathbf{s}$ $T = 10^{10}\,\mathbf{K}$ $\rho \sim 10^5\,\mathbf{g\,cm^{-3}}$

Neutrinos decouple, e^--e^+ pairs start to annihilate. No nuclei.

25% n, 75% p

$t = 13\,\mathbf{s}$ $T = 3 \times 10^9\,\mathbf{K}$ $\rho \sim 10^5\,\mathbf{g\,cm^{-3}}$

Reheating of photons, pairs annihilate, ν fully decoupled, deuterium still cannot form.

17% n, 83% p

$t = 3\,\mathbf{min}$ $T = 10^9\,\mathbf{K}$ $\rho \sim 10^5\,\mathbf{g\,cm^{-3}}$

Pairs are gone, neutron decay becomes important, start of nucleosynthesis

14% n, 86% p

Summary

$t = 35\,\mathbf{min}$ $T = 3 \times 10^{8}\,\mathbf{K}$ $\rho \sim 0.1\,\mathbf{g\,cm^{-3}}$

game over

Next important event: $t \sim 300000\,\text{years}$: Interaction CMB/matter stops ("last scattering", recombination).

Before we look at this, we look at

the fi rst 0.01 s: the very early universe

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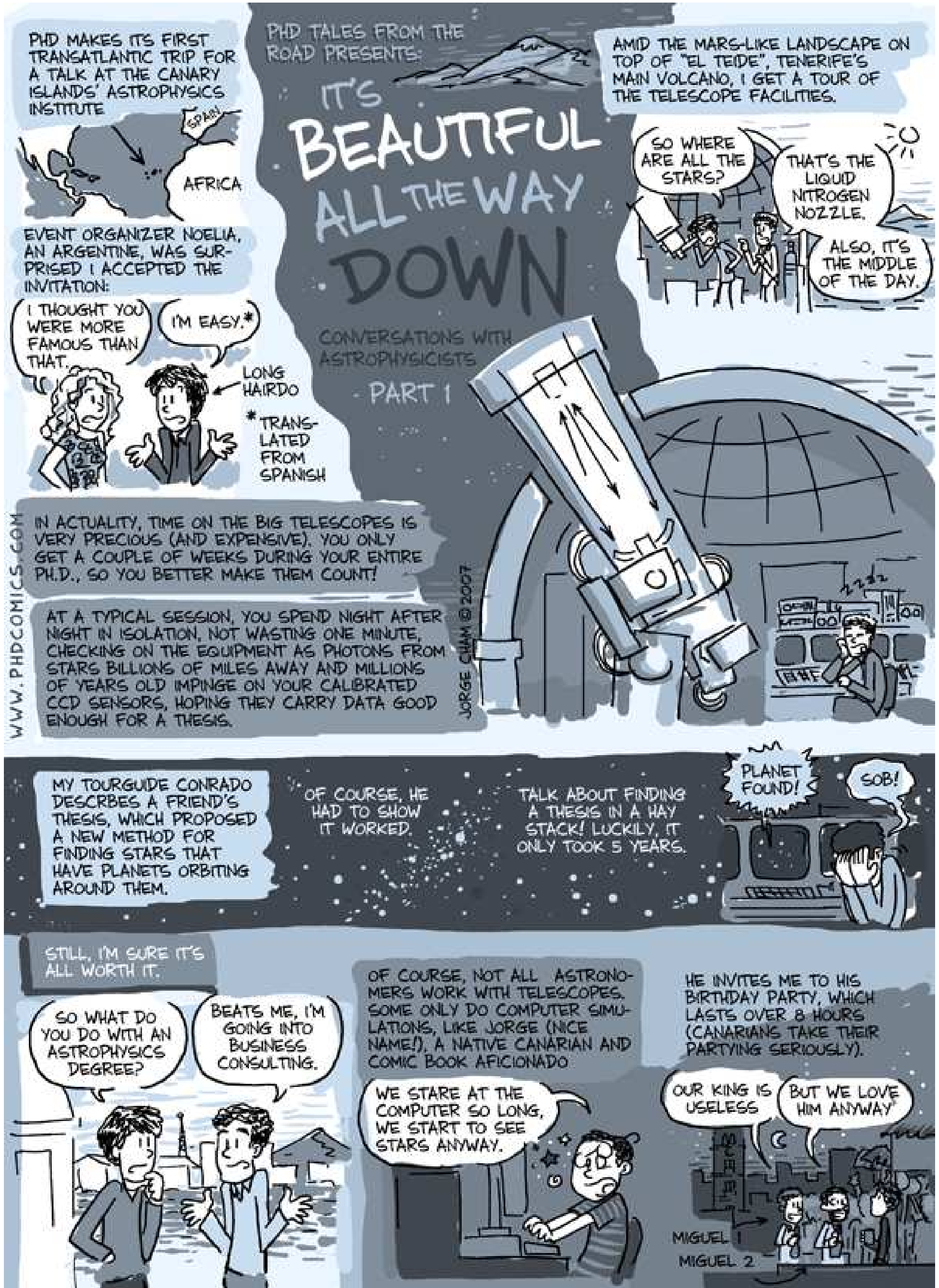

PHD TALES FROM THE ROAD PRESENTS:
IT'S BEAUTIFUL ALL THE WAY DOWN
CONVERSATIONS WITH ASTROPHYSICISTS
PART 1
PHD MAKES ITS FIRST TRANSATLANTIC TRIP FOR A TALK AT THE CANARY ISLANDS' ASTROPHYSICS INSTITUTE
SPAIN
AFRICA
EVENT ORGANIZER NOELIA, AN ARGENTINE, WAS SURPRISED I ACCEPTED THE INVITATION:
I THOUGHT YOU WERE MORE FAMOUS THAN THAT.
I'M EASY.*
LONG HAIRDO
* TRANSLATED FROM SPANISH
AMID THE MARS-LIKE LANDSCAPE ON TOP OF "EL TEIDE", TENERIFE'S MAIN VOLCANO, I GET A TOUR OF THE TELESCOPE FACILITIES.
SO WHERE ARE ALL THE STARS?
THAT'S THE LIQUID NITROGEN NOZZLE.
ALSO, IT'S THE MIDDLE OF THE DAY.
IN ACTUALITY, TIME ON THE BIG TELESCOPES IS VERY PRECIOUS (AND EXPENSIVE). YOU ONLY GET A COUPLE OF WEEKS DURING YOUR ENTIRE PH.D., SO YOU BETTER MAKE THEM COUNT!
AT A TYPICAL SESSION, YOU SPEND NIGHT AFTER NIGHT IN ISOLATION, NOT WASTING ONE MINUTE, CHECKING ON THE EQUIPMENT AS PHOTONS FROM STARS BILLIONS OF MILES AWAY AND MILLIONS OF YEARS OLD IMPINGE ON YOUR CALIBRATED CCD SENSORS, HOPING THEY CARRY DATA GOOD ENOUGH FOR A THESIS.
JORGE CHAM © 2007
WWW.PHDCOMICS.COM
MY TOURGUIDE CONRADO DESCRBES A FRIEND'S THESIS, WHICH PROPOSED A NEW METHOD FOR FINDING STARS THAT HAVE PLANETS ORBITING AROUND THEM.
OF COURSE, HE HAD TO SHOW IT WORKED.
TALK ABOUT FINDING A THESIS IN A HAY STACK! LUCKILY, IT ONLY TOOK 5 YEARS.
PLANET FOUND!
SOB!
STILL, I'M SURE IT'S ALL WORTH IT.
SO WHAT DO YOU DO WITH AN ASTROPHYSICS DEGREE?
BEATS ME, I'M GOING INTO BUSINESS CONSULTING.
OF COURSE, NOT ALL ASTRONOMERS WORK WITH TELESCOPES. SOME ONLY DO COMPUTER SIMULATIONS, LIKE JORGE (NICE NAME!), A NATIVE CANARIAN AND COMIC BOOK AFICIONADO
WE STARE AT THE COMPUTER SO LONG, WE START TO SEE STARS ANYWAY.
HE INVITES ME TO HIS BIRTHDAY PARTY, WHICH LASTS OVER 8 HOURS (CANARIANS TAKE THEIR PARTYING SERIOUSLY).
OUR KING IS USELESS
BUT WE LOVE HIM ANYWAY
MIGUEL 1
MIGUEL 2


IT'S BEAUTIFUL ALL THE WAY DOWN
CONVERSATIONS WITH ASTROPHYSICISTS, PART 2
PHD TALES FROM THE ROAD
WWW.PHDCOMICS.COM
JORGE CHAM © 2007
RECENTLY AT PRINCETON, ANOTHER GROUP OF ASTROPHYSICISTS TAKE ME OUT TO DINNER (AND ICE CREAM OF COURSE).
THEY DESCRIBE TO ME WHAT THEY DO ON LONG FLIGHTS WHEN ASKED THE DREADED:
SO, WHAT DO YOU DO?
UH...
IF YOU FEEL LIKE TALKING, ANSWER:
I'M AN ASTRONOMER.
WOW! TELL ME MORE!
TO KILL THE CONVERSATION:
I'M A PHYSICIST.
ZZZZZ!
RISING ASTROPHYSICS STAR (NO PUN INTENDED) KATIE DESCRIBES HER RESEARCH: MINI BLACK HOLES, WHICH HAVE THE MASS OF A MOUNTAIN IN THE SIZE OF A PEA.
APPARENTLY, THESE MINI BLACK HOLES ARE ZOOMING ALL AROUND US, AND COULD ANNIHILATE YOU INSTANTLY.
I'M NOT WORRIED THOUGH.
OF COURSE, SHE'S NEVER SEEN ONE. IT'S ALL THEORETICAL. I PRESS THEM ON THIS POINT.
SO MUCH OF WHAT ASTROPHYSICISTS CLAIM, NOBODY'S EVER SEEN!
WE CAN'T SEE IT, BUT WE KNOW IT'S THERE.
FOR EXAMPLE, TO EXPLAIN THE GRAVITATIONAL TRAJECTORIES OF CERTAIN GALAXIES, THEY INVENTED "DARK MATTER". THE UNIVERSE IS EXPANDING FASTER THAN PREDICTED? BLAME IT ON "DARK ENERGY".
SO, IF YOU HAVE NO CLUE ABOUT SOMETHING...
WE PUT "DARK" IN FRONT OF IT. IT'S EASIER TO GET FUNDING.
I HAVE A DARK THESIS.
FROM OBSERVATIONS OF WHOLE GALAXIES AND NEBULAE, THEORIES ARE BUILT ABOUT THE ORIGIN AND BUILDING BLOCKS OF OUR UNIVERSE.
IS THERE A GRAND THEORY OF EVERYTHING? SOME PHYSICISTS SAY "WHY BOTHER?", SUBSCRIBING TO THE ANTHROPIC PRINCIPLE:
THINGS (QUARKS, LEPTONS) ARE THE WAY THEY ARE SIMPLY BECAUSE IF THEY WEREN'T, WE WOULDN'T BE HERE TO ASK THIS QUESTION.
WHY ME?
WHY NOT?
THE BIG BANG? WE KNOW IT HAPPENED BECAUSE OF ONE MEASUREMENT OF THE UNIVERSE'S BACKGROUND NOISE.
KATIE ADAMANTLY REFUSES TO ACCEPT THIS:
IN OTHER WORDS, THERE ISN'T A GRAND EQUATION AT THE HEART OF EVERYTHING, SO WE SHOULD STOP LOOKING.
THAT'S IT?
Waves / centimeter
SO FAR, ALL OF REALITY SEEMS TO BE DESCRIBED BY EXQUISITE, ELEGANT MATHEMATICAL EQUATIONS.
WE CAN'T GIVE UP NOW. IT'S GOTTA BE BEAUTIFUL ALL THE WAY DOWN.

Inflation

Inflation

So far, have seen that BB works remarkably well in explaining the observed universe.

There are, however, many problems with the classical BB theories:

Horizon problem: CMB looks too isotropic $\Longrightarrow$ Why?

Flatness problem: Density close to BB was very close to $\Omega = 1$ (deviation $\sim 10^{-16}$ during nucleosynthesis) $\Longrightarrow$ Why?

Hidden relics problem: There are no observed magnetic monopoles, although predicted by GUT, neither gravitinos and other exotic particles $\Longrightarrow$ Why?

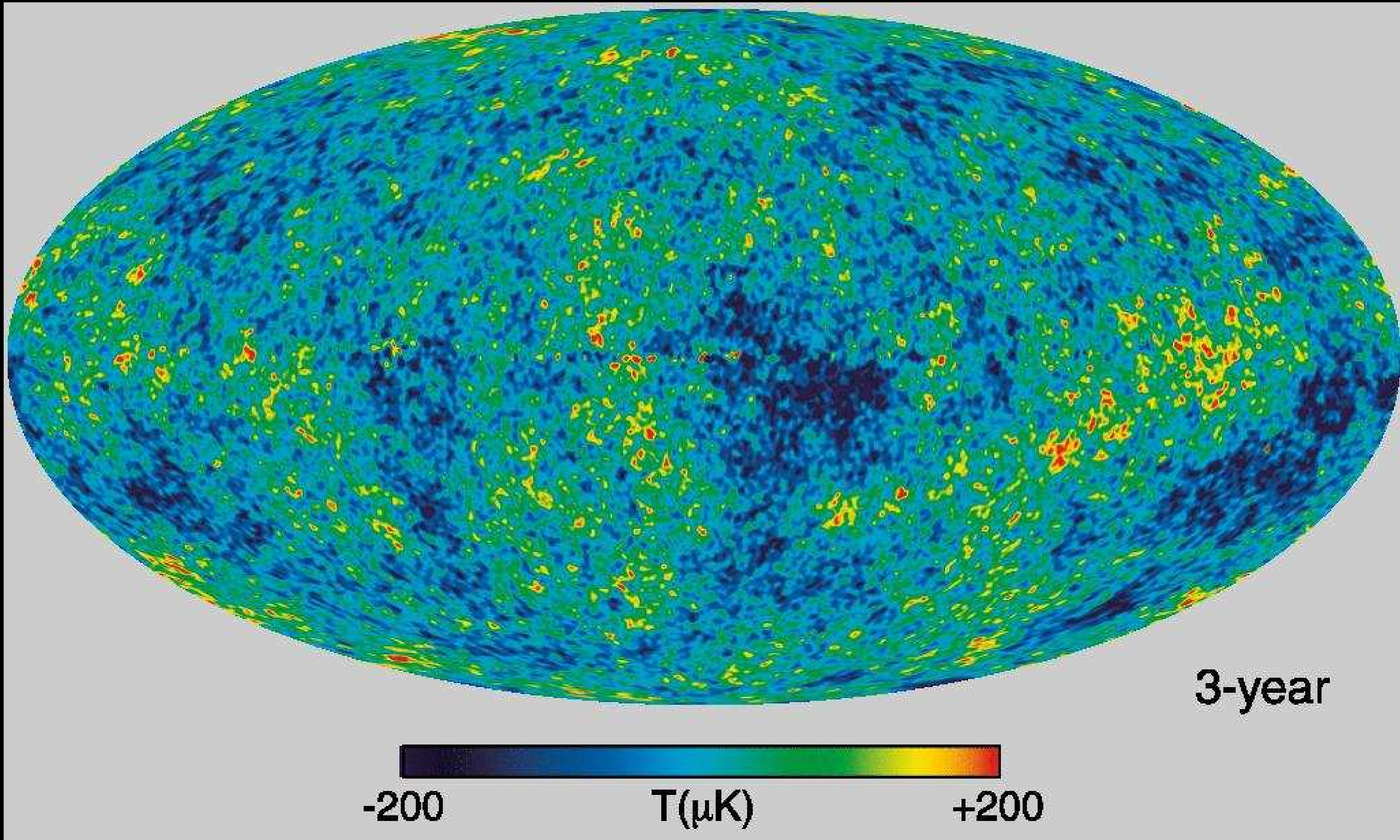
Vacuum energy problem: Energy density of vacuum is 10^{120} times smaller than predicted $\Longrightarrow$ Why?

Expansion problem: The universe expands $\Longrightarrow$ Why?

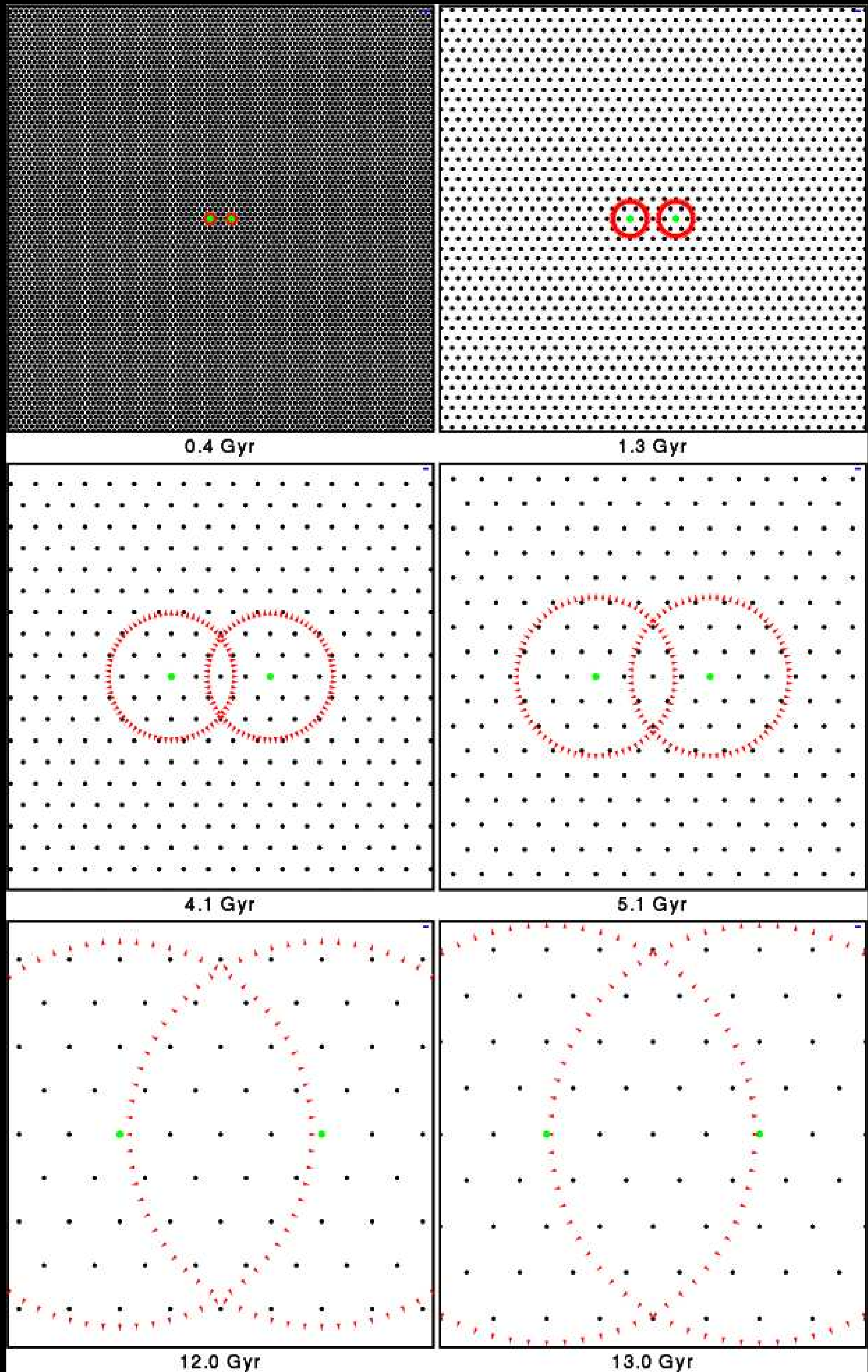
Baryogenesis: There is virtually no antimatter in the universe $\Longrightarrow$ Why?

Structure formation: Standard BB theory produces no explanation for lumpiness of universe.

Inflation attempts to answer all of these questions.



(WMAP; Page et al., 2007)



courtesy E. Wright.

Horizon problem, III

COBE and WMAP: There are temperature fluctuations in CMB on 10° scales:

$$\frac{\Delta T_{\text{CMB}}}{T_{\text{CMB}}} \sim 2 \times 10^{-5} \tag{7.1}$$

Size of observable universe at given epoch ("particle horizon") is given by coordinate distance traveled by photons since the big bang (Eq. 4.43):

$$d_{\text{h}} = R_0 \cdot r_{\text{H}}(t) = \int_0^t \frac{c\,\mathrm{d}t}{a(t)} \tag{7.2}$$

For a matter dominated universe with $\Omega = 1$,

$$a(t) = \left(\frac{3H_0}{2}\,t\right)^{2/3} \tag{4.72}$$

such that for $t = t_0 = 2/(3H_0)$ (Eq. 4.73):

$$d_{\text{h}}(t_0) = \frac{3c}{(3H_0/2)^{2/3}}\,t_0^{1/3} = \frac{2c}{H_0} \tag{7.3}$$

Horizon problem, IV

For matter dominated universes at redshift z, Eq. (7.3) works out to

$$d_\mathrm{h} \approx \frac{6000\,\mathrm{Mpc}}{h\sqrt{\Omega z}} \tag{7.4}$$

(Peacock, 1999, eq. 11.2)

Since CMB decoupled at $z \sim 1000$, at that time $d_\mathrm{h} \sim 200\,\mathrm{Mpc}$, while today $d_\mathrm{h} \sim 6000\,\mathrm{Mpc}$

$\Longrightarrow$ current observable volume $\sim 30000\times$ larger!

Note: we use $a \Longrightarrow$ all scales refer to what they are *now*, not what they *were* when the photons started!

Horizon problem: Why were causally disconnected areas on the sky so similar when CMB last interacted with matter?

Note that the horizon distance is larger than Hubble length:

$$d_\mathrm{h} = \frac{2c}{H_0} > \frac{2c}{3H_0} = c \cdot t_0 = d_\mathrm{H} \tag{7.5}$$

Reason for this is that universe expanded while photons traveled towards us
$\Longrightarrow$ Current observable volume larger than volume expected in a non-expanding universe.

Flatness problem, I

Current observations of density of universe roughly imply

$$0.01 \lesssim \Omega \lesssim 2 \quad \text{i.e., } \Omega \sim 1 \tag{7.6}$$

(will be better constrained later)

$\Omega \sim 1$ imposes very strict conditions on initial conditions of universe:

The Friedmann equation (e.g., Eq. 4.57) can be written in terms of Ω:

$$\Omega - 1 = \frac{k}{a^2 H^2} = \frac{ck}{\dot{a}^2} \tag{7.7}$$

For a nearly flat, matter dominated universe, $a(t) \propto t^{2/3}$, such that

$$\frac{\Omega(t) - 1}{\Omega(t_0) - 1} = \left(\frac{t}{t_0}\right)^{2/3} \tag{7.8}$$

while for the radiation dominated universe with $a(t) \propto t$,

$$\frac{\Omega(t) - 1}{\Omega(t_0) - 1} = \frac{t}{t_0} \tag{7.9}$$

Flatness problem, II

Today: $t_0 = 3.1 \times 10^{17}\,h^{-1}$ s, i.e., observed flatness predicts for era of nucleosynthesis ($t = 1$ s):

$$\frac{\Omega(1\,\mathrm{s}) - 1}{\Omega(t_0) - 1} \sim 10^{-12} \ldots 10^{-16} \tag{7.10}$$

i.e., very close to unity.

Flatness problem: It is very unlikely that Ω was so close to unity at the beginning without a physical reason.

Had Ω been different from 1, the universe would immediately have been collapsed or expanded too fast $\Longrightarrow$ Anthropocentric point of view *requires* $\Omega = 1$.

Hidden relics problem

Modern theories of particle physics predict the following particles to exist:

Gravitinos: From supergravity, spin $3/2$ particle with $mc^2 \sim 100\,\text{GeV}$, if it exists, then nucleosynthesis would not work if BB started at $kT > 10^9\,\text{GeV}$.

Moduli: Spin-0 particles from superstring theory, contents of vacuum at high energies.

Magnetic Monopoles: Predicted in grand unifying theories, but not observed.

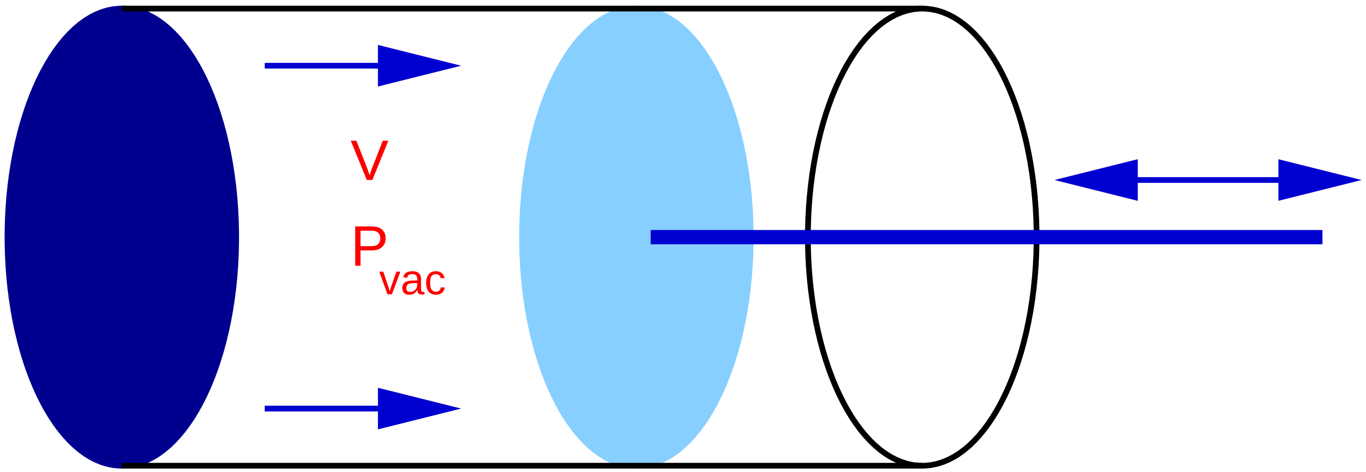
Hidden relics problem: If there was a normal big bang, then strange particles should exist, which are not observed today.

Vacuum, Λ, I

What is vacuum? *Not empty space* but rather ground state of some physical theory

(Reviews: Carroll, Press & Turner 1992, Carroll 2001)

Since ground state should be same in all coordinate systems $\Longrightarrow$ Vacuum is Lorentz invariant.



(after Peacock, 1999, Fig. 1.3)

Equation of state (Zeldovich, 1967):

$$P_{\text{vac}} = -\rho_{\text{vac}} c^2 \tag{7.11}$$

This follows directly from 1st law of thermodynamics: ρ_{vac} should be constant if compressed or expanded, which is true only for this type of equation of state:

$$\mathrm{d}E = \mathrm{d}U + P\,\mathrm{d}V = \rho_{\text{vac}} c^2\,\mathrm{d}V - \rho_{\text{vac}} c^2\,\mathrm{d}V = 0 \tag{7.12}$$

Vacuum, Λ, II

ρ_{vac} defines Einstein's cosmological constant

$$\Lambda = -\frac{8\pi G \rho_{\text{vac}}}{c^4} \tag{7.13}$$

Adding ρ_{vac} to the Friedmann equations allows to define

$$\Omega_\Lambda = \frac{\rho_{\text{vac}}}{\rho_{\text{crit}}} = \frac{\rho_{\text{vac}}}{3H^2/8\pi G} = \frac{c^4\Lambda}{3H^2} \tag{7.14}$$

Classical physics: Particles have energy

$$E = T + V \tag{7.15}$$

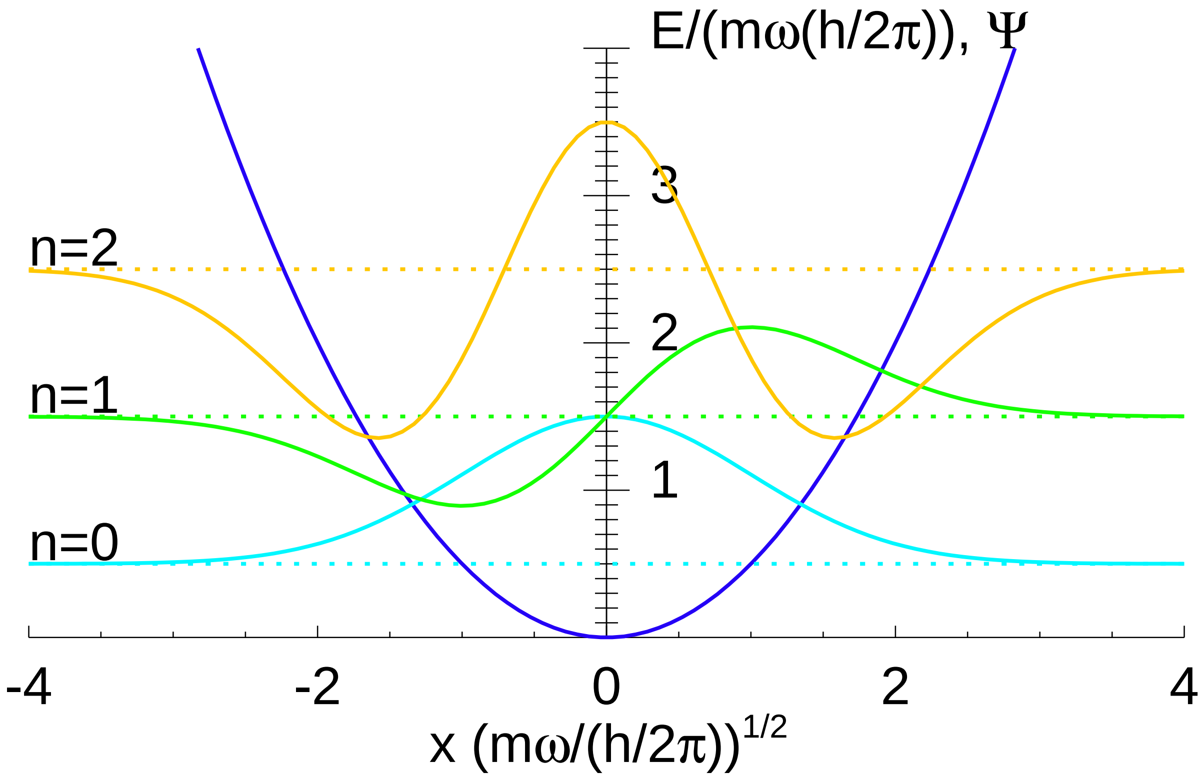
and force is $F = -\nabla V$, i.e., can add constant without changing equation of motion

$\Longrightarrow$ In classical physics, we are able to define $\rho_{\text{vac}} = 0$!

Quantum mechanics is (as usual) more difficult.

Vacuum, Λ, III

Vacuum in quantum mechanics:



Simplest case: harmonic oscillator:

$$V(x) = \frac{1}{2}m\omega^2 x^2 \quad \text{i.e.,} \quad V(0) = 0 \tag{7.16}$$

However, particles can only have energies

$$E_n = \frac{1}{2}\hbar\omega + n\hbar\omega \quad \text{where } n \in \mathbb{N} \tag{7.17}$$

$\Longrightarrow$ Vacuum state has zero point energy

$$E_0 = \frac{1}{2}\hbar\omega \tag{7.18}$$

Simple consequence of uncertainty principle!

In QM, we could normalize $V(x)$ such that $E_0 = 0$, important here is that vacuum state energy *differs* from classical expectation!

Vacuum, Λ, IV

Quantum field theory: Field as collection of harmonic oscillators of all frequencies. Simplest case: spinless boson ("scalar field", ϕ).

$\Longrightarrow$ Vacuum energy is the sum of all contributing ground state modes:

$$E_0 = \sum_j \frac{1}{2}\hbar\omega_j \tag{7.19}$$

Calculate sum by putting system in box with volume L^3, and then $L \longrightarrow \infty$.

Box $\Longrightarrow$ periodic boundary conditions:

$$\lambda_i = L/n_i \quad \Longleftrightarrow \quad k_i = 2\pi/\lambda_i = 2\pi n_i/L \tag{7.20}$$

for $n_i \in \mathbb{N} \Longrightarrow$ there are $\mathrm{d}k_i L/2\pi$ discrete wavenumbers in $[k_i, k_i + \mathrm{d}k_i]$, such that

$$E_0 = \frac{1}{2}\hbar L^3 \int \frac{\omega_{\mathbf{k}}}{(2\pi)^3}\mathrm{d}^3\mathbf{k} \quad \text{where} \quad \omega_k^2 = k^2 + m^2/\hbar^2 \tag{7.21}$$

Imposing cutoff k_{max}:

$$\rho_{\text{vac}}c^2 = \lim_{L\to\infty} \frac{E_0}{L^3} = \hbar\frac{k_{\text{max}}^4}{16\pi^3} \tag{7.22}$$

Divergent for $k_{\text{max}} \longrightarrow \infty$ ("ultraviolet divergence").

Not worrisome as we expect simple QM to break down at large energies anyway (ignored collective effects, etc.).

Vacuum, Λ, V

When does classical quantum mechanics break down?

Estimate: Formation of "Quantum black holes":

$$\lambda_{\text{de Broglie}} = \frac{2\pi\hbar}{mc} < \frac{2Gm}{c^2} = r_{\text{Schwarzschild}} \tag{7.23}$$

$\Longrightarrow$ Defines Planck mass:

$$m_{\text{P}} = \sqrt{\frac{\hbar c}{G}} \mathrel{\hat{=}} 1.22 \times 10^{19}\,\text{GeV} \tag{7.24}$$

Corresponding length scale: Planck length:

$$l_{\text{P}} = \frac{\hbar}{m_{\text{P}}} = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-37}\,\text{cm} \tag{7.25}$$

...and time scale (Planck time):

$$t_{\text{P}} = \frac{l_{\text{P}}}{c} = \sqrt{\frac{\hbar G}{c^5}} \sim 10^{-47}\,\text{s} \tag{7.26}$$

$\Longrightarrow$ Limits of current physics until successful theory of quantum gravity.

The system of units based on l_{P}, m_{P}, t_{P} is called the system of Planck units.

Vacuum, Λ, VI

To calculate the QFT vacuum energy density, choose

$$k_{\mathrm{max}} = m_{\mathrm{P}} c^2/\hbar \tag{7.27}$$

Inserting into Eq. (7.22) gives

$$\rho_{\mathrm{vac}} c^2 = 10^{74}\,\mathrm{GeV}\,\hbar^{-3} \quad \text{or} \quad \rho_{\mathrm{vac}} \sim 10^{92}\,\mathrm{g\,cm}^{-3} \tag{7.28}$$

a tad bit on the high side ($\sim 10^{120}$ higher than observed).

Inserting ρ_{vac} in Friedmann equation: $T < 3\,\mathrm{K}$ at $t = 10^{-41}\,\mathrm{s}$ after Big Bang.

To obtain current universe we require $k_{\mathrm{max}} = 10^{-2}\,\mathrm{eV} \Longrightarrow$ Less than binding energy of Hydrogen, where QM definitively works!

Vacuum energy problem: Contributions from virtual fluctuations of all particles must cancel to very high precision to produce observable universe.

Casimir effect: force between conducting plates of area A and distance a in vacuum is $F_{\mathrm{Casimir}} = \hbar c A \pi^2/(240 a^4) \Longrightarrow$ caused by incomplete cancellation of quantum fluctuations. Confirmed by Lamoreaux in 1996 at 5% level.

Expansion problem

Cosmological Expansion:

GR predicts expansion of the universe, but initial conditions for expansion are not set!

Classical cosmology: "The unverse expands since it has expanded in the past"
$\Longrightarrow$ Hardly satisfying...

> Cosmological Expansion Problem: What is the physical mechanism responsible for the expansion of the universe?

To put it more bluntly:
"The Big Bang model explains nothing about the origin of the universe as we now perceive it, because all the most important features are 'predestined' by virtue of being built into the assumed initial conditions near to $t = 0$." (Peacock, 1999, p. 324)

Baryogenesis

Quantitatively: Today:

$$\frac{N_{\mathrm{p}}}{N_{\gamma}} \sim 10^{-9} \quad \text{but} \quad \frac{N_{\bar{\mathrm{p}}}}{N_{\gamma}} \sim 0 \qquad (7.29)$$

Assuming isotropy and homogeneity, this is puzzling: Violation of Copernican principle!

> Antimatter problem: There are more particles than antiparticles in the observable universe.

Sakharov (1968): Asymmetry implies three fundamental properties for theories of particle physics:

1. CP violation (particles and antiparticles must behave differently in reactions, observed, e.g., in the K^0 meson),
2. Baryon number violating processes (more baryons than antibaryons $\Longrightarrow$ Prediction by GUT),
3. Deviation from thermal equilibrium in early universe (CPT theorem: $m_{\mathrm{X}} = m_{\bar{\mathrm{X}}} \Longrightarrow$ *same* number of particles and antiparticles in thermal equilibrium).

Structure formation

Final problem: structure formation

> In the classical BB picture, the initial conditions for structure formation observed are not explained. Furthermore, assuming the observed Ω_{baryons}, the observed structures (=us) cannot be explained.

The theory of inflation attempts to explain all of the problems mentioned by invoking phase of exponential expansion in the very early universe ($t \lesssim 10^{-16}$ s).

Basic Idea, I

Use the Friedmann equation with a cosmological constant:

$$H^2(t) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3} \qquad (7.30)$$

Basic assumption of inflationary cosmology:

> During the big bang there was a phase where Λ dominated the Friedmann equation.

$$H(t) = \frac{\dot{a}}{a} = \sqrt{\frac{\Lambda}{3}} = \text{const.} \qquad (7.31)$$

since $\Lambda =$ const. (probably...). Solution of Eq. (7.31):

$$a \propto \mathrm{e}^{Ht} \qquad (7.32)$$

and inserting into Eq. (7.7) shows that

$$\Omega - 1 = \frac{k}{a^2 H^2} \propto \mathrm{e}^{-2Ht} \qquad (7.33)$$

Basic Idea, II

When did inflation happen?

Typical assumption: Inflation = phase transition of a scalar fi eld ("inflaton") associated with Grand Unifying Theories.

Therefore the assumptions:

- temperature $kT_{\text{GUT}} = 10^{15}$ GeV, when $1/H \sim 10^{-34}\,\text{sec}$ ($t_{\text{start}} \sim 10^{-34}$ s).
- inflation lasted for 100 Hubble times, i.e., for $\Delta T = 10^{-32}$ s.

With Eq. (7.32): Inflation: Expansion by factor $e^{100} \sim 10^{43}$.

. . . corresponding to a volume expansion by factor $\sim 10^{130}$

$\Longrightarrow$ solves hidden relics problem!

Furthermore, Eq. (7.33) shows

$$\Omega - 1 = 10^{-86} \tag{7.34}$$

$\Longrightarrow$ solves flatness problem!

Basic Idea, III

Temperature behavior: *During* inflation universe supercools:

Remember: entropy density

$$s = \frac{\rho c^2 + P}{T} \tag{6.72}$$

But for Λ:

$$p = -\rho c^2 \tag{7.11}$$

so that the entropy density of vacuum

$$s_{\text{vac}} = 0 \tag{7.35}$$

Trivial result since vacuum is just one quantum state $\Longrightarrow$ *very* low entropy.

Inflation produces no entropy $\Longrightarrow S$ existing *before* inflation gets diluted, since entropy *density* $s \propto a^{-3}$.

But for relativistic particles $s \propto T^3$ (Eq. 6.74), such that

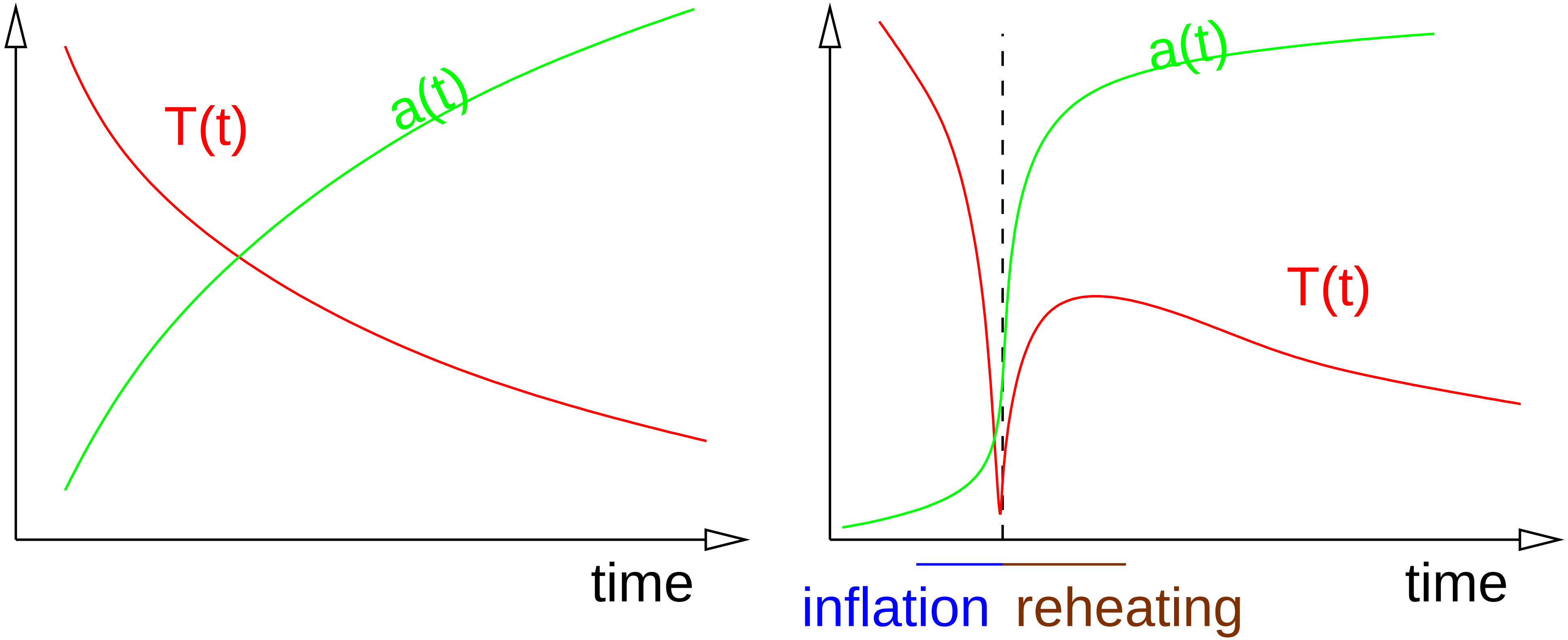
$$aT = \text{const.} \quad \Longrightarrow \quad T_{\text{after}} = 10^{-43} T_{\text{before}} \tag{7.36}$$

When inflation stops: vacuum energy of inflaton field transferred to normal matter

$\Longrightarrow$ "Reheating" to temperature

$$T_{\text{reheating}} \sim 10^{15}\,\text{GeV} \tag{7.37}$$

Summary



(after Bergström & Goobar, 1999, Fig. 9.1, and Kolb & Turner, Fig. 8.2)

Scalar Fields, I

For inflation to work: need short-term cosmological constant, i.e., need particles with negative pressure.

Basic idea (Guth, 1981): phase transition where suddenly a large Λ happens.

How? $\Longrightarrow$ Quantum Field Theory!

Describe hypothetical particle with a time-dependent quantum field, $\phi(t)$, and potential, $V(\phi)$.

Simplest example from QFT ($\hbar = c = 1$):

$$V(\phi) = \frac{1}{2} m^2 \phi^2 \tag{7.38}$$

where m: "mass of field". Particle described by ϕ: "inflaton".

For all scalar fields, particle physics shows:

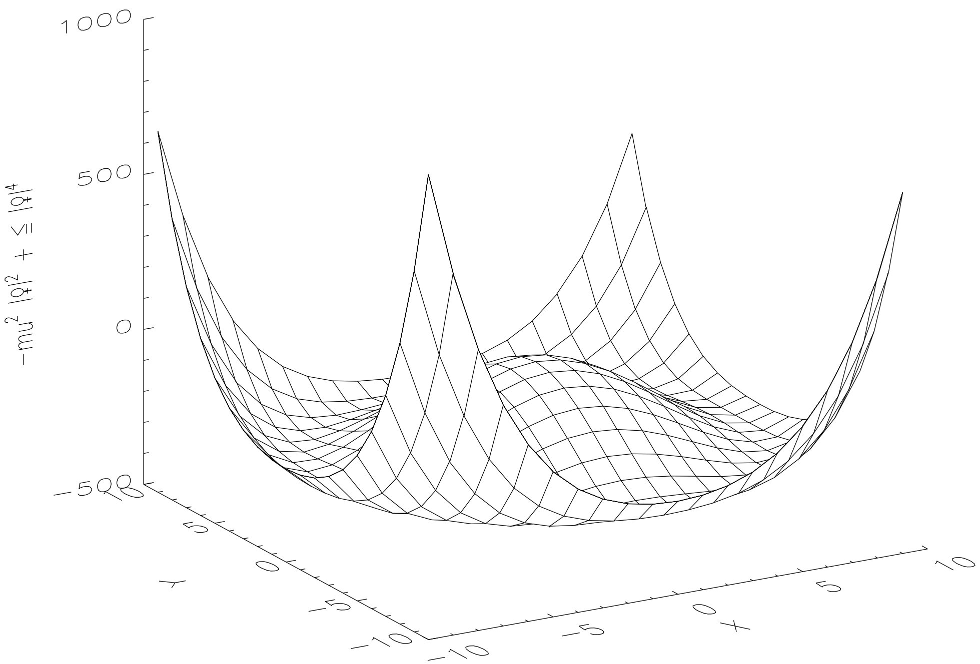
$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{7.39}$$

$$P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{7.40}$$

i.e., obeys vacuum equation of state!

"Vacuum": particle "sits" at minimum of V.

Scalar Fields, II



Typically: potential looks more complicated.

Due to symmetry, after harmonic oscillator, 2nd simplest potential: Mexican hat potential ("Higgs potential"),

$$V(\phi) = -\mu^2\phi^2 + \lambda\phi^4 \qquad (7.41)$$

$\Longrightarrow$ Minimum of V still determines vacuum value.

For $T \neq 0$, we need to take interaction with thermal bath into account

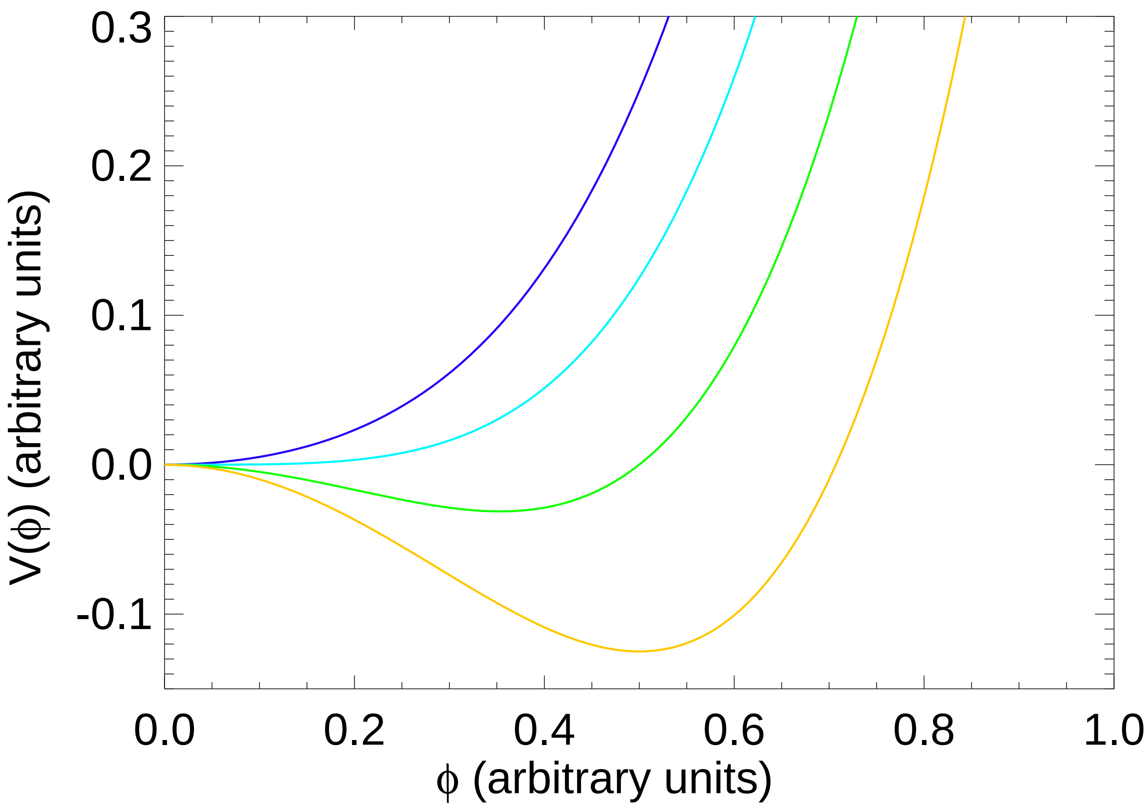
$\Longrightarrow$ Temperature dependent potential!

$$V_{\text{eff}}(\phi) = -(\mu^2 - aT^2)\phi^2 + \lambda\phi^4 \qquad (7.42)$$

where a some constant.

(minimization of Helmholtz free energy, see Peacock, 1999, , p. 329ff., for details)

Scalar Fields, III



The minimum of V is at

$$\phi = \begin{cases} 0 & \text{for } T > T_c \\ \sqrt{\frac{\mu^2 - aT^2}{2\lambda}} & \text{for } T < T_c \end{cases} \tag{7.43}$$

where the critical temperature

$$T_c = \mu/\sqrt{a} \tag{7.44}$$

and

$$V_{\text{min}} = \begin{cases} 0 & \text{for } T > T_c \\ -\frac{(\mu^2 - aT^2)^2}{4\lambda} & \text{for } T < T_c \end{cases} \tag{7.45}$$

Since switch happens suddenly: phase transition

Scalar Fields, IV

Minimum V_{min} for $T > T_{\text{c}}$ smaller than "vacuum minimum"

$\Longrightarrow$ Behaves like a cosmological constant!

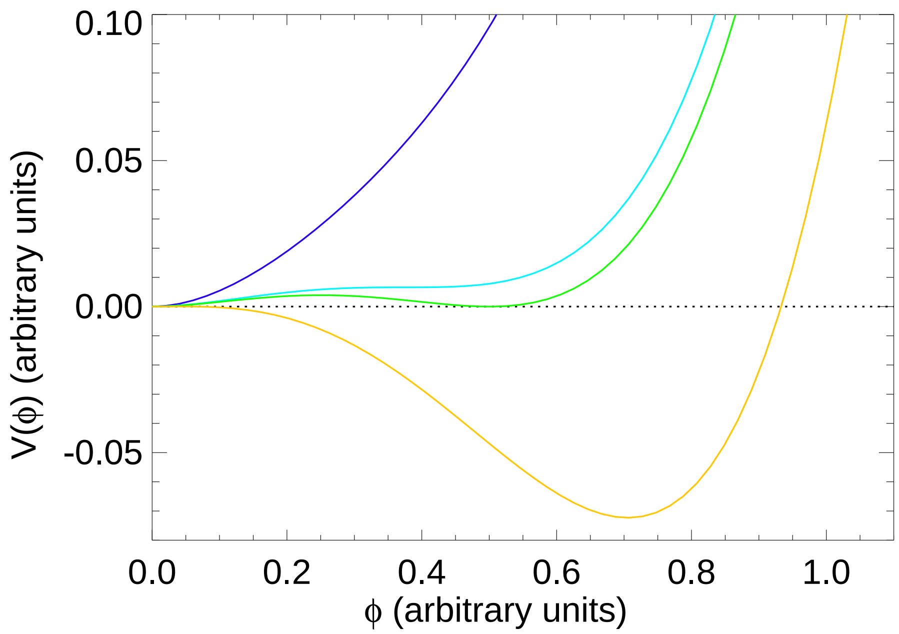
Since $T_{\text{c}} \propto \mu$,

Inflation sets in at mass scale of whatever scalar fi eld produces inflation.

Grand Unifying Theories: $m \sim 10^{15}$ GeV.

The problem is, what $V(\phi)$ to use. . .

First-Order Inflation



(after Peacock, 1999, Fig. 11.2)

Original idea (Guth, 1981):

$$V(\phi, T) = \lambda|\phi|^4 - b|\phi|^3 + aT^2|\phi|^2 \quad (7.46)$$

has two minima for T greater than a critical temperature:

$V_{\text{min}}(\phi = 0)$: false vacuum

$V_{\text{min}}(\phi > 0)$: true vacuum iff < 0.

Particle can tunnel between both vacua: first order phase transition $\Longrightarrow$ first order inflation.

Problem: vacuum tunnels between false and true vacua $\Longrightarrow$ formation of bubbles.

Outside of bubbles: inflation goes infinitely ("graceful exit problem").

First order inflation is not feasible.

Summary

First order inflation does not work

$\Longrightarrow$ Potentials derived from GUTs do not work.

$\Longrightarrow$ However, many empirical potentials do not suffer from these problems.

$\Longrightarrow$ inflation is *still* theory of choice for early universe.

Catchphrases (Liddle & Lyth, 2000, Ch. 8):

- chaotic inflation,
- supersymmetry/-gravitation $\Longrightarrow$ tree-level potentials,
- renormalizable global susy,
- power-law inflation,
- hybrid inflation (combination of *two* scalar fields) $\Longrightarrow$ spontaneous or dynamical susy breaking,
- scalar-tensor gravity

and many more...
All are somewhat *ad hoc*, and have more or less foundations in modern theories of QM and gravitation.

Information on what model is correct comes from

1. predicted seed to structure formation, and
2. values of Ω and Λ.

$\Longrightarrow$ Determine Ω and Λ!

Bergström, L., & Goobar, A., 1999, Cosmology and Particle Physics, (Chichester: Wiley)

Carroll, S. M., 2001, Living Rev. Rel., 4, 2001

Carroll, S. M., Press, W. H., & Turner, E. L., 1992, ARA&A, 50, 499

Liddle, A. R., & Lyth, D. H., 2000, Cosmological Inflation and Large-Scale Structure, (Cambridge: Cambridge Univ. Press)

Page, L., et al., 2007, Astrophys. J., Suppl. Ser., 170, 335

Peacock, J. A., 1999, Cosmological Physics, (Cambridge: Cambridge Univ. Press)

Sakharov, A. D., 1968, Dokl. Akad, Nauk SSSR (Soviet Phys. Dokl.), 12, 1040

Determination of Ω and Λ

Inflation

Previous lectures: Inflation requires

$$\Omega = \frac{\rho}{\rho_{\text{crit}}} = \Omega_{\text{m}} + \Omega_{\Lambda} = 1 \qquad (8.1)$$

Here,

Ω_{m}: Ω due to gravitating stuff,

Ω_{Λ}: Ω due to vacuum energy or other exotic stuff.

To decide whether that is true:

- need inventory of gravitating material in the universe,
- need to search for evidence of non-zero Λ

Also search for evidence in structure formation $\Longrightarrow$ Later...

Inflation

Remember that

$$\Omega_\mathrm{m} = \frac{\rho_\mathrm{m}}{\rho_\mathrm{crit}} = \frac{8\pi G\rho}{3H^2} \tag{4.58}$$

and

$$\Omega_\Lambda = \frac{\rho_\mathrm{vac}}{\rho_\mathrm{crit}} = \frac{\rho_\mathrm{vac}}{3H^2/8\pi G} = \frac{c^4\Lambda}{3H^2} \tag{7.14}$$

As for a typical ensemble of stars,

$$\frac{M}{L} \approx \text{const.} \tag{8.2}$$

we often express Ω in terms of a mass to luminosity ratio:

Using canonical luminosity density of universe, one can show (Peacock, 1999, p. 368, for the B-band):

$$\left.\frac{M}{L}\right|_\mathrm{crit} = 1390\,h\frac{M_\odot}{L_\odot} \tag{8.3}$$

. . . which means that there *must* be lots of dark matter.

Introduction

Constituents of Ω_{m}:

- Radiation (CMBR)
- Neutrinos
- Baryons ("normal matter", Ω_{b})
- Other, non-radiating, gravitating material ("dark matter")

Radiation: From temperature of CMBR, using $u = \rho c^2 = a_{\text{rad}} T^4$:

$$\Omega_\gamma h^2 = 2.480 \times 10^{-5} \quad (8.4)$$

for $h = 0.72$, $\Omega_\gamma = 4.8 \times 10^{-5}$

Massless Neutrinos have

$$\Omega_\nu = 3 \cdot \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} \Omega_\gamma = 0.68\, \Omega_\gamma \quad (8.5)$$

Photons and massless neutrinos are unimportant for todays Ω.

Massive Neutrinos

Sudbury Neutrino Observator (SNO) and Super-Kamiokande: Neutrinos are not massless.

From neutrino decoupling and expansion:

Current neutrino density: 113 neutrinos cm^{-3} per neutrino family.

In terms of Ω:

$$\Omega_\nu h^2 = \frac{\sum m_i}{93.5\,\text{eV}} \tag{8.6}$$

$\Longrightarrow$ For $h = 0.72$, $m \sim 16\,\text{eV}$ would be suffi cient to close universe

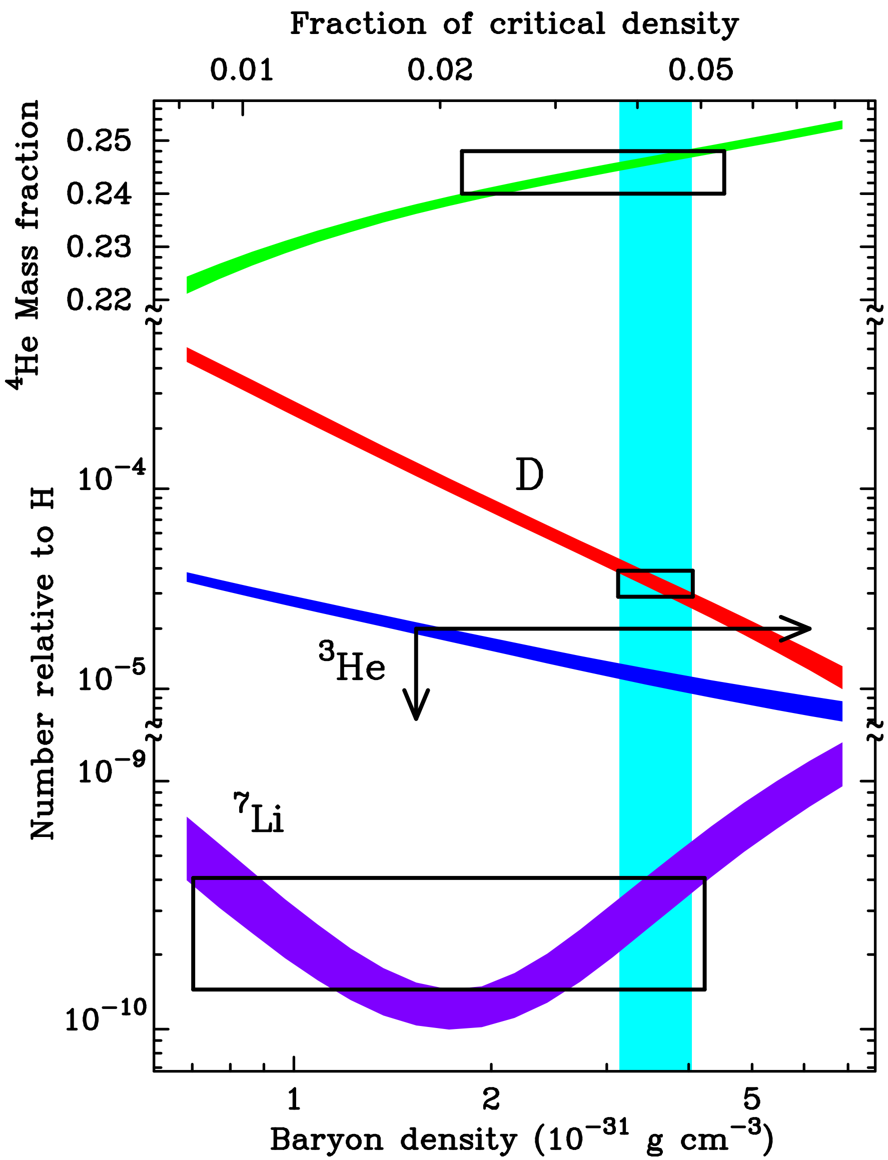
Current mass limits: $m_{\nu_\text{e}} < 2.2\,\text{eV}$, $m_{\nu_\mu} < 170\,\text{keV}$, and $m_{\nu_\tau} < 15.5\,\text{MeV}$

Source: `http://cupp.oulu.fi/neutrino/nd-mass.html`

Note that solar neutrino oscillations imply Δm between ν_e and ν_μ is $\sim 10^{-4}\,\text{eV}$, i.e., most probable mass for ν_μ is much smaller than the direct experimental limit.

Structure formation shows that $\sum m_\nu < 0.7\,\text{eV}$ (Spergel et al., 2007).

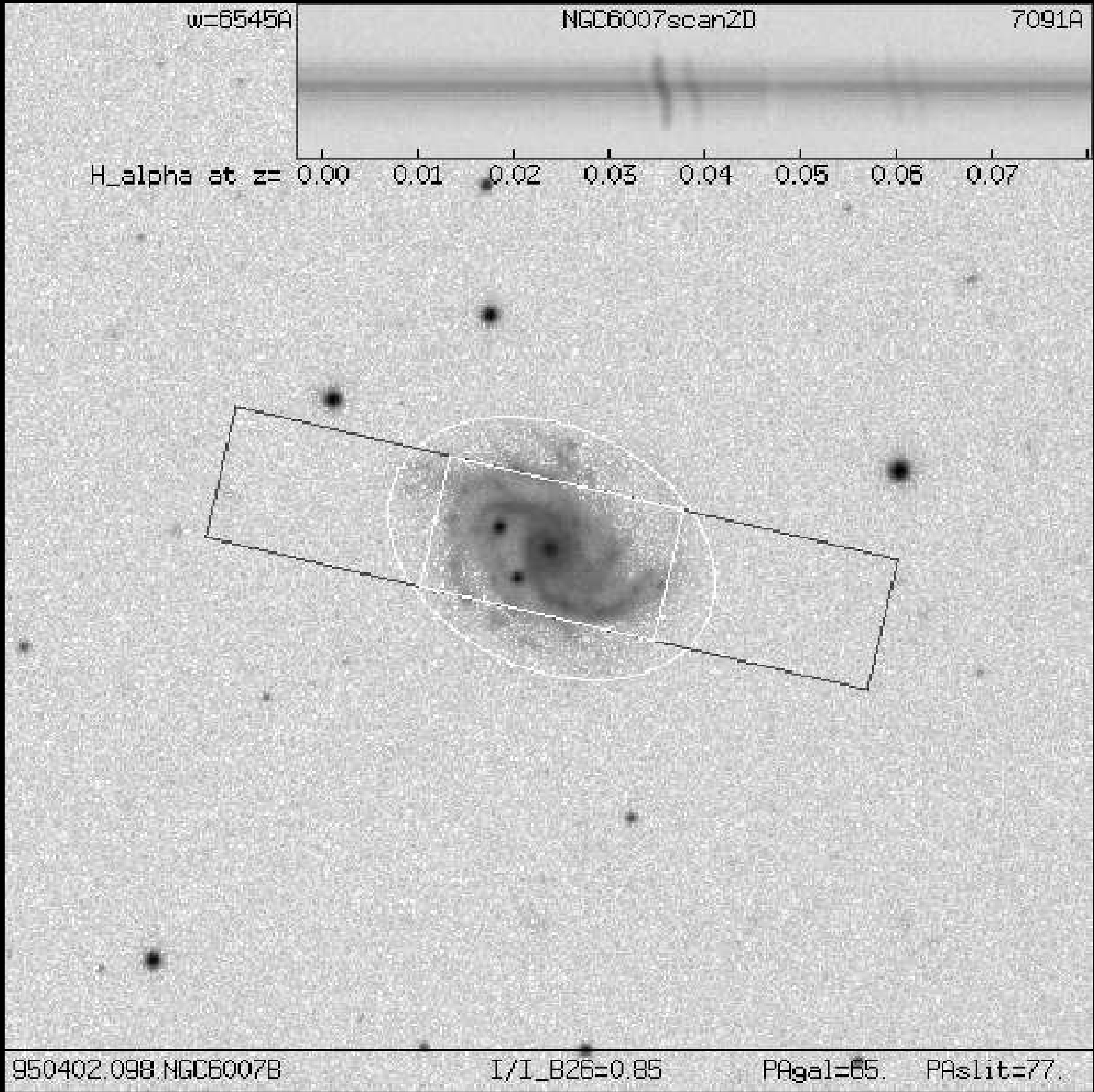
Baryons



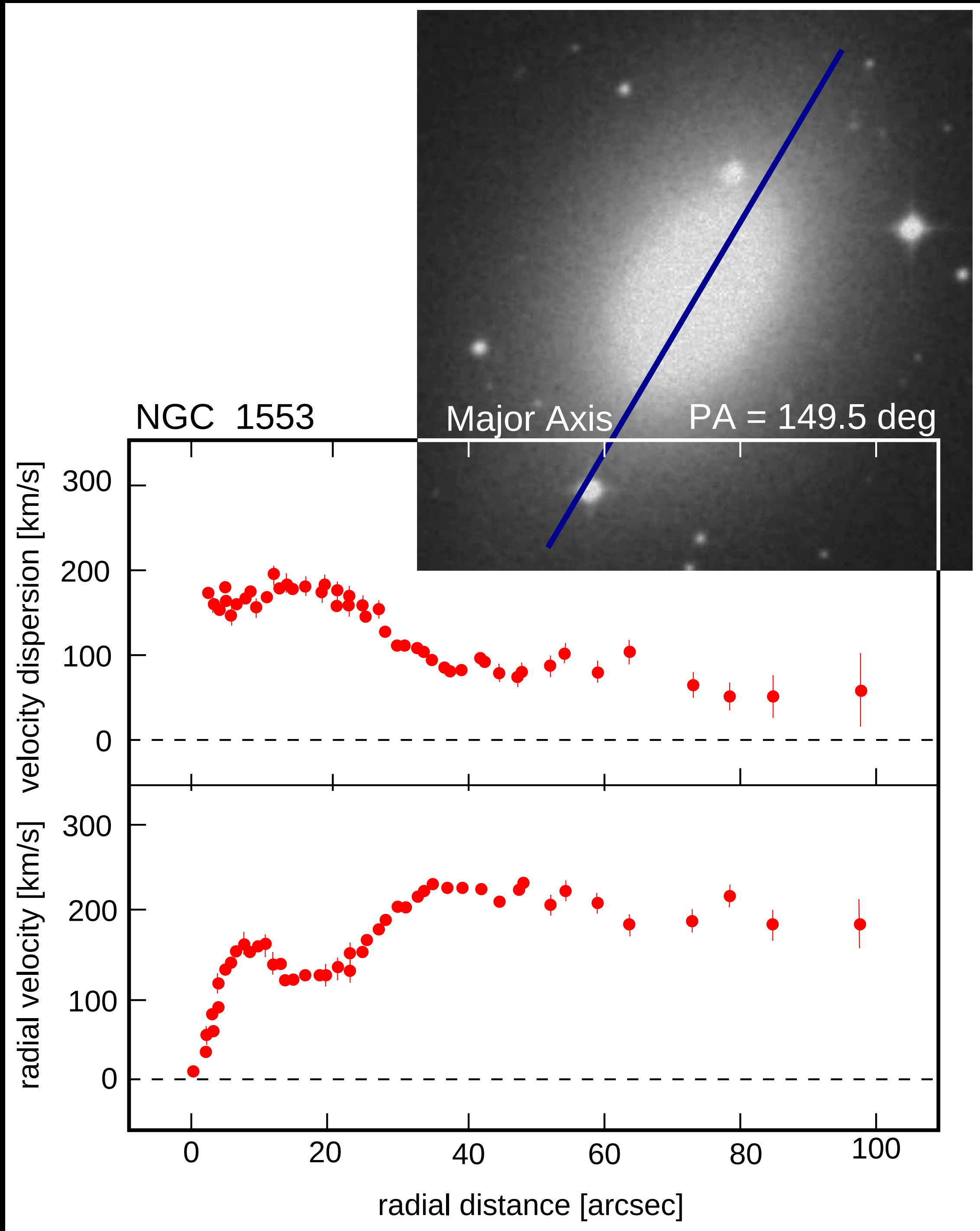
Best evidence for mass in baryons, Ω_{b}: primordial nucleosynthesis.

$$\Omega_{\mathrm{b}} h^2 = 0.02 \pm 0.002 \qquad (8.7)$$

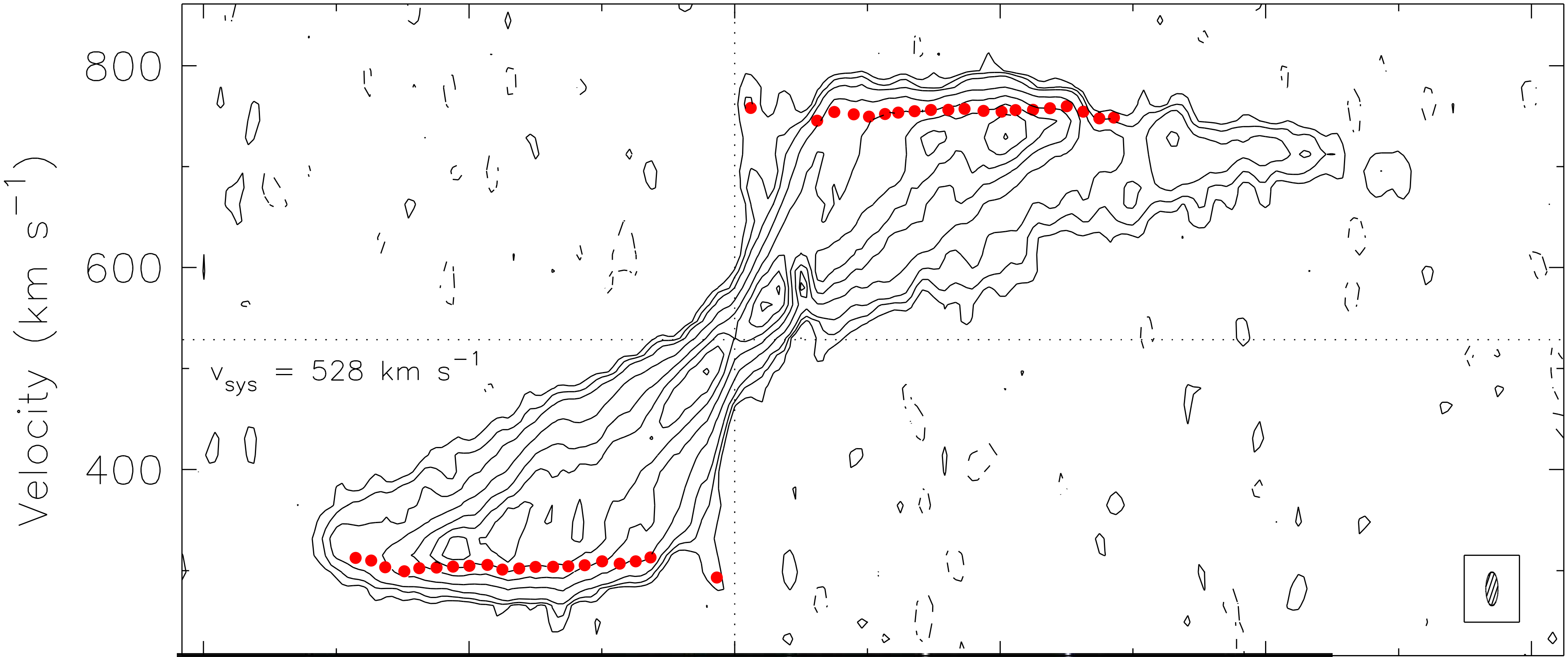
(Burles, Nollett & Turner, 1999, Fig. 1)



NGC 6007 (Jansen; `http://www.astro.rug.nl/~nfgs/`)



NGC 1553 (S0) after Kormendy (1984, ApJ 286, 116)



NGC 891 (Swaters et al., 1997, ApJ 491, 140 / Paul LeFevre, S&T Nov. 2002)

Galaxy Rotation Curves, IV

Stellar motion due to mass within r:

$$\frac{GM(\leq r)}{r^2} = \frac{v_{\text{rot}}^2(r)}{r}$$

$$\Longrightarrow M(\leq r) = \frac{v_{\text{rot}}^2 r}{G}$$

therefore:

$v \sim$ const. $\Longrightarrow M(\leq r) \propto r$.

NGC 891, KPNO 1.3 m
Barentine & Esquerdo

Galaxy Rotation Curves, V

For disk in spiral galaxies, $I(r) = I_0 \exp(-r/h)$ such that

$$L(r < r_0) = I_0 \int_0^{r_0} 2\pi r \exp(-r/h)\,\mathrm{d}r \propto h^2 - h(r+h)\exp(-r/h) \tag{8.8}$$

such that for $r \longrightarrow \infty$:

$$L(r < r_0) \to \text{const.} \tag{8.9}$$

If $M/L \sim$ const. $\Longrightarrow$ contradiction with observations! (we would expect $v \propto r^{-1/2}$)

Result for galaxies compared to stars

$$\left.\frac{M}{L}\right|_{\text{galaxies}} = 10\ldots20\,\frac{M_\odot}{L_\odot} \quad \text{vs.} \quad \left.\frac{M}{L}\right|_{\text{stars}} = 1\ldots3\,\frac{M_\odot}{L_\odot}$$

Only about 10% of the gravitating matter in universe radiates.

Coma Cluster (O. Lopez-Cruz, I. K. Shelton, & KPNO)

Galaxy Clusters, II

For mass of galaxy clusters, make use of the virial theorem:

$$E_{\text{kin}} = -E_{\text{pot}}/2 \quad (8.10)$$

in statistical equilibrium.

Measurement: assume isotropy, such that

$$\left\langle v^2 \right\rangle = \left\langle v_x^2 \right\rangle + \left\langle v_y^2 \right\rangle + \left\langle v_z^2 \right\rangle = 3 \left\langle v_\parallel^2 \right\rangle \quad (8.11)$$

Assuming that the velocity dispersion is independent of m_i gives:

$$E_{\text{kin}} = \frac{1}{2} \sum_i m_i \boldsymbol{v}_i^2 = \frac{3}{2} M \left\langle v_\parallel^2 \right\rangle \quad (8.12)$$

where M is the total mass.

If the cluster is spherically symmetric $\Longrightarrow$ Define weighted mean separation R_{cl}, such that

$$E_{\text{pot}} = \frac{GM^2}{R_{\text{cl}}} \quad (8.13)$$

From Eqs. (8.12) and (8.13):

$$M = \frac{3}{G} \left\langle v_\parallel^2 \right\rangle R_{\text{cl}} \quad (8.14)$$

E.g.: $v_\parallel \sim 1000\,\text{km}\,\text{s}^{-1}$, $R \sim 1\,\text{Mpc} \Longrightarrow M = 1.4 \times 10^{48}\,\text{g} = 7 \times 10^{14}\,M_\odot$ (MW: $6 \times 10^{11}\,M_\odot$).

Derivation of the Virial Theorem

Assume system of particles, each with mass m_i. Acceleration on particle i:

$$\ddot{\mathbf{r}} = \sum_{j \neq i} \frac{G m_j (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \tag{8.15}$$

. . . scalar product with $m_i \mathbf{r}_i$

$$m_i \mathbf{r}_i \cdot \ddot{\mathbf{r}}_i = \sum_{j \neq i} \frac{G m_i m_j \mathbf{r}_i \cdot (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \tag{8.16}$$

. . . since

$$\frac{1}{2} \frac{\mathrm{d}^2 \mathbf{r}_i^2}{\mathrm{d}t^2} = \frac{\mathrm{d}}{\mathrm{d}t} (\dot{\mathbf{r}}_i \cdot \mathbf{r}_i) = \ddot{\mathbf{r}}_i \cdot \mathbf{r}_i + \dot{\mathbf{r}}_i \cdot \mathbf{r}_i \tag{8.17}$$

. . . therefore Eq. (8.16)

$$\frac{1}{2} \frac{\mathrm{d}^2}{\mathrm{d}t^2} (m_i \mathbf{r}_i^2) - m_i \dot{\mathbf{r}}_i{}^2 = \sum_{j \neq i} \frac{G m_i m_j \mathbf{r}_i \cdot (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \tag{8.18}$$

Summing over all particles in the system gives

$$\frac{1}{2} \sum_i \frac{\mathrm{d}^2}{\mathrm{d}t^2} (m_i \mathbf{r}_i^2) - \sum_i m_i \dot{\mathbf{r}}_i{}^2 = \sum_i \sum_{j \neq i} \frac{G m_i m_j \mathbf{r}_i \cdot (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \tag{8.19}$$

$$= \frac{1}{2} \left(\sum_i \sum_{j \neq i} G m_i m_j \frac{\mathbf{r}_i \cdot (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_i - \mathbf{r}_j|^3} + \sum_j \sum_{i \neq j} G m_j m_i \frac{\mathbf{r}_j \cdot (\mathbf{r}_i - \mathbf{r}_j)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \right) \tag{8.20}$$

$$= \frac{1}{2} \left(\sum_i \sum_{j \neq i} G m_i m_j \frac{\mathbf{r}_i \cdot \mathbf{r}_j - \mathbf{r}_i^2}{|\mathbf{r}_i - \mathbf{r}_j|^3} + \sum_j \sum_{i \neq j} G m_j m_i \frac{\mathbf{r}_j \cdot \mathbf{r}_i - \mathbf{r}_j^2}{|\mathbf{r}_j - \mathbf{r}_i|^3} \right) \tag{8.21}$$

$$= -\frac{1}{2} \sum_{\substack{i,j \\ i \neq j}} \frac{G m_i m_j}{|\mathbf{r}_i - \mathbf{r}_j|} \tag{8.22}$$

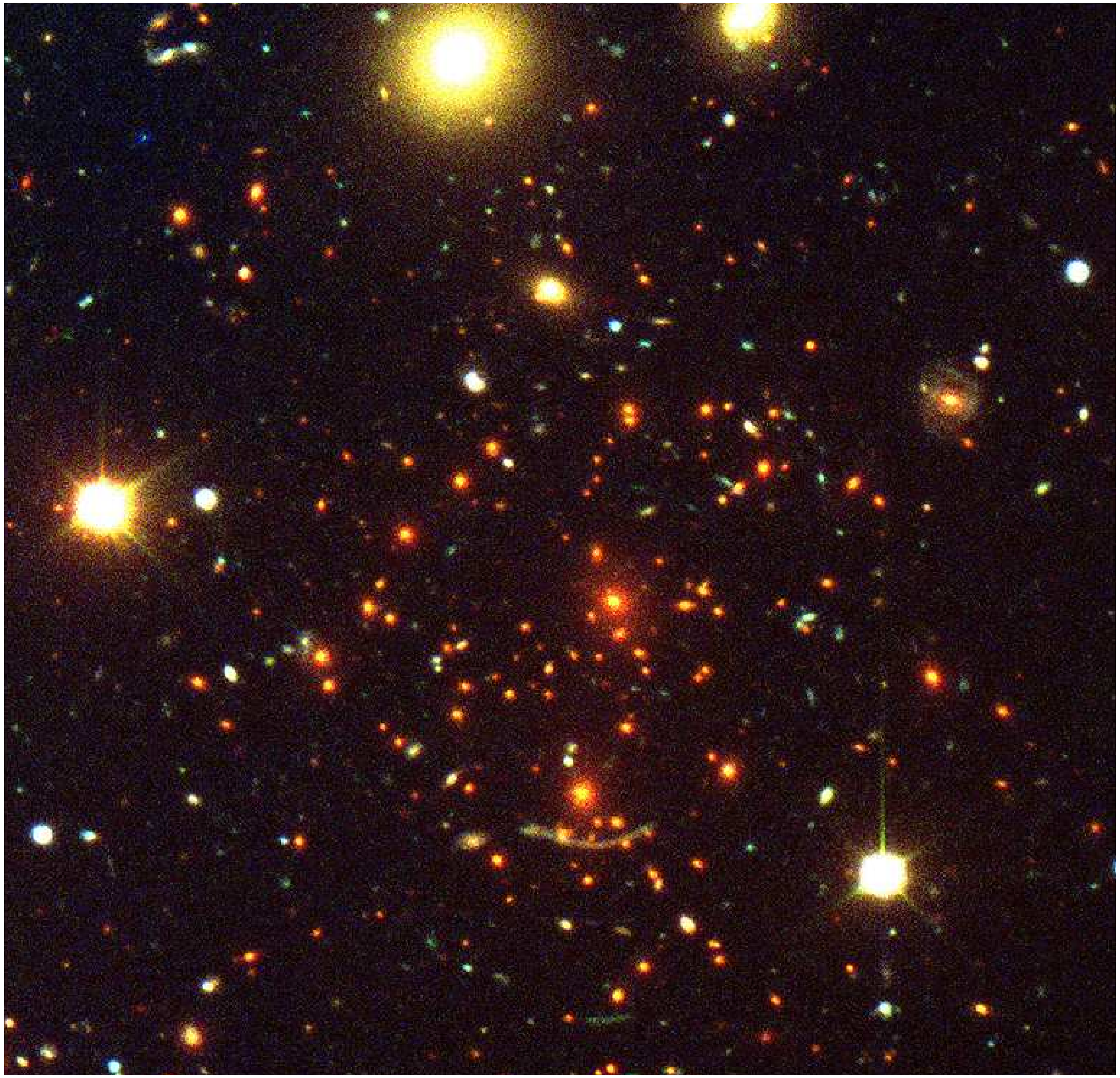
Thus, identifying the total kinetic energy, T, and the gravitational potential energy, U, gives

$$2T - U = \frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\sum_i m_i \mathbf{r}_i^2 = 0 \tag{8.23}$$

in statistical equilibrium.

Thus we fi nd the virial theorem: $T = \frac{1}{2}|U|$

Galaxy Clusters, III



More detailed analysis using more complicated mass models gives (Merritt, 1987):

$$\frac{M}{L} \sim 350h^{-1}\frac{M_\odot}{L_\odot} \qquad (8.24)$$

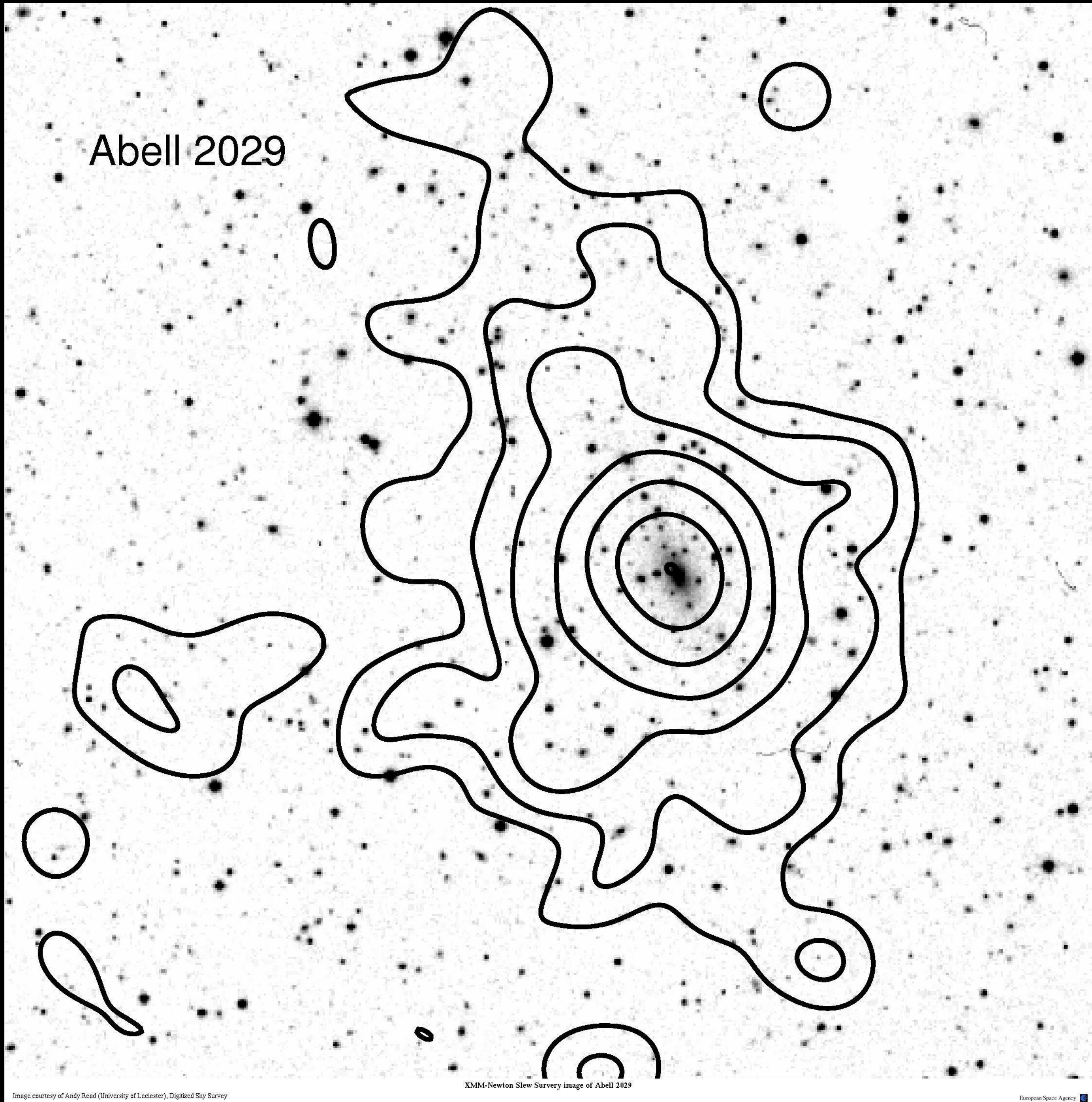
while we would have expected $M/L = 10\ldots20$ as for galaxies

Dark matter is an important constituent in galaxy clusters

Abell 370 (VLT UT1+FORS)

Abell 2029, Palomar Schmidt [DSS]

Abell 2029, Soft X-rays (*Chandra*; NASA/CXC/UCI/A.Lewis et al.)



Abell 2029, Optical and X-rays (*XMM-Newton*; Andy Read [Leicester]/DSS/ESA; larger FoV)

X-ray emission, IV

X-ray emission from galaxy clusters gives mass to higher precision:

Assume gas in potential of galaxy cluster. If gas is in hydrostatic equilibrium:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM_r\rho}{r^2} \tag{8.25}$$

where the pressure P can be determined from the equation of state:

$$P = nkT = \frac{\rho kT}{\mu m_\mathrm{H}} \tag{8.26}$$

where m_H: mass of H-atom, μ mean molecular weight of gas ($\mu = 0.6$ for fully ionized).

Differentiating Eq. (8.26) wrt r gives

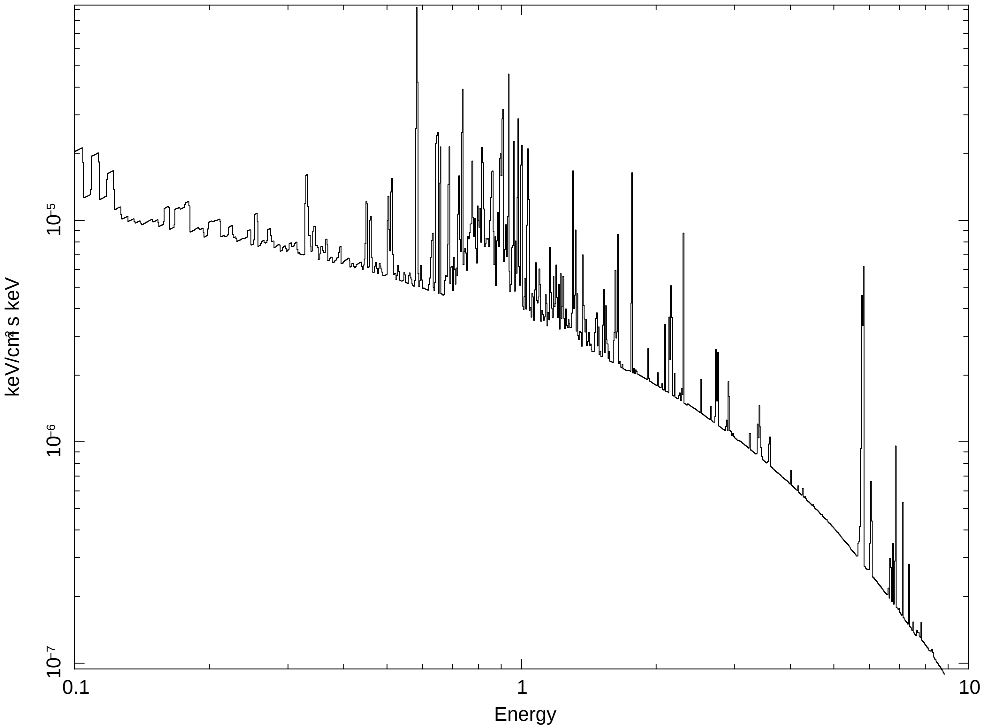
$$\frac{\mathrm{d}P}{\mathrm{d}r} = \frac{k}{\mu m_\mathrm{H}}\left(T\frac{\mathrm{d}\rho}{\mathrm{d}r} + \rho\frac{\mathrm{d}T}{\mathrm{d}r}\right) = \frac{\rho kT}{\mu m_\mathrm{H}}\left(\frac{\mathrm{d}\log\rho}{\mathrm{d}r} + \frac{\mathrm{d}\log T}{\mathrm{d}r}\right) \tag{8.27}$$

Inserting $\mathrm{d}P/\mathrm{d}r$ into Eq. (8.25) and solving for M_r gives

$$M_r = -\frac{kTr^2}{G\mu m_\mathrm{H}}\left(\frac{\mathrm{d}\log\rho}{\mathrm{d}r} + \frac{\mathrm{d}\log T}{\mathrm{d}r}\right) \tag{8.28}$$

X-ray emission, V

To determine M_r, we need to measure $T(r)$ and $\rho(r)$. These quantities can be obtained from the observed X-ray spectrum:



Theoretical X-ray spectrum of a cluster.

Cluster gas mainly radiates by bremsstrahlung emission, with a spectral continuum shape

$$\epsilon(E) \propto \left(\frac{m_\mathrm{e}}{kT}\right)^{1/2} g(E,T)\, n\, n_\mathrm{e} \exp\left(-\frac{E}{kT}\right) \tag{8.29}$$

where

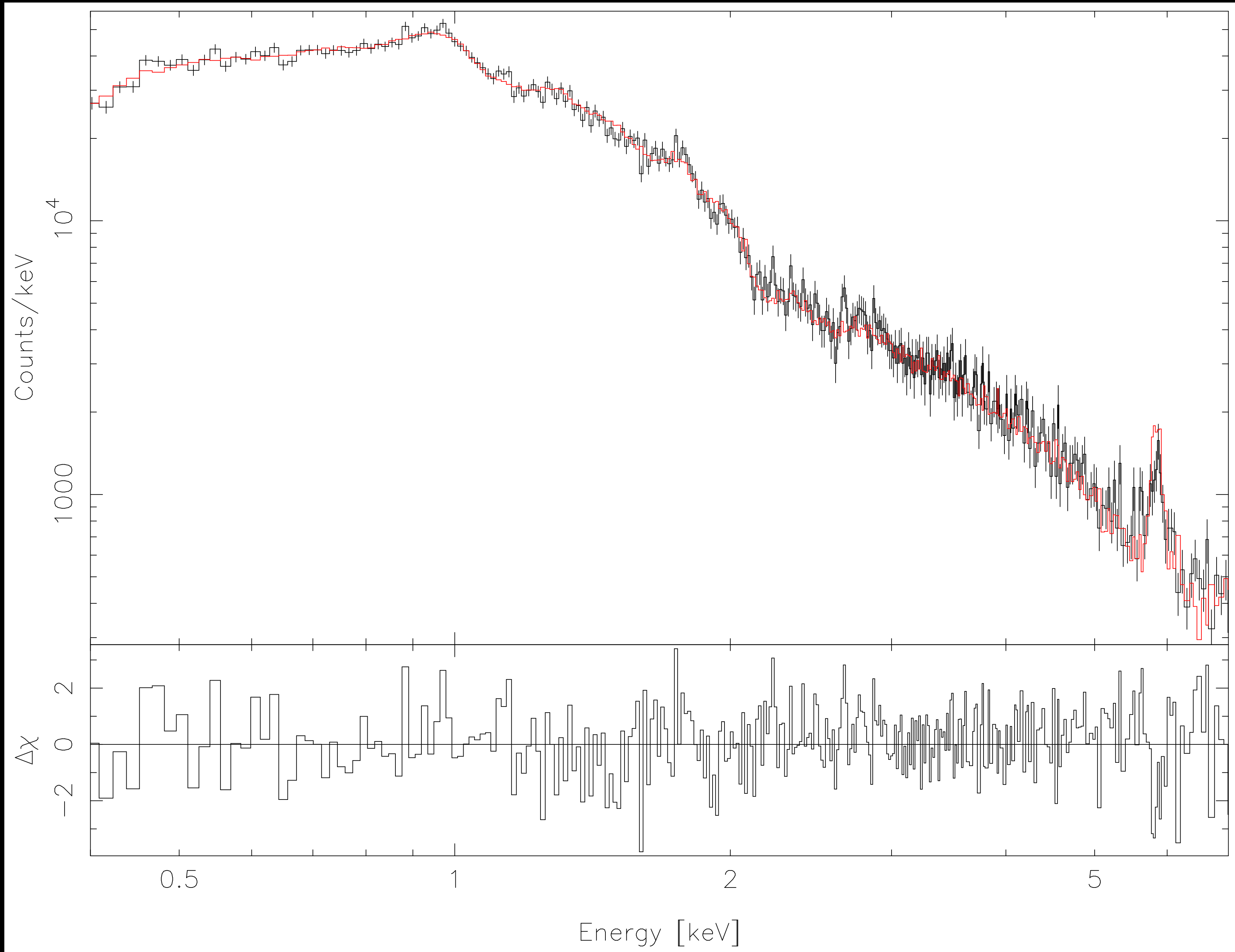
n: number density of nuclei,

n_e: number density of electrons,

$g(E,T)$: Gaunt factor (QM correction factor, roughly constant).

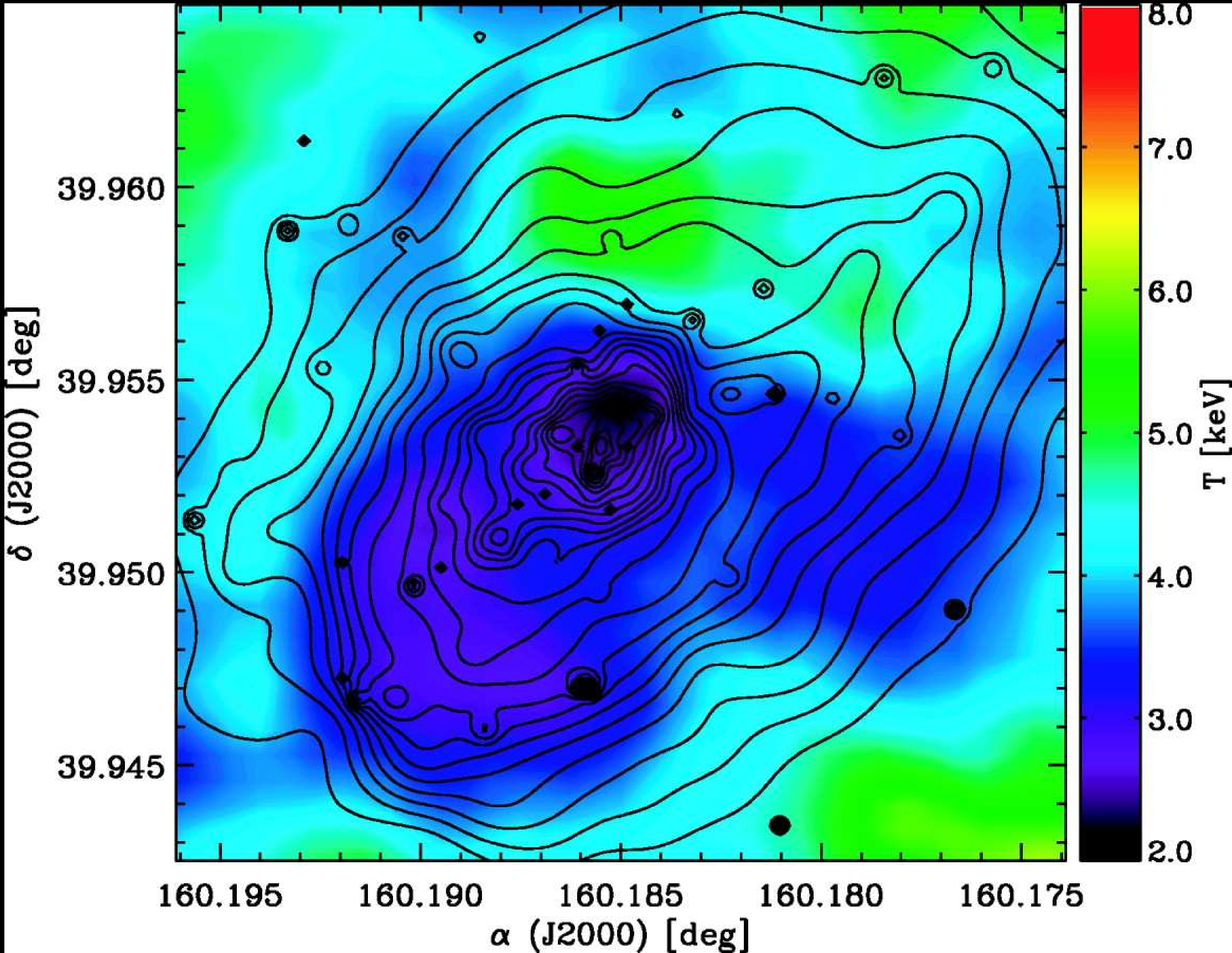
plus emission lines. . .

$\Longrightarrow$ $T(r)$ can be obtained from the X-ray spectral shape, n and n_e from the measured flux $\Longrightarrow$ M_r.



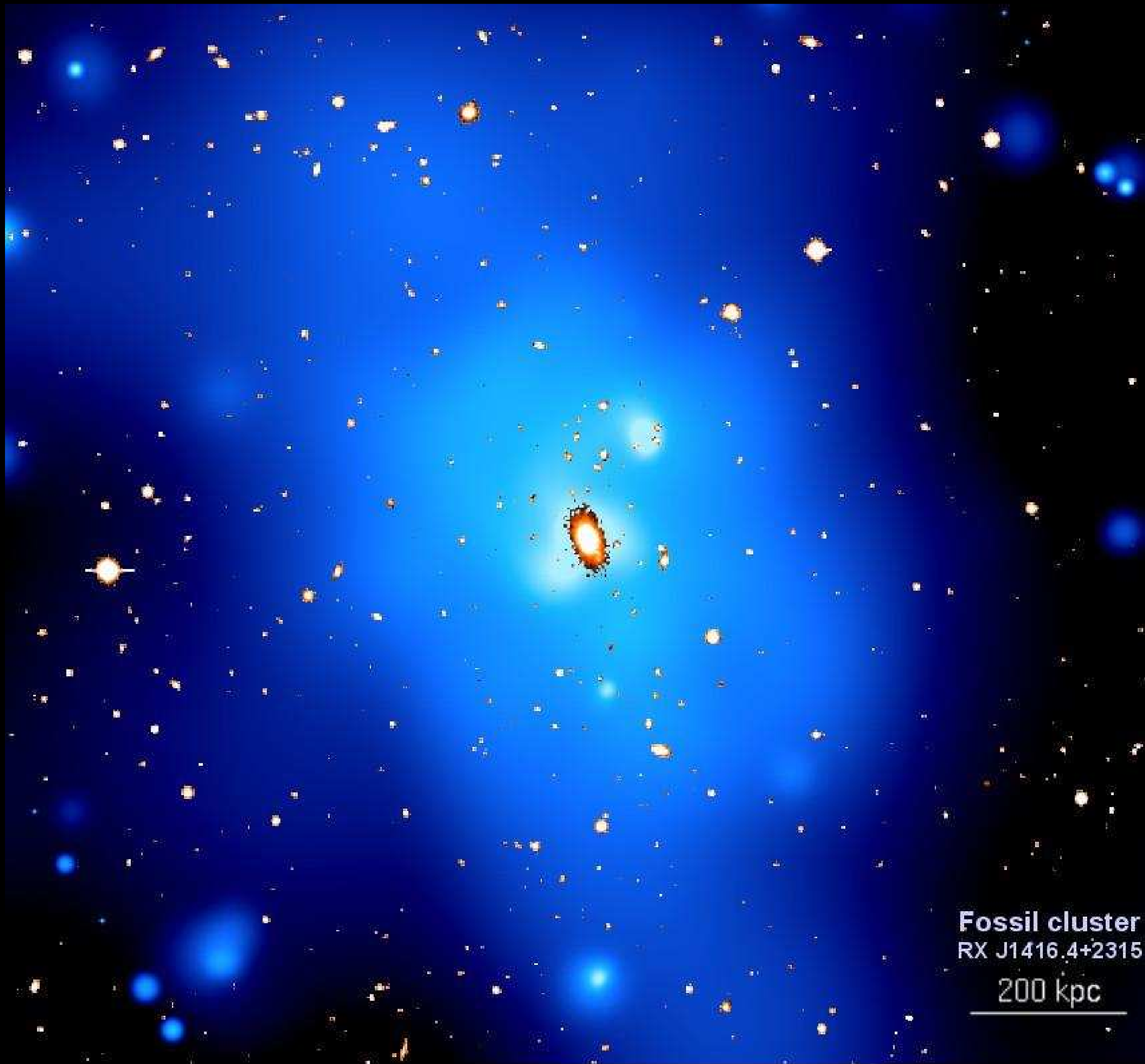
(Wise, McNamara & Murray, 2004, Fig. 2)

X-ray spectrum of A1068 obtained from *Chandra*



(Wise, McNamara & Murray, 2004, Fig. 8)

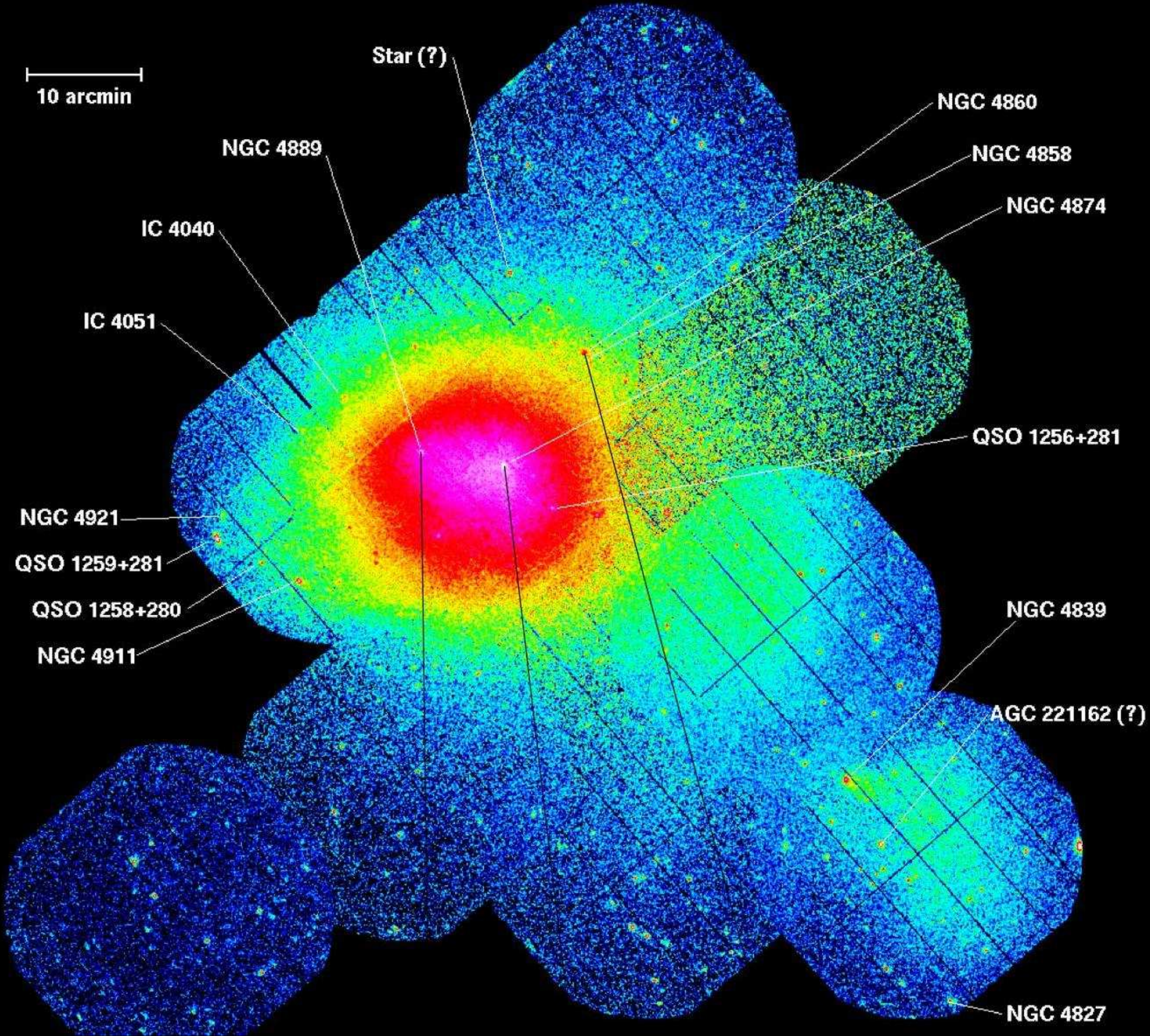
Temperature distribution in A1068 obtained with *Chandra*



XMM-Newton explores the fossil galaxy cluster RX J1416.4+2315

Image courtesy of Habib Khosroshahi (University of Birmingham)

European Space Agency



Technical problems:

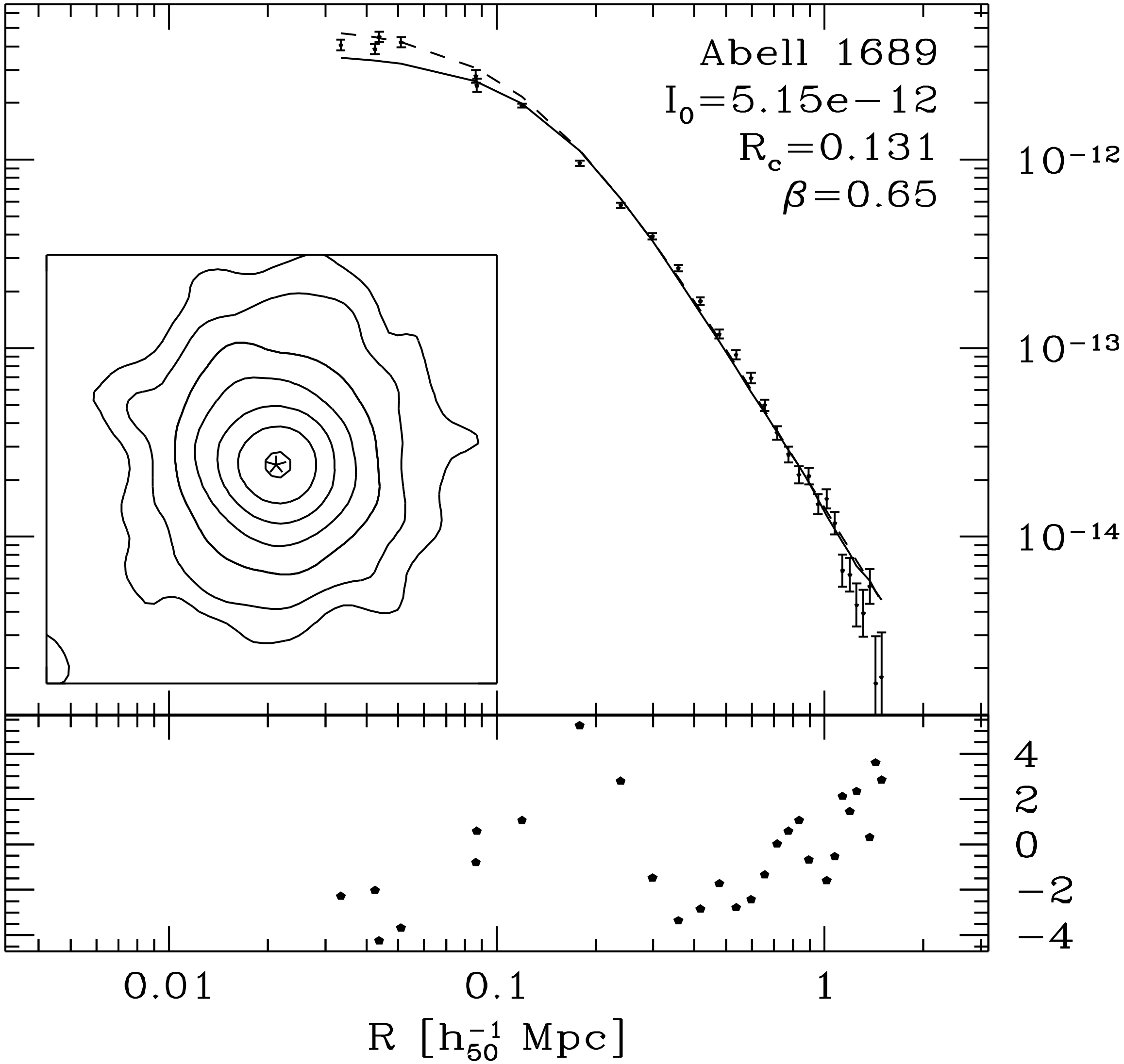
- see through cluster $\Longrightarrow$ integrate over line of sight, assuming spherical geometry.
- spherical geometry is assumed
- it is unclear whether gas is in hydrostatic equilibrium (cooling flows? – but note, there is sparse evidence for a "flow")

XMM-Newton, EPIC-pn

Result for Coma:

$$\frac{M_\mathrm{B}}{M_\mathrm{tot}} = 0.01 + 0.05\,h^{-3/2} \qquad (8.30)$$

X-ray emission, X



(Mohr, Mathiesen & Evrard, 1999)

Generally: assume intensity profile from β-model,

$$\frac{I(r)}{I_0} = \left(1 + \left(\frac{r}{R_c}\right)^2\right)^{-3\beta+\frac{1}{2}} \qquad (8.31)$$

and obtain T from fitting X-ray spectra to "shells" $\Longrightarrow$ technically complicated...

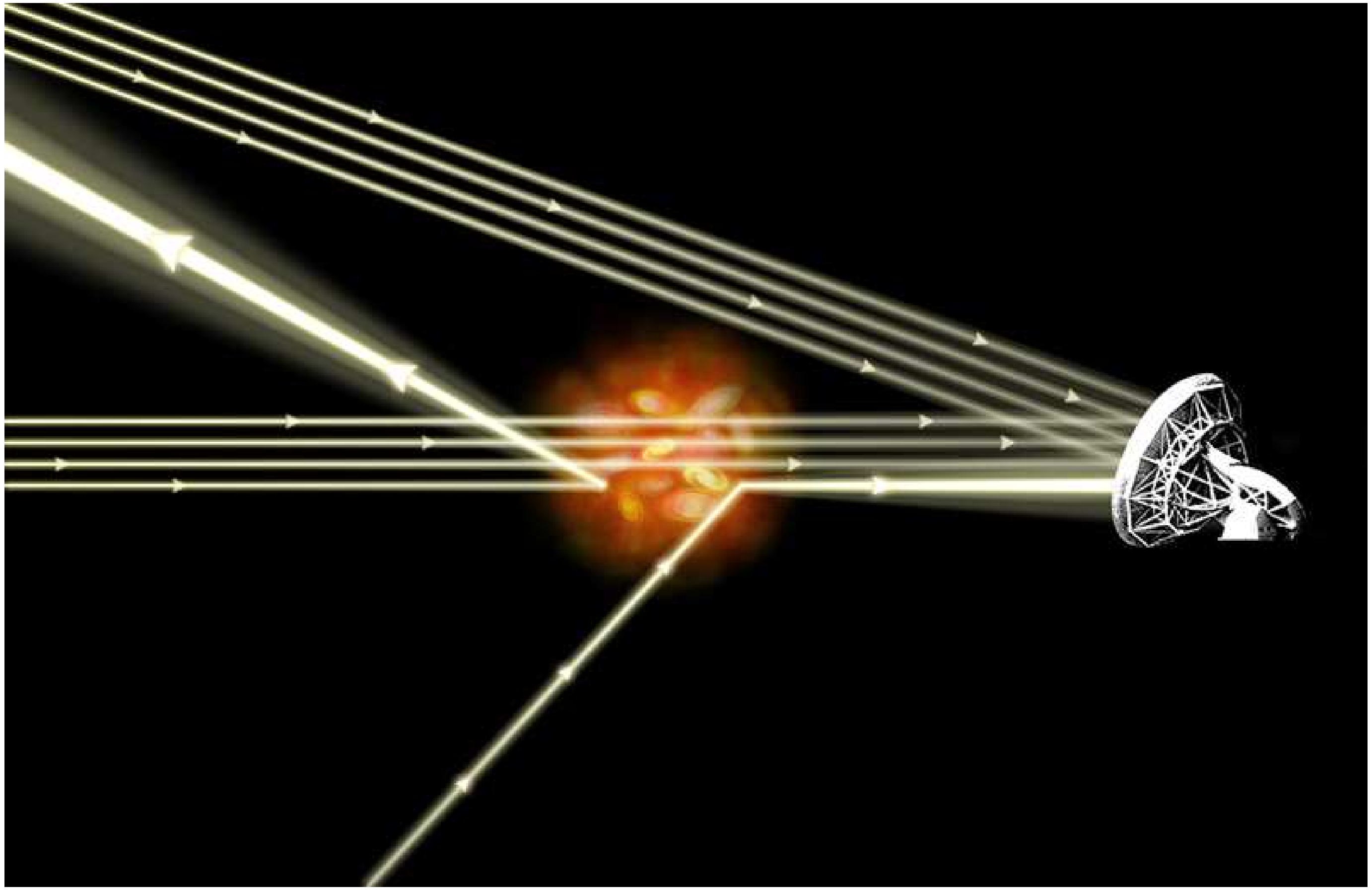
Summary for X-ray mass determination for 45 clusters (Mohr, Mathiesen & Evrard, 1999):

$$f_{\text{gas}} = (0.07 \pm 0.002)h^{-3/2} \qquad (8.32)$$

resulting in

$$\Omega_{\text{m}} = \Omega_{\text{b}}/f_{\text{gas}} = (0.3 \pm 0.05)\,h^{-1/2} \qquad (8.33)$$

Sunyaev-Zeldovich, I



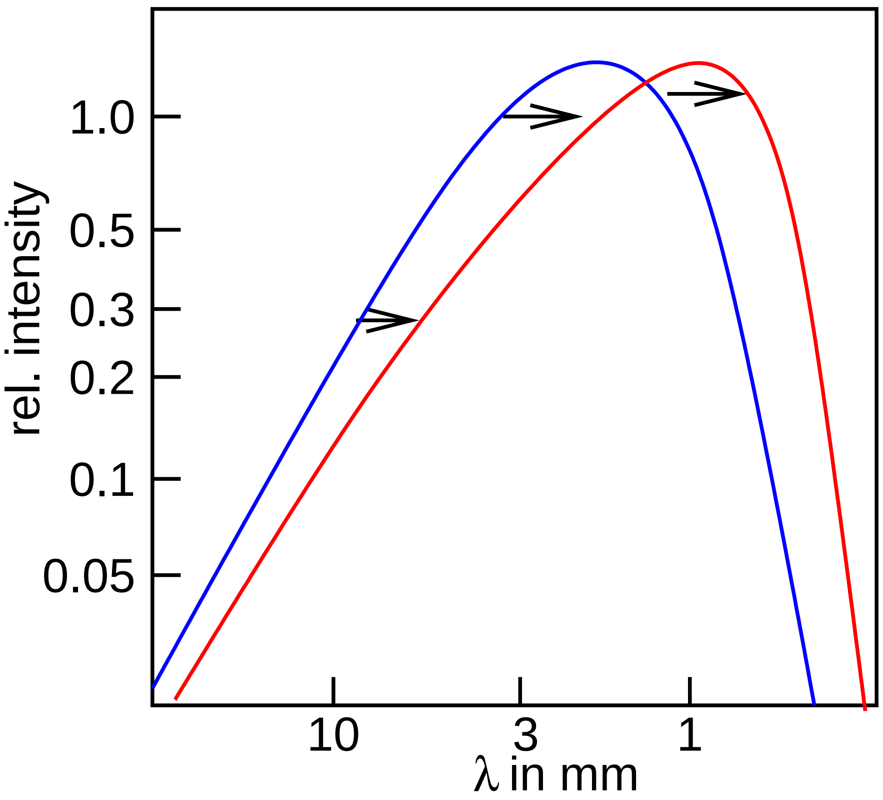
NASA/CXC/M.Weiss

Gas in cooling flow influences CMBR by Compton upscattering

$\Longrightarrow$ Sunyaev-Zeldovich effect (1970).

Sunyaev-Zeldovich, II

The quantitative derivation of the SZ-effect is difficult, basically, one sets up the Fokker-Planck equation for the photon gas and from this derives the so-called Kompaneets equation, see, e.g., Peacock (1999, p. 375ff.).



after Schneider

The basic ingredients are the optical depth for Compton scattering (Compton y-parameter):

$$y = \int \left(\frac{kT_e}{m_e c^2}\right) \sigma_T N_e \, dl \qquad (8.34)$$

From this follows in the Rayleigh-Jeans regime that the intensity due to Compton upscattering changes as follows:

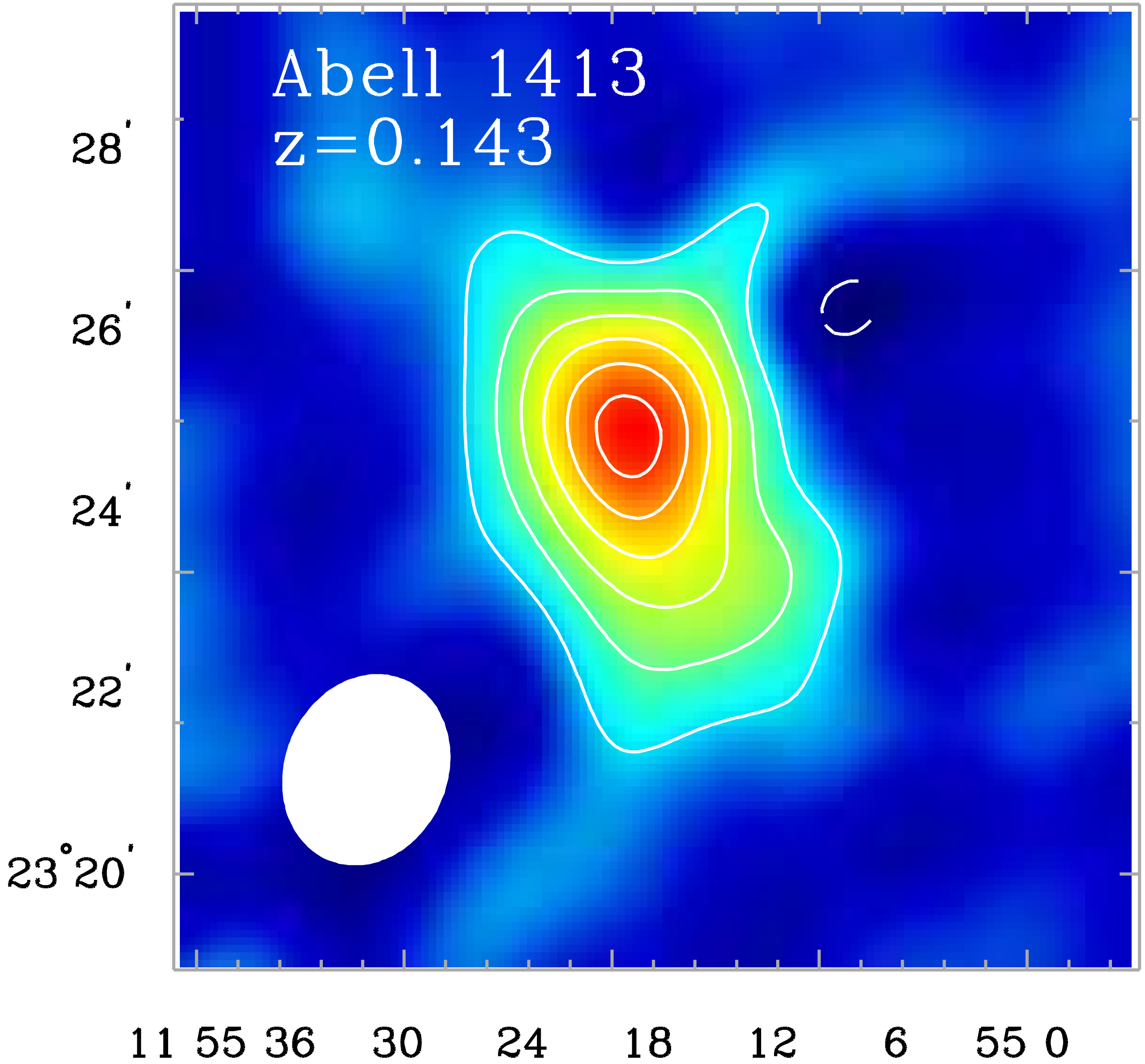
$$\frac{\Delta I}{I} = -2y \sim 10^{-4} \qquad (8.35)$$

(for typical parameters).

$\Longrightarrow$ ΔI allows to measure of $\int N_e T_e \, dl$

$\Longrightarrow$ Mass!

T is known from X-ray spectrum.



(temperature decrement from 3 K background, Carlstrom et al., 2000, Fig. 3)

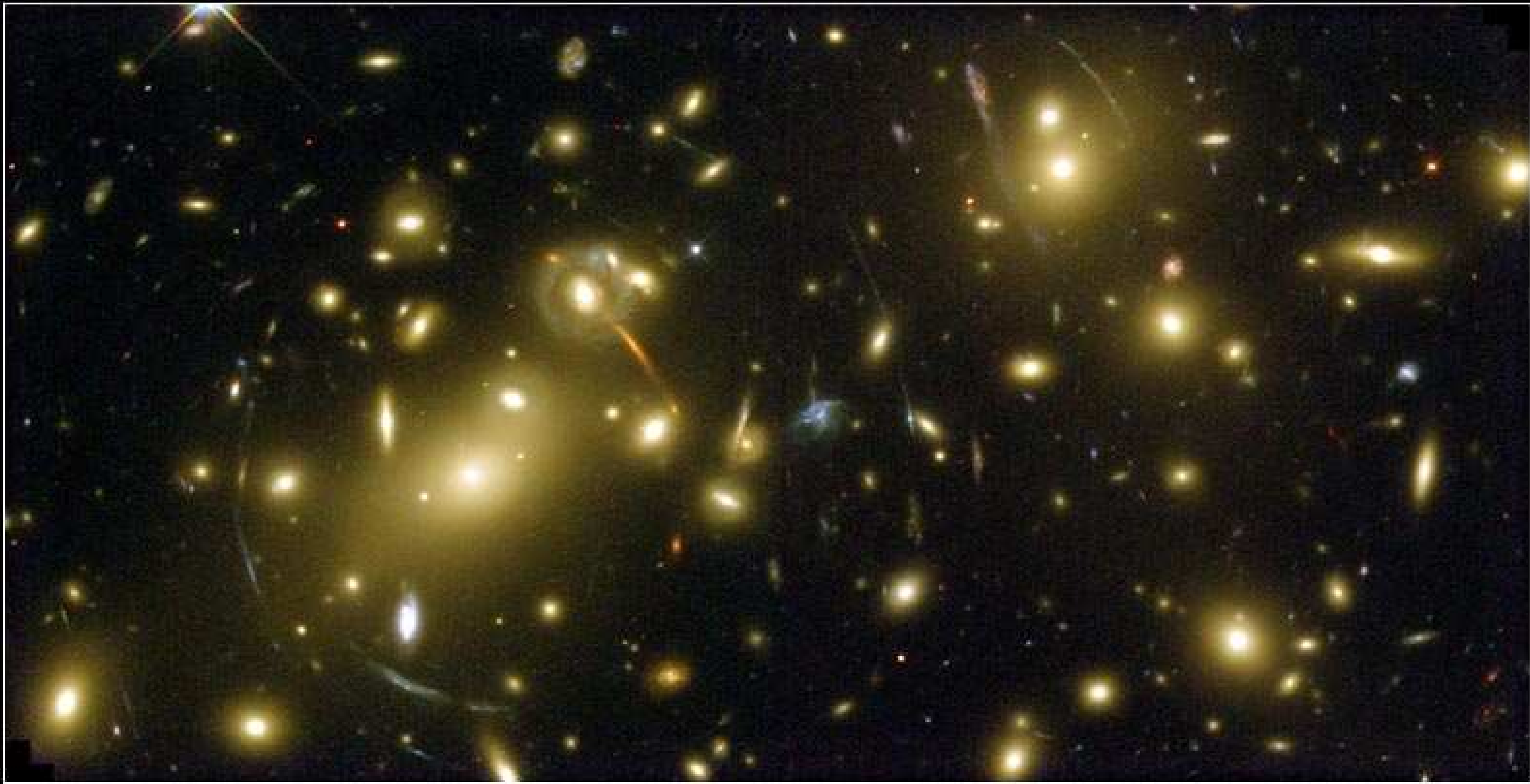
SZ analysis gives gas fraction for 27 clusters

$$f_{\mathrm{gas}} = (0.06 \pm 0.006)h^{-3/2} \tag{8.36}$$

remarkably similar to X-ray result $\Longrightarrow$ clumping of gas does not influence results! (SZ only traces real gas...)

f_{gas} translates to

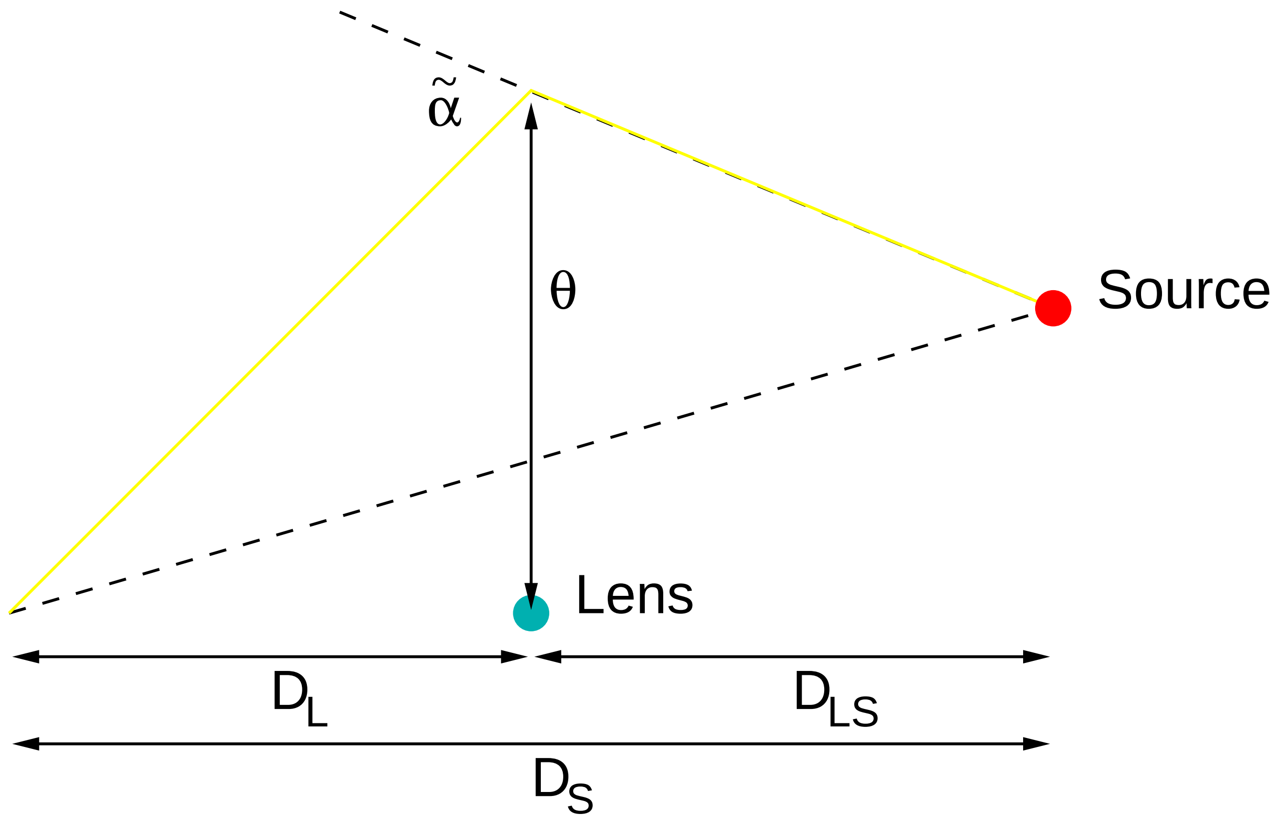
$$\Omega_{\mathrm{m}} = (0.25 \pm 0.04)\,h^{-1} \tag{8.37}$$



Galaxy Cluster Abell 2218 — **HST** • WFPC2

NASA, A. Fruchter and the ERO Team (STScI) • STScI-PRC00-08

Gravitational Lenses, II



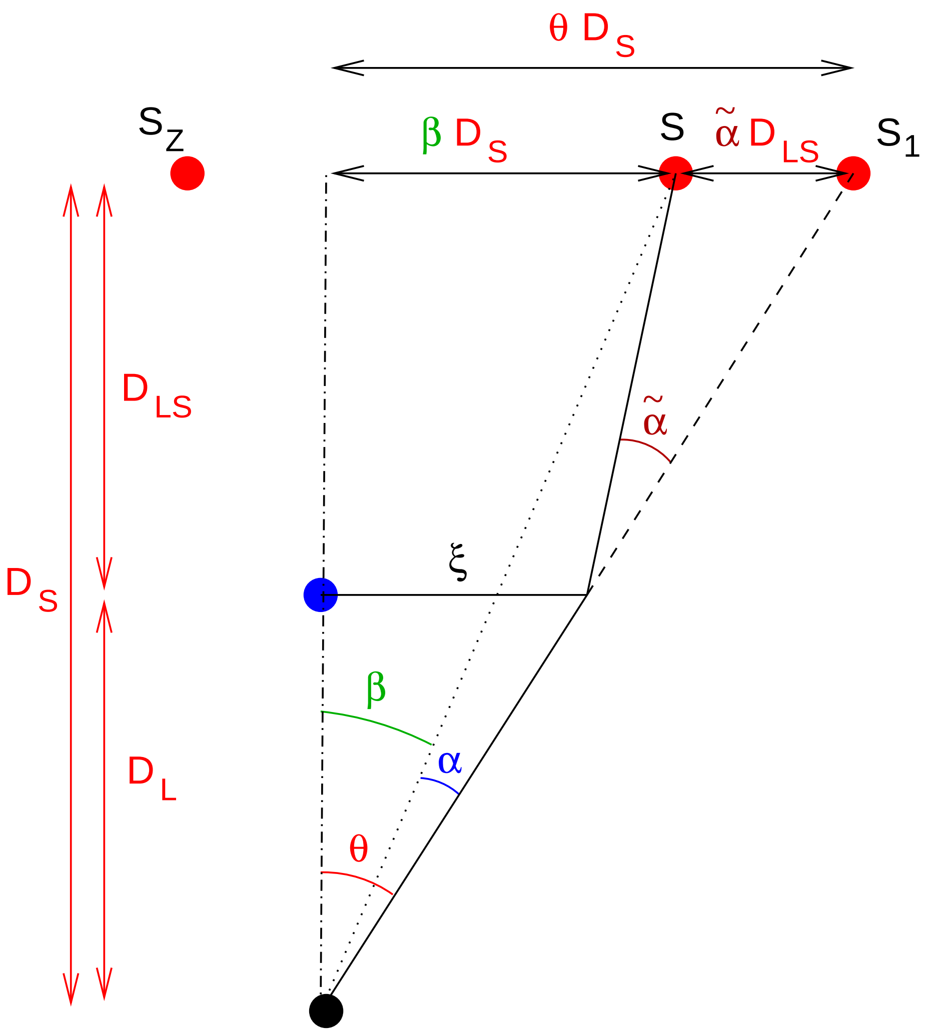
(after Longair, 1998, Fig. 4.8a)

GR: Angular deflection of light due to presence of mass M:

$$\tilde{\alpha} = \frac{4GM}{\theta c^2} = \frac{2}{c^2} \cdot \frac{2GM}{\theta} \tag{8.38}$$

where θ: distance of closest approach (*twice* the classical result).

Gravitational Lenses, III



after Wambsganss (1998, Fig. 3)

In the small angle approximation:

$$\theta D_\mathsf{S} = \beta D_\mathsf{S} + \tilde{\alpha} D_\mathsf{LS} \tag{8.39}$$

such that

$$\beta = \theta - \frac{D_\mathsf{LS}}{D_\mathsf{S}} \tilde{\alpha} \tag{8.40}$$

Defining the reduced deflection angle,

$$\alpha = \frac{D_\mathsf{LS}}{D_\mathsf{S}} \tilde{\alpha} = \frac{D_\mathsf{LS}}{D_\mathsf{S}} \cdot \frac{2}{c^2} \cdot \frac{2GM}{\theta} \tag{8.41}$$

then gives the lens equation

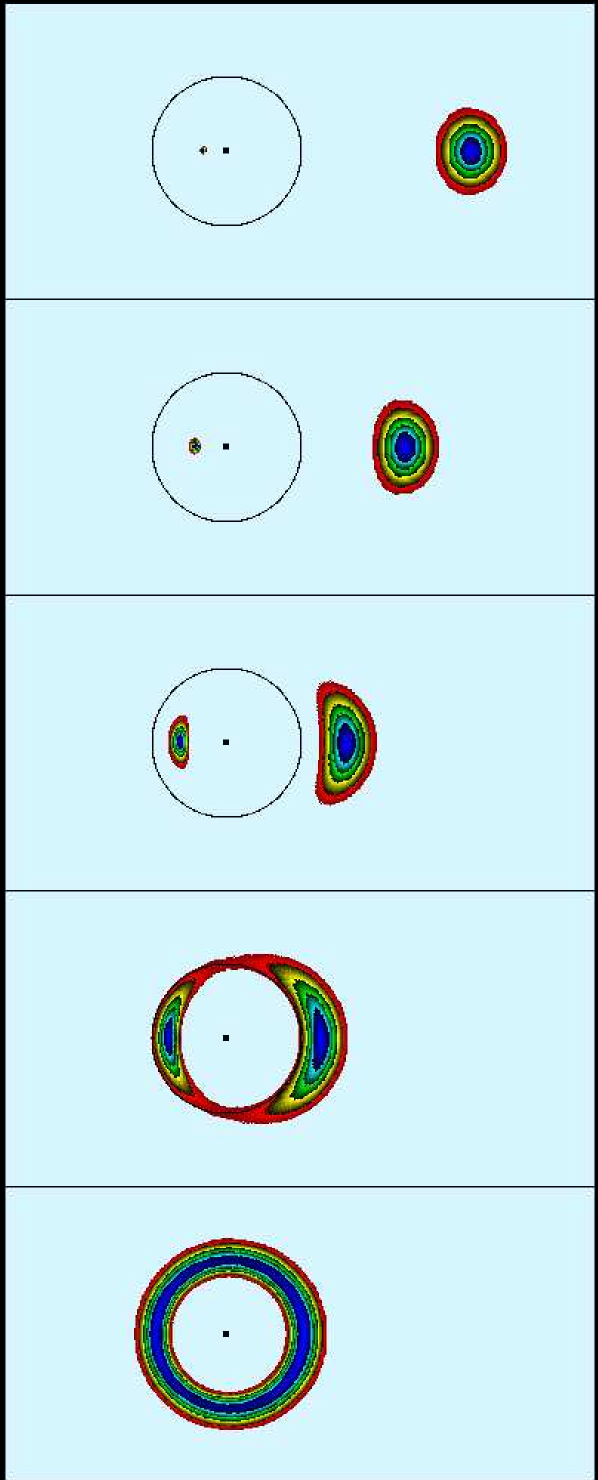
$$\beta = \theta - \alpha = \theta - \frac{D_\mathsf{LS}}{D_\mathsf{L} D_\mathsf{S}} \cdot \frac{4GM}{c^2\theta}$$

$$= \theta - \frac{1}{D} \cdot \frac{4GM}{c^2\theta} \tag{8.42}$$

where

$$D = \frac{D_\mathsf{L} D_\mathsf{S}}{D_\mathsf{LS}} \tag{8.43}$$

(last expression valid for a point-mass)



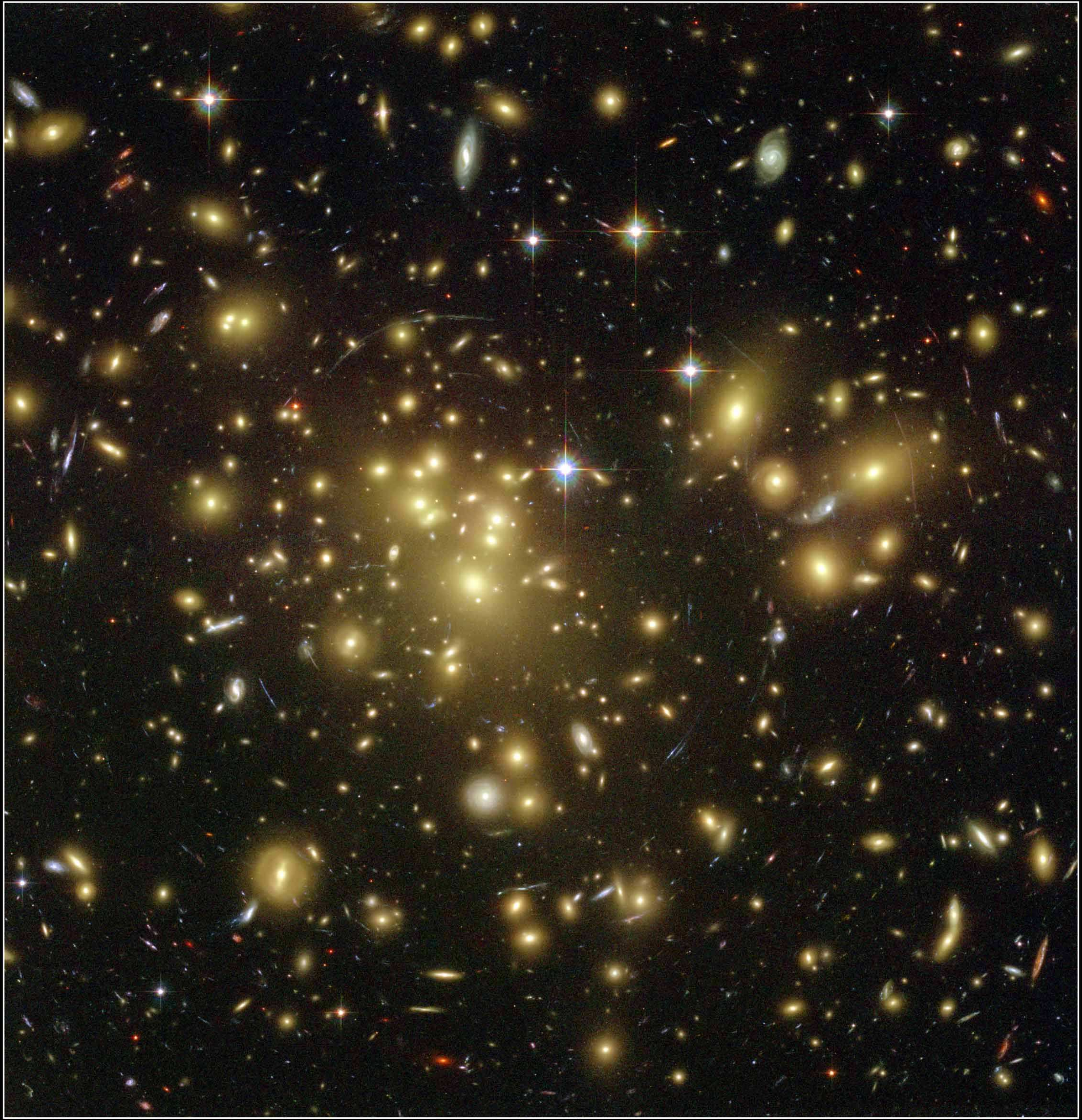
Einstein ring: source directly behind lens,
Obtain radius by setting $\beta = 0$ in lens-equation (Eq. 8.42):

$$\theta_{\mathrm{E}}^2 = \frac{4GM}{c^2}\frac{1}{D} \qquad (8.44)$$

i.e.,

$$\theta_{\mathrm{E}} = 98.9'' \left(\frac{M}{10^{15}\,M_\odot}\right)^{1/2} \frac{1}{(D/1\,\mathrm{Gpc})^{1/2}} \qquad (8.45)$$

Mass measurements possible by observing "giant luminous arcs" and Einstein rings.



Galaxy Cluster Abell 1689
Hubble Space Telescope • Advanced Camera for Surveys

NASA, N. Benitez (JHU), T. Broadhurst (The Hebrew University), H. Ford (JHU), M. Clampin(STScI), G. Hartig (STScI), G. Illingworth (UCO/Lick Observatory), the ACS Science Team and ESA
STScI-PRC03-01a

General results of mass determinations from lensing agree with other methods.

Summary

So far, we have seen:

Photons:

$$\Omega_\gamma h^2 = 2.480 \times 10^{-5} \tag{8.46}$$

Neutrinos:

$$\Omega_\nu h^2 = 1.69 \times 10^{-5} \tag{8.47}$$

Baryons (from nucleosynthesis):

$$\Omega_{\mathrm{b}} h^2 = 0.02 \quad \text{where} \quad \Omega_{\mathrm{stars}} \sim 0.005 \ldots 0.01 \tag{8.48}$$

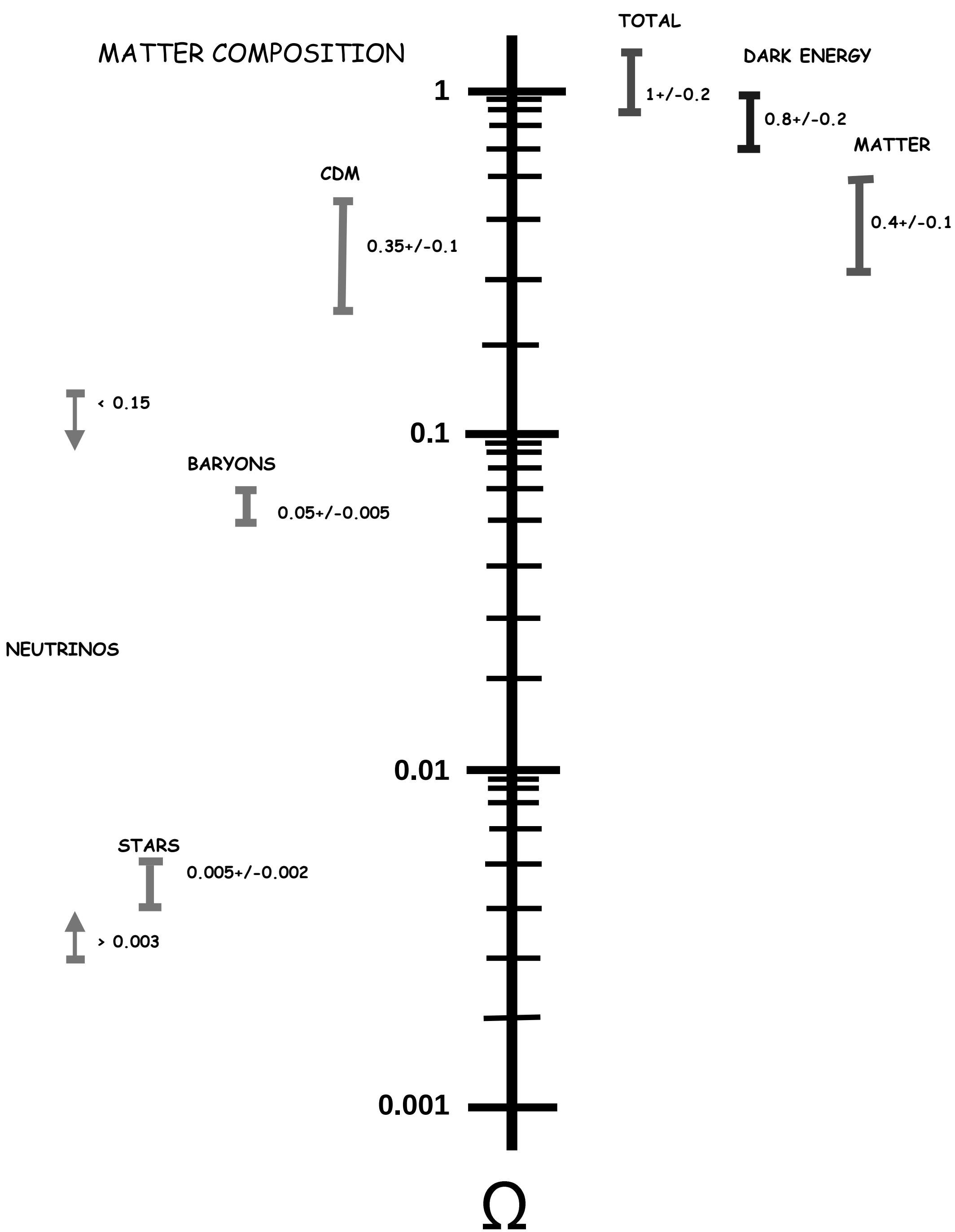
Baryons+dark matter (from clusters):

$$\Omega_{\mathrm{m}} \sim 0.25 \tag{8.49}$$

(of which $\sim 10\%$ in baryons)

If we believe in $\Omega_{\mathrm{total}} \equiv 1 \Longrightarrow \Omega_\Lambda \sim 0.7$.

MATTER / ENERGY in the UNIVERSE



(Turner, 1999, Fig. 1, numbers slightly different to ours ...)

Introduction

Clusters and galaxies: $\Omega_m \sim 0.3$, but for baryons $\Omega_b \sim 0.02$

$\Longrightarrow$ Rest of gravitating material is dark matter.

$\Longrightarrow$ There are two dark matter problems:

$$\Omega_m \xleftarrow{\text{nonbaryonic dark matter}} \Omega_b \xleftarrow{\text{baryonic dark matter}} \Omega_{stars}$$

baryonic dark matter= undetected baryons:

- diffuse hot gas?
- MACHOs (Massive compact halo objects; white dwarfs, neutron stars, black holes, brown dwarfs, jupiters,. . .)

nonbaryonic dark matter= exotic stuff:

- massive neutrinos
- axions
- neutralinos

Baryonic Dark Matter, I

Intra Cluster Gas:

Pro:

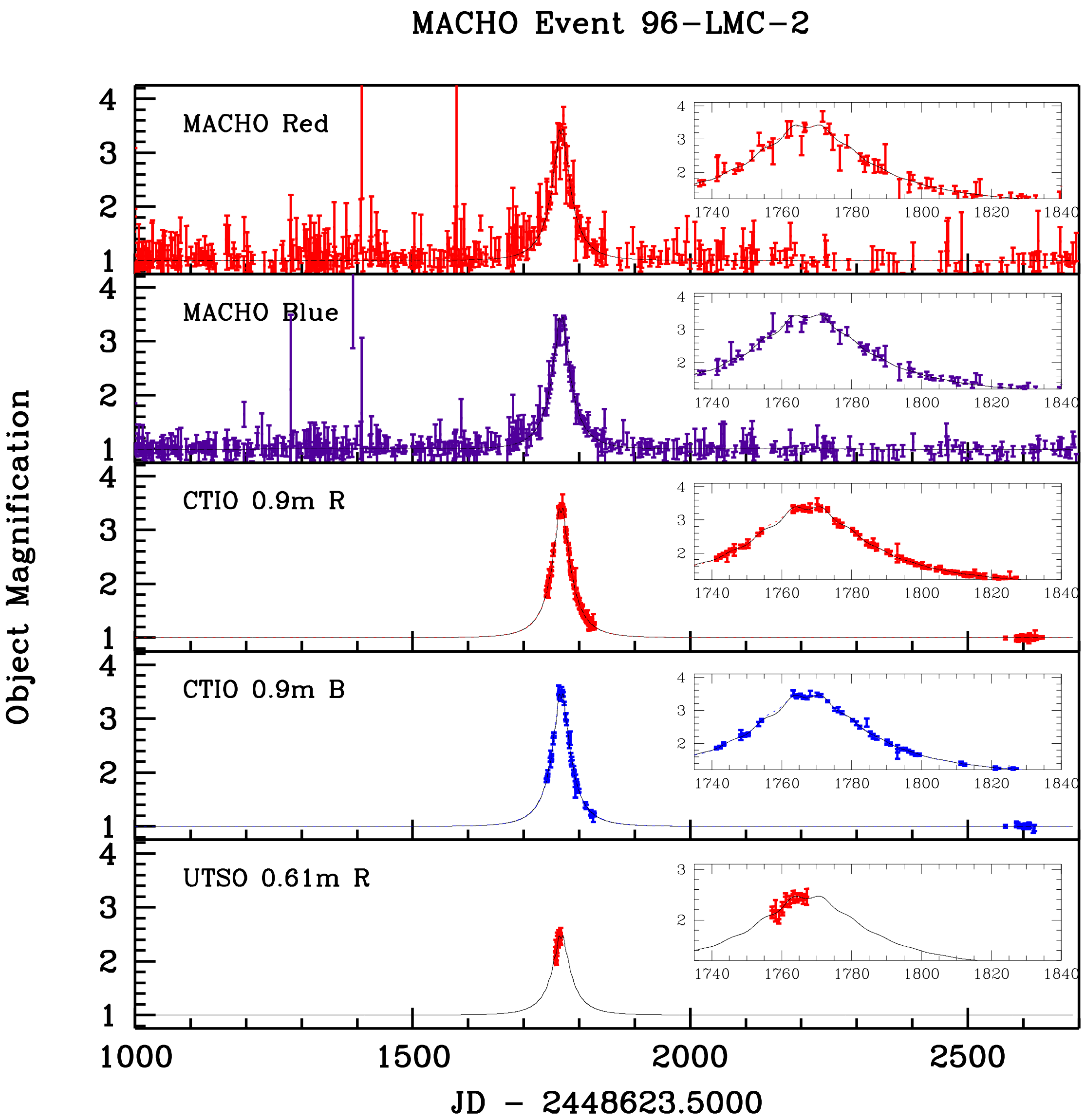
1. same location where the hot gas in clusters also found,
2. structure formation suggests most baryons are *not* in structures today

Contra:

1. 90% of the universe is *not* in clusters...
2. gas has not been detected at any wavelength

If gas cold enough, would not expect it to be detectable, so point 2 is not really valid.

Baryonic Dark Matter, II



MACHOS:

Pro:

1. detected by microlensing towards SMC and LMC (see figure) $\Longrightarrow$ MW halo consists of 50% WD

Contra:

1. possible “self-lensing” (by stars in MW or SMC/LMC; confirmed for a few cases)
2. where are white dwarfs?
3. WD formation rate too high (100 year^{-1} Mpc^{-3})

(Alcock et al., 2001, Fig. 2)

Nonbaryonic Dark Matter

Nonbaryonic dark matter:

Requirements: must be gravitating and non-interacting with baryons

$\Longrightarrow$ Grab-box of elementary particle physics:

1. Neutrinos with non-zero mass

   Pro: It exists, mass limits are a few eV, need only $\langle m_\nu \rangle \sim 10\,\text{eV}$

   Contra: ν are relativistic $\Longrightarrow$ Hot dark matter $\Longrightarrow$ Forces top down structure formation, contrary to what is believed to have happened.

2. Axion

   (=Goldstone boson from QCD, invented to prevent strong CP violation in QCD; $m \sim 10^{-5\ldots-2}\,\text{eV}$)

   Pro: It *could* exist, would be in Bose-Einstein condensate due to inflation ($\Longrightarrow$ Cold dark matter!), might be detectable in the next 10 years

   Contra: We do not know it exists...

3. Neutralino or other WIMPs (weakly interacting massive particles; masses $m \sim \text{GeV}$)

   Pro: Also is CDM

   Contra: We do not know they exist...

Friedmann with $\Lambda \neq 0$, I

$\Longrightarrow$ Need to study cosmology with $\Lambda \neq 0$.

Reviews: Carroll, Press & Turner (1992), Carroll (2001)

Friedmann equation with $\Lambda \neq 0$:

$$H^2(t) = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2} + \frac{\Lambda c^2}{3} \tag{7.30}$$

And defi ne the Ω's (Eqs.4.58, 7.14):

$$\Omega_\mathrm{m} = \frac{8\pi G\rho_\mathrm{m}}{3H_0^2}, \qquad \Omega_\Lambda = \frac{\Lambda c^4}{3H_0^2}, \qquad \Omega_k = -\frac{kc^2}{R_0^2 H_0^2} \tag{8.50}$$

Because of Eq. (7.30),

$$\Omega_\mathrm{m} + \Omega_\Lambda + \Omega_k = \Omega + \Omega_k = 1 \tag{8.51}$$

Friedmann with $\Lambda \neq 0$, II

It is easier to set $c = 1$ and to work with the dimensionless scale factor,

$$a = \frac{R(t)}{R_0} \tag{4.29}$$

$\Longrightarrow$ Friedmann:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\frac{\rho_{\mathrm{m},0}}{a^3} - \frac{k}{a^2 R_0^2} + \frac{\Lambda}{3} \tag{8.52}$$

since $\rho_\mathrm{m} = \rho_{\mathrm{m},0} a^{-3}$ (Eq. 4.63).

Inserting the Ω's

$$\left(\frac{\dot{a}/H_0}{a}\right)^2 = \frac{\Omega_\mathrm{m}}{a^3} + \frac{1 - \Omega_\mathrm{m} - \Omega_\Lambda}{a^2} + \Omega_\Lambda \tag{8.53}$$

Substituting the time in units of todays Hubble time,

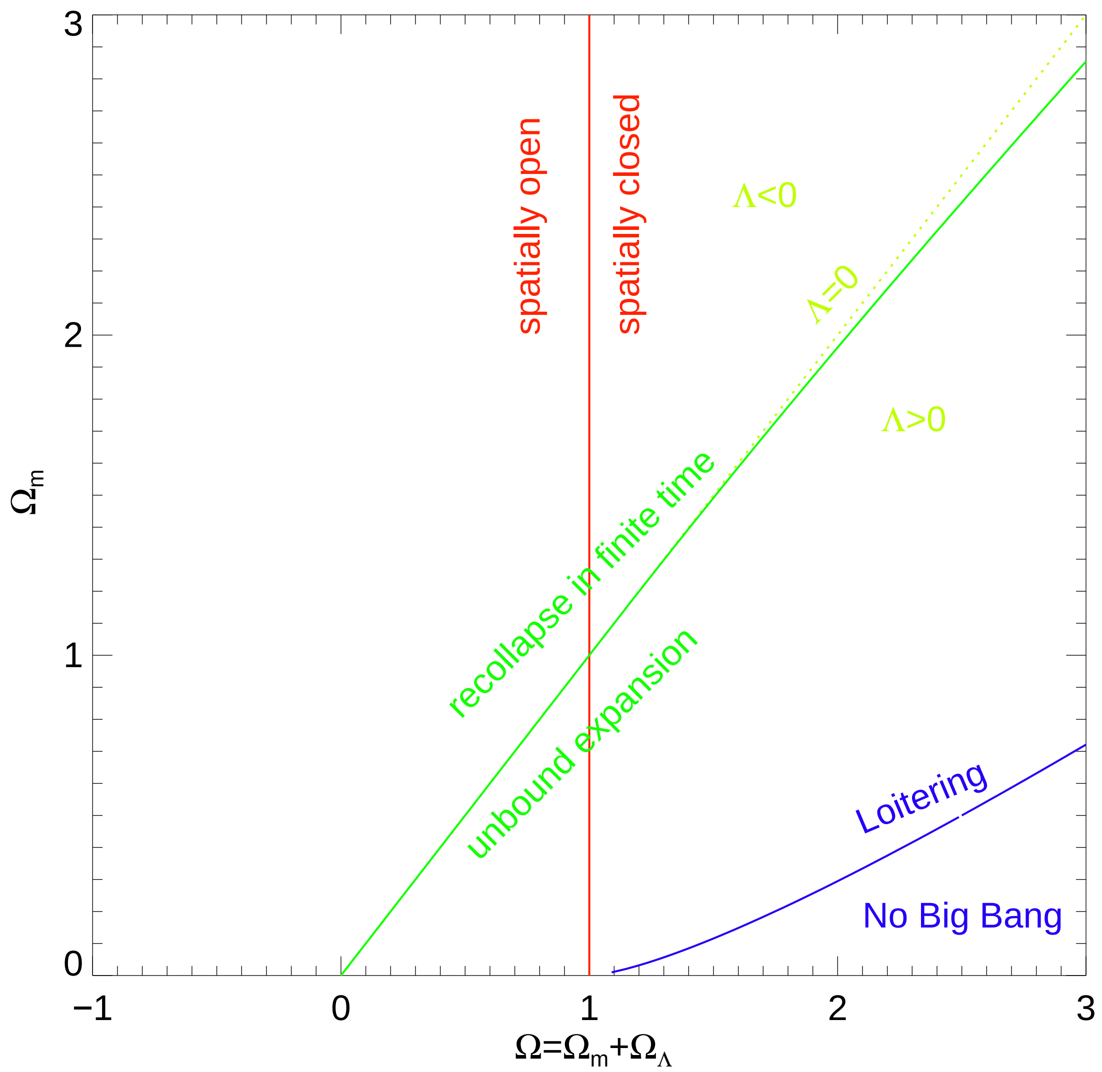
$$\tau = H_0 \cdot t \tag{8.54}$$

results in

$$\left(\frac{\mathrm{d}a}{\mathrm{d}\tau}\right)^2 = 1 + \Omega_\mathrm{m}\left(\frac{1}{a} - 1\right) + \Omega_\Lambda(a^2 - 1) \quad \text{where} \quad a(\tau = 1) = 1 \quad \text{and} \quad \left.\frac{\mathrm{d}a}{\mathrm{d}\tau}\right|_{\tau=1} = 1 \tag{8.55}$$

For most combinations of Ω_m and Ω_Λ, need to solve numerically.

Friedmann with $\Lambda \neq 0$, III

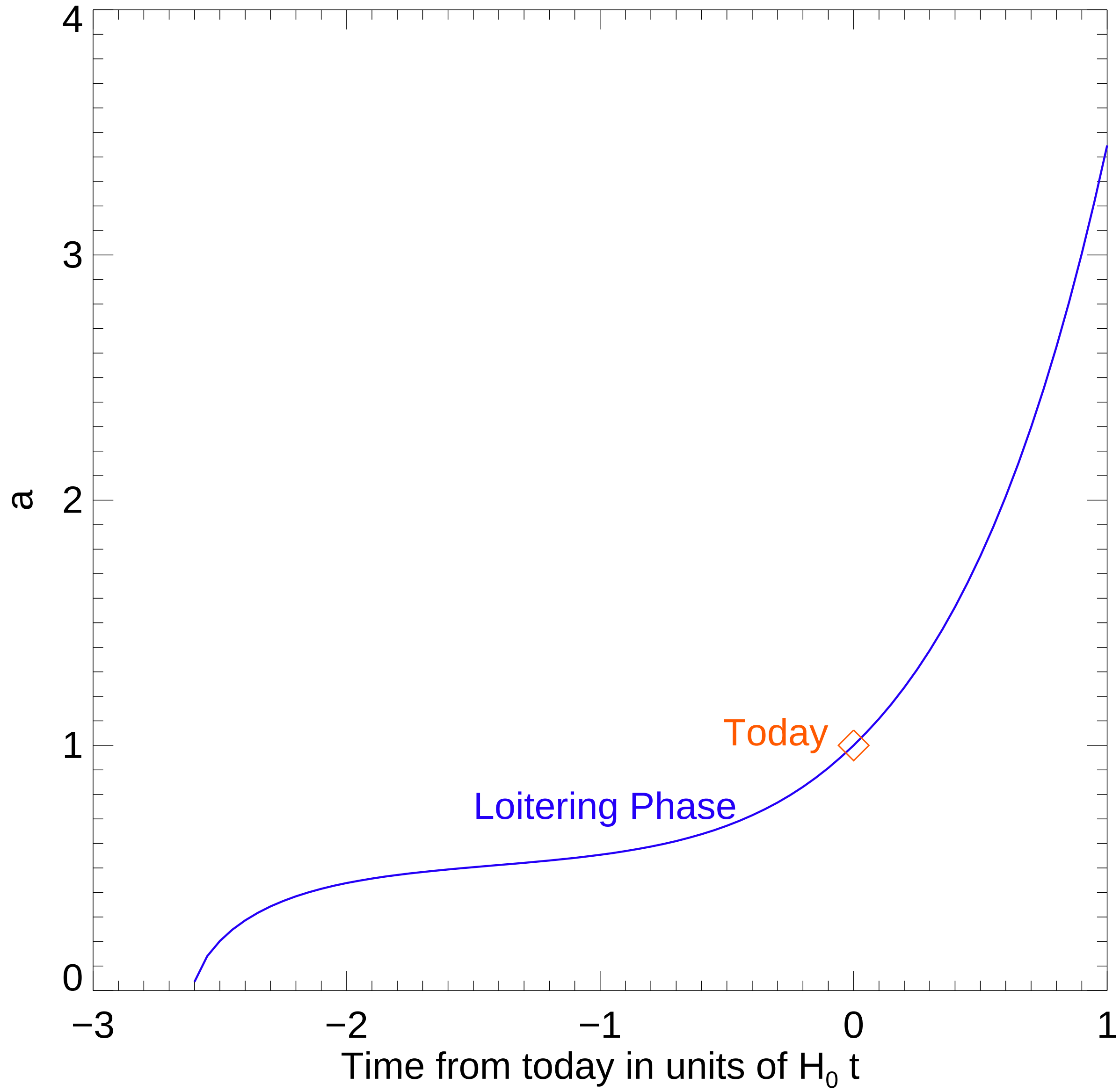


(after Carroll, Press & Turner, 1992, Fig. 1)

With Λ, evolution of universe is more complicated than without:

- unbound expansion possible for $\Omega < 1$,
- For Ω_Λ large: no big bang!
- For Ω_Λ large: possible "loitering phase"

$\Omega_\Lambda > 1$, I



For large Ω_Λ: contraction from $+\infty$ and reexpansion

$\Longrightarrow$ no big bang.

For slightly smaller Ω_Λ: phase where $\dot{a} \sim 0$ in the past

$\Longrightarrow$ loitering universe.

"Loitering universe" with $\Omega_m = 0.55$, $\Omega_\Lambda = 2.055$

$\Omega_\Lambda > 1$, II

QSO at $z = 5.82$, courtesy SDSS

Threshold for presence of a turning-point (Carroll, Press & Turner, 1992, Eq. 12):

$$\Omega_\Lambda \geq \Omega_{\Lambda,\text{thresh}} = 4\Omega_\text{m} \left\{ C_\kappa \left[\frac{1}{3} C_\kappa^{-1} \left(\frac{1-\Omega_\text{m}}{\Omega_\text{m}} \right) \right] \right\}^3 \tag{8.56}$$

where $\kappa = \text{sgn}(0.5 - \Omega_\text{m})$ and $C_\kappa(\theta)$ was defined in Eq. (4.24).

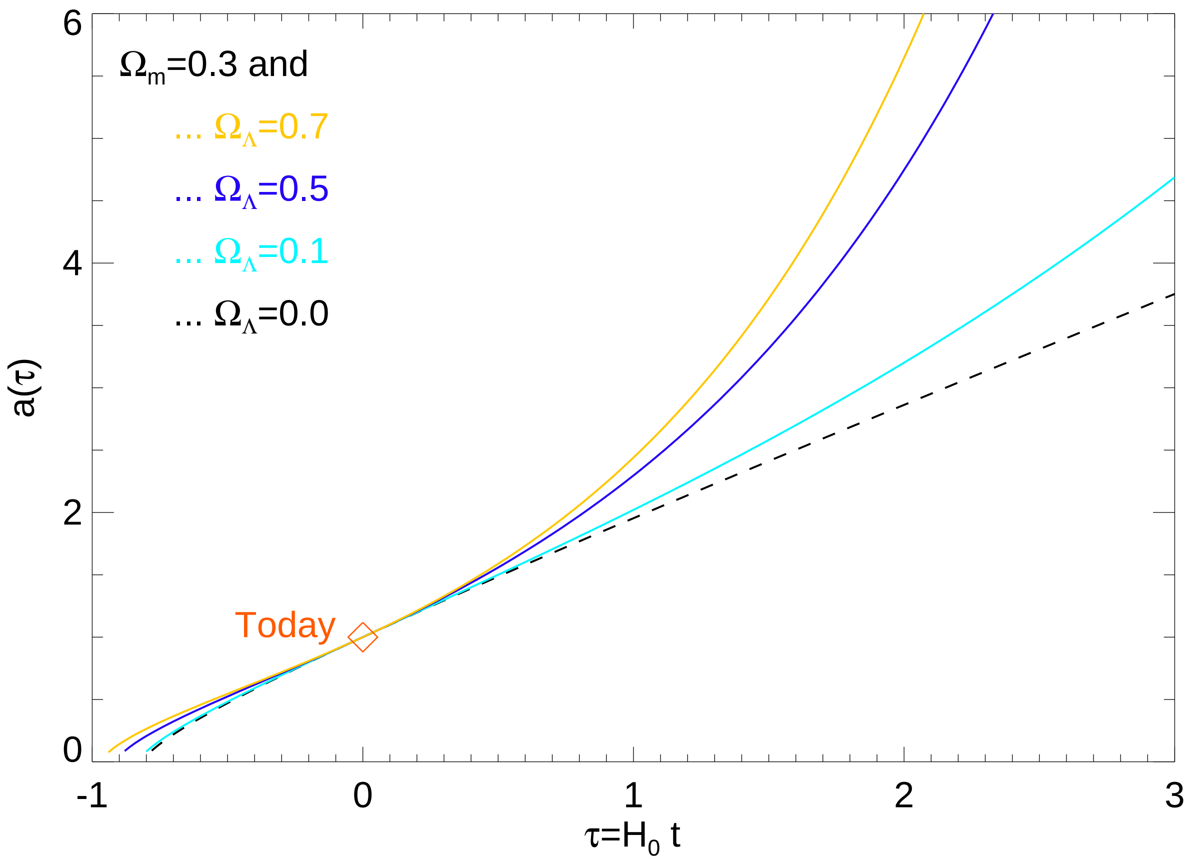
For $\Omega_\Lambda = \Omega_{\Lambda,\text{thresh}}$: turning-point, i.e., there is a minimal a.

Since $1 + z = 1/a$ (Eq. 4.40), existence of turning-point $\Longrightarrow$ maximal possible z:

$$z \leq 2C_\kappa \left(\frac{1}{3} C_\kappa^{-1} \left\{ \frac{1-\Omega_\text{m}}{\Omega_\text{m}} \right\} \right) - 1 \tag{8.57}$$

(Carroll, Press & Turner, 1992, Eq. 14). Since quasars are observed with $z > 5.82$, this means that $\Omega_\text{m} < 0.007$, clearly not what is observed $\Longrightarrow$ $\Omega_\Lambda < 1$.

$\Omega_\Lambda < 1$



For $\Omega_\Lambda < 1$ evolution has two parts:

- matter domination, similar to earlier results
- Λ domination, exponential rise.

Exponential rise called by some workers the "second inflationary phase"...

$\Omega_\Lambda < 1$

Calculation of age of universe is similar to $\Omega_\Lambda = 0$ case (see, e.g., Eq. 4.81), but generally only possible numerically.

Result:

Universes with $\Omega_\Lambda > 0$ are *older* than those with $\Omega_\Lambda = 0$.

This solves the age problem, that some globular clusters have age comparable to age of universe if $\Omega_\Lambda = 0$.

Analytical formula for age (Carroll, Press & Turner, 1992, Eq. 17):

$$t = \frac{2}{3H_0}\frac{\sinh^{-1}\left(\sqrt{(1-\Omega_a)/\Omega_a}\right)}{\sqrt{1-\Omega_a}} \tag{8.58}$$

for $\Omega_a < 1$, where

$$\Omega_a = 0.7\Omega_\mathrm{m} + 0.3(1-\Omega_\Lambda) \tag{8.59}$$

For $\Omega_\mathrm{m} = 0.3$, $\Omega_\Lambda = 0.7$, $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$: $t = 13.5\,\mathrm{Gyr}$.

Remember that for $\Omega_\mathrm{m} = 1$, $t = 3/2H_0$!

Luminosity Distance, I

Influence of Λ is most prominent at large distances!

$\Longrightarrow$ Expect influence on Hubble Diagram.

$\Longrightarrow$ Need to find relation between measured flux, emitted luminosity, and redshift.

Assume source with luminosity L at comoving coordinate r, emitting isotropically into 4π sr.

At time of detection today, photons are

- on sphere with proper radius $R_0 r$,
- redshifted by factor $1 + z$,
- spread in time by factor $1 + z$.

$\Longrightarrow$ observed flux is

$$F = \frac{L}{4\pi R_0^2 r^2 (1+z)^2} \tag{8.60}$$

Luminosity Distance, II

Because the observed flux is

$$F = \frac{L}{4\pi R_0^2 r^2 (1+z)^2} \tag{8.60}$$

in analogy to the inverse square law one defi nes the luminosity distance as

$$d_\mathrm{L} = R_0 \cdot r \cdot (1+z) \tag{8.61}$$

The calculation of d_L is somewhat technical, one can show that (Carroll, Press & Turner, 1992):

$$d_\mathrm{L} = \frac{c}{H_0} |\Omega_k|^{-1/2} \cdot S_{-\mathrm{sgn}(\Omega_k)} \left\{ |\Omega_k|^{1/2} \int_0^z \left[(1+z)^2 (1+\Omega_\mathrm{m} z) - z(2+z)\Omega_\Lambda \right]^{1/2} \,\mathrm{d}z \right\} \tag{8.62}$$

Supernovae

Best way to determine Ω_Λ:

Type Ia supernovae

Remember: SN Ia = CO WD collapse. . . (Hoyle, Fowler, Colgate, Wheeler,. . .)

The distance modulus is

$$m - M = 5 \log \left(\frac{d_\mathrm{L}}{1\,\mathrm{Mpc}} \right) + 25 \qquad (8.63)$$

Use SNe as standard candles $\Longrightarrow$ Deviations from $d_\mathrm{L} \propto z$ indicative of Λ.

Two projects:

- High-z Supernova Team (STSCI, Riess et al.)
- Supernova Cosmology Project (LBNL, Perlmutter et al.)

Both fi nd SNe out to $z \sim 1$.

Present mainly Perlmutter et al. results here, Riess et al. (1998) are similar.

Supernovae

Basic observations: easy:

- Detect SN in rise $\Longrightarrow$ CTIO 4 m
- Follow SN for $\sim$ 2–3 months with 2–4 m class telescopes, HST, Keck. . .

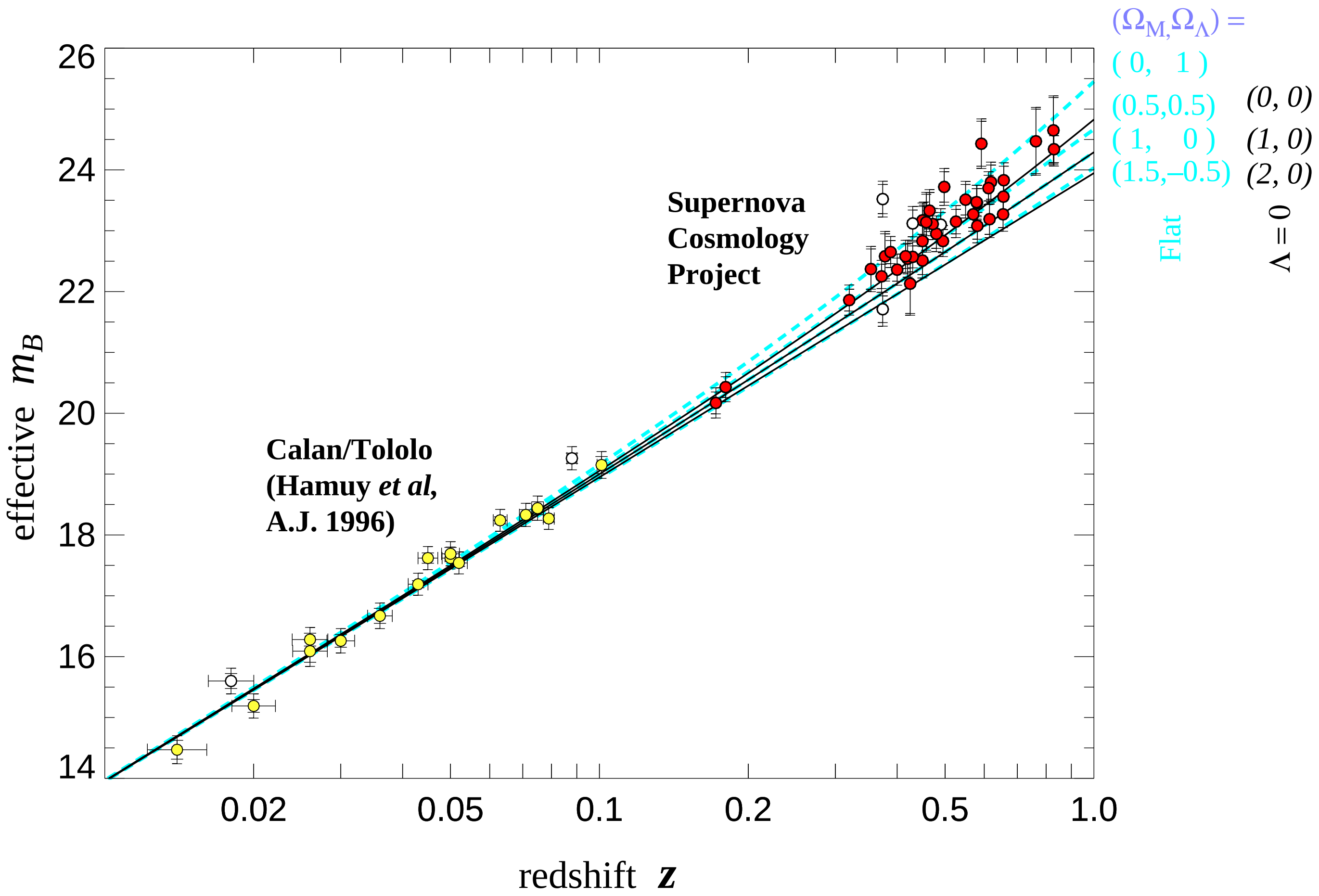
More technical problems in data analysis: Conversion into source frame:

- Correction of photometric flux for redshift: “K-correction”
- Correct for time dilatation in SN light curve

Further things to check

- SN internal extinction
- Galactic extinction
- Galactic reddening
- Photometric cross calibration
- Peculiar motion of SN

Supernovae



Vertical error bars:

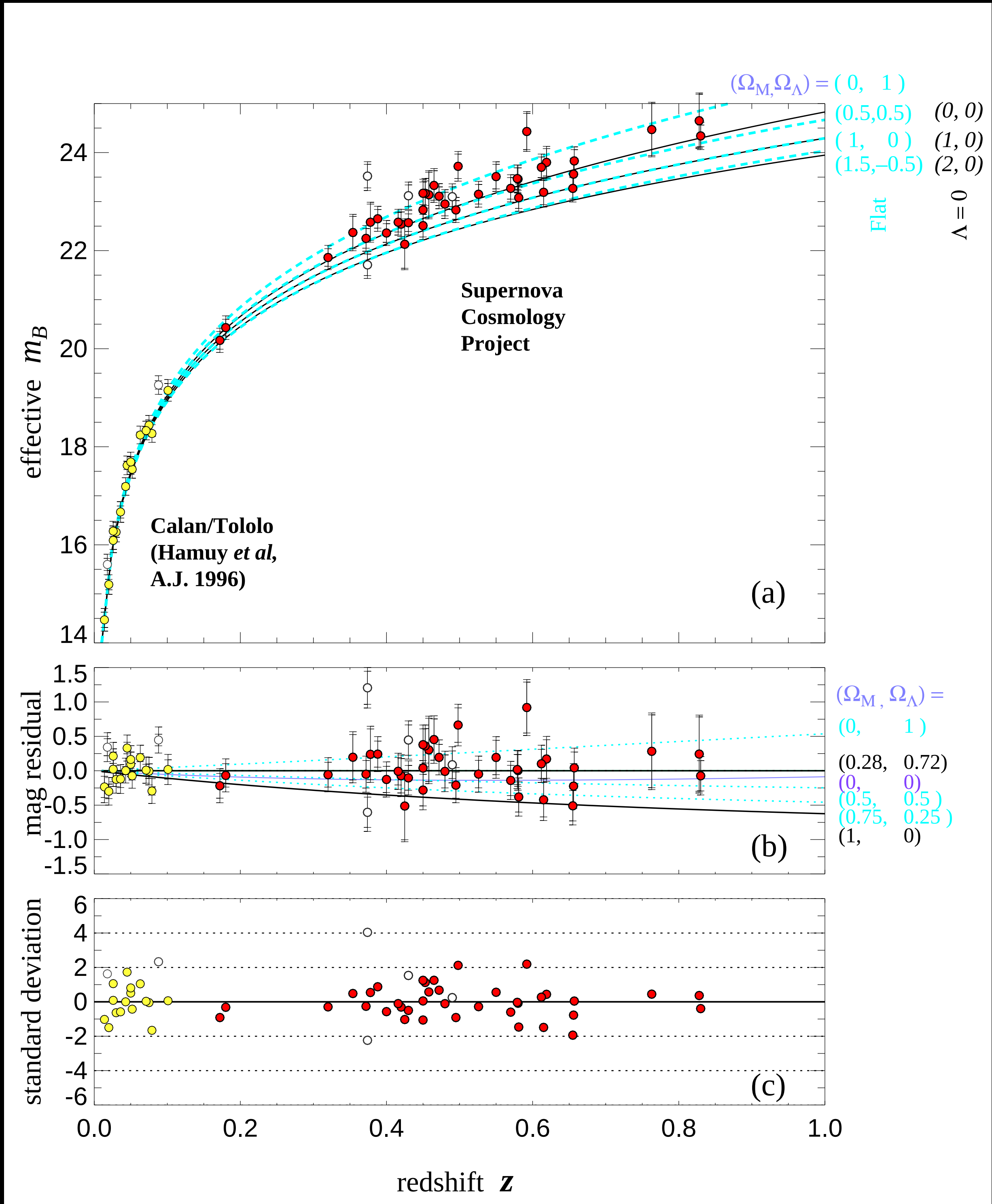
measurement uncertainty *plus* 0.17 mag intrinsic mag. dispersion

Horizontal error bars:

300 km s^{-1} peculiar velocity uncertainty

(Perlmutter et al., 1999, Fig. 1)

42 SNe from SCP, 18 low redshift from Calán/Tololo SN Survey



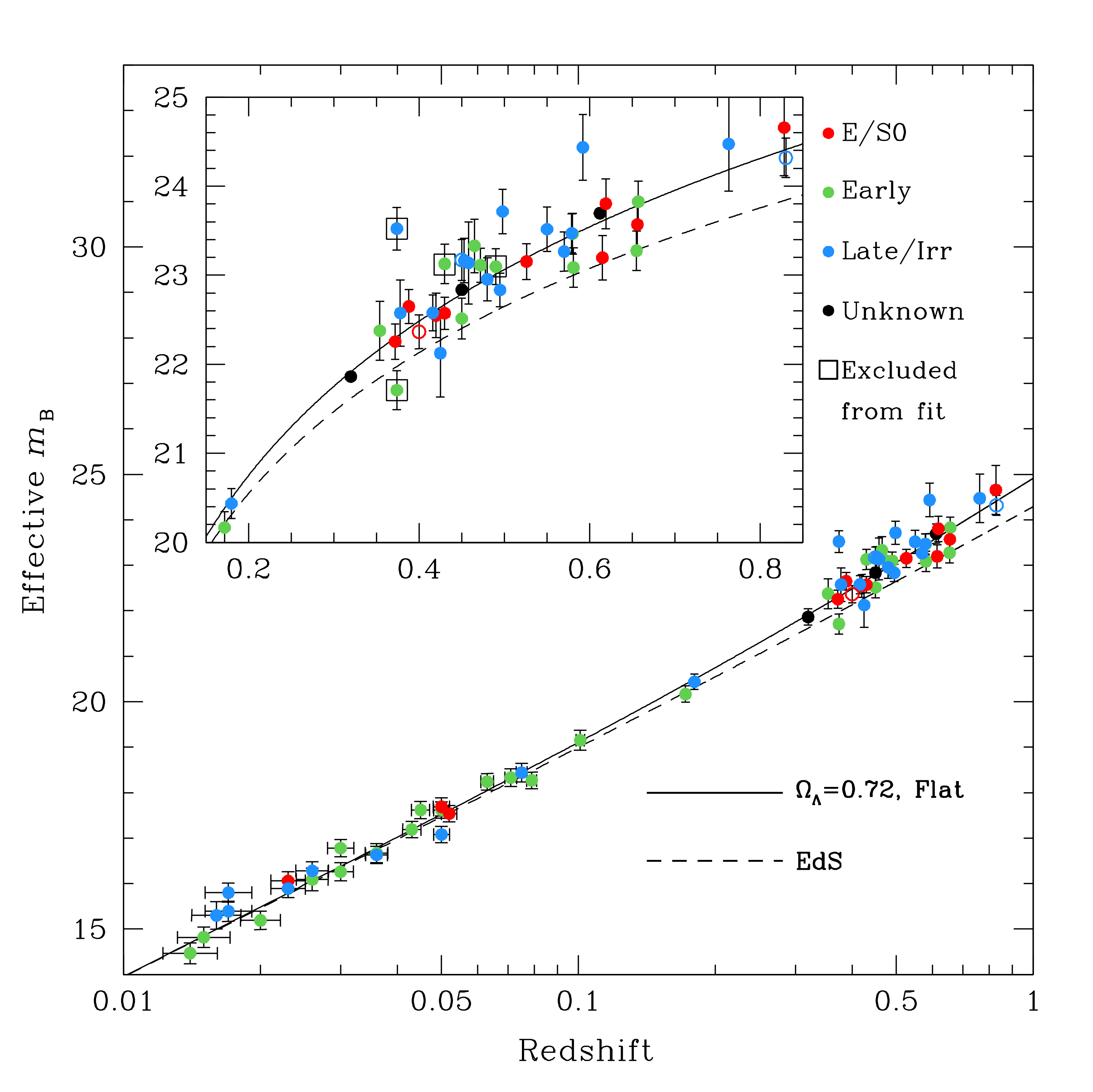
Best fit: $\Omega_{m,\,\text{flat}} = 0.28^{+0.09}_{-0.08}$,

$\chi^2/\text{DOF} = 56/50$

corresponding best free fit:

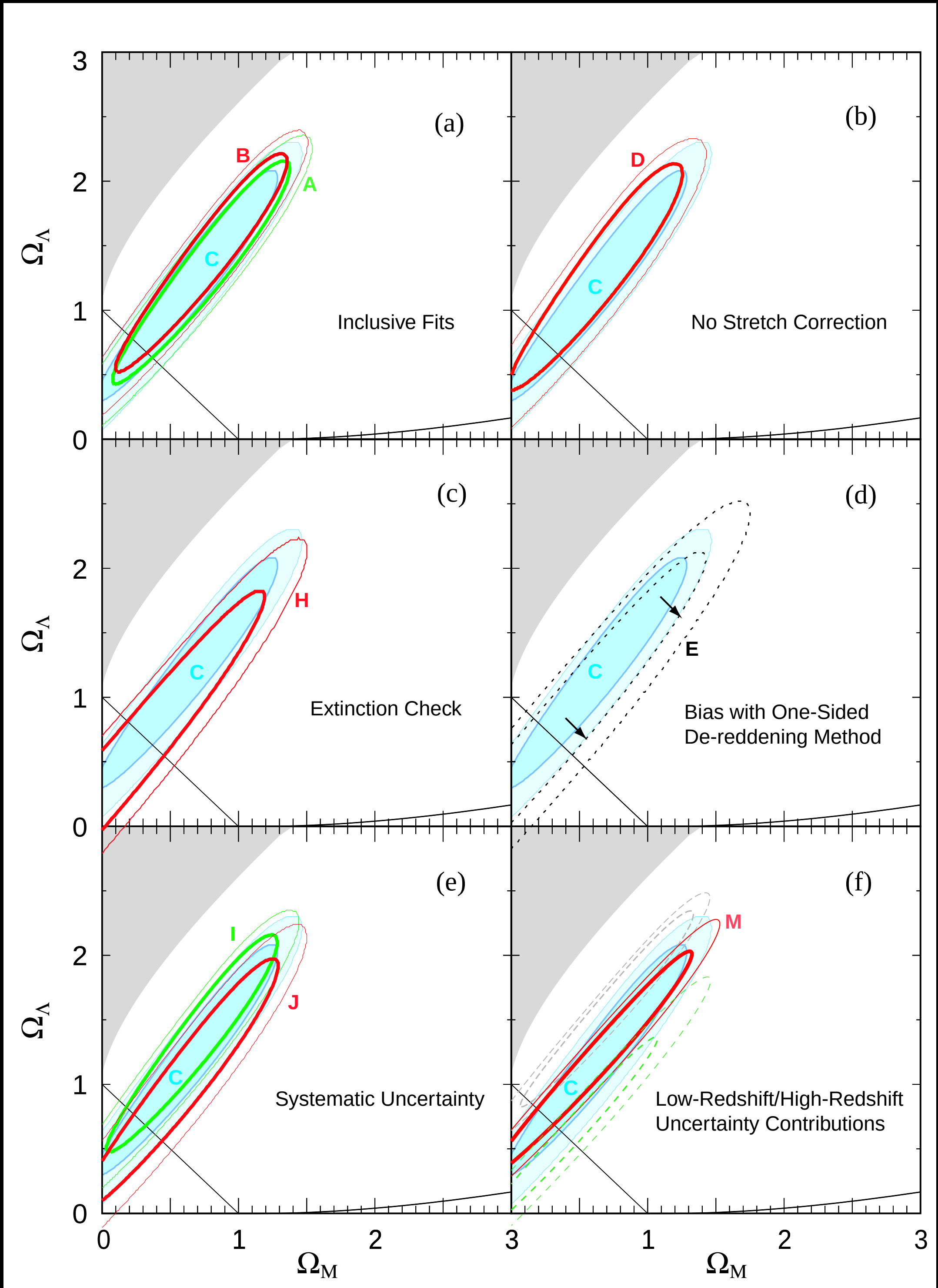
$(\Omega_m, \Omega_\Lambda) = (0.73, 1.32)$.

(Perlmutter et al., 1999, Fig. 2)

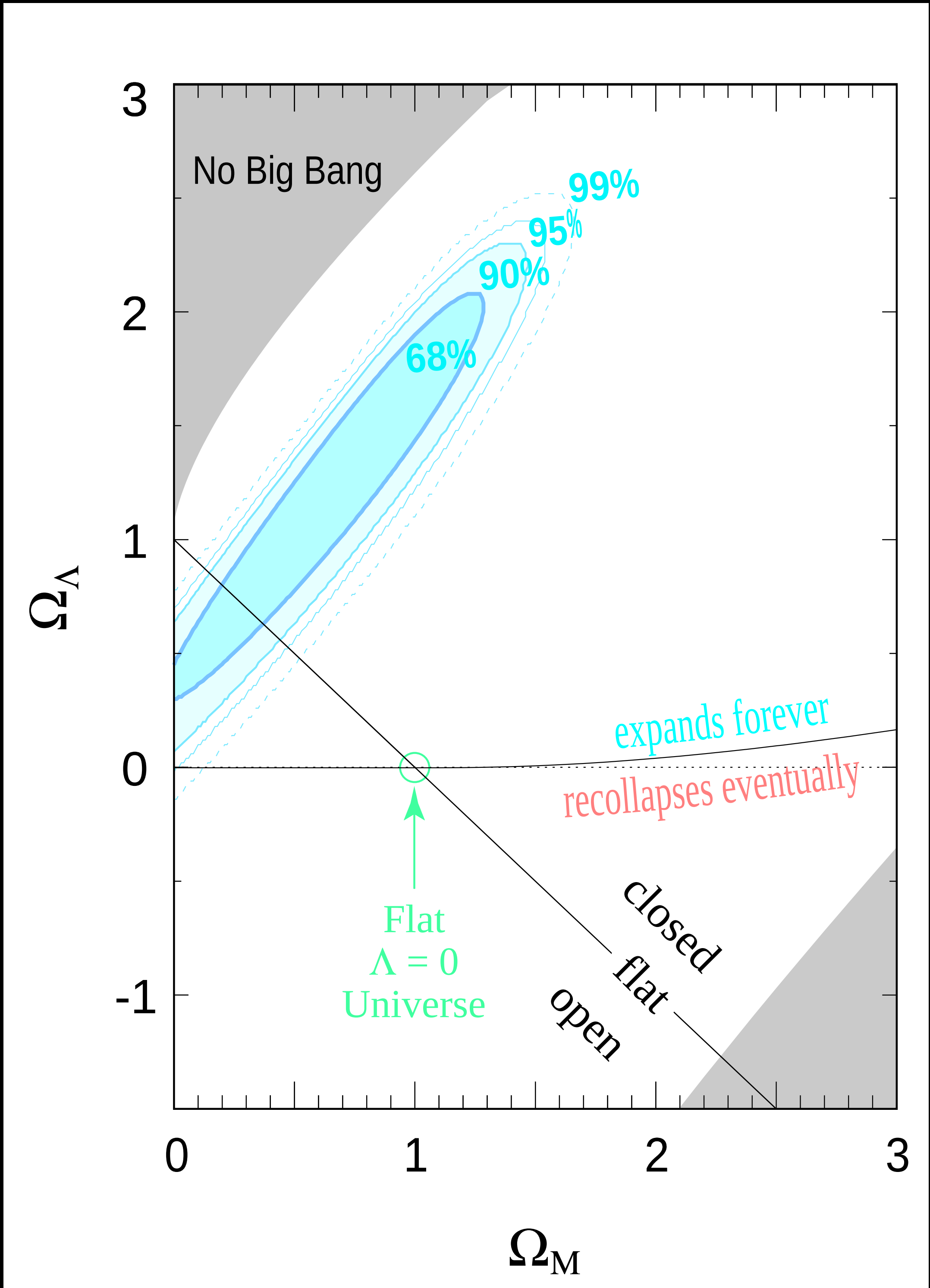


Updated 2002 Hubble diagram for SN Iae confi rms Perlmutter 1999.

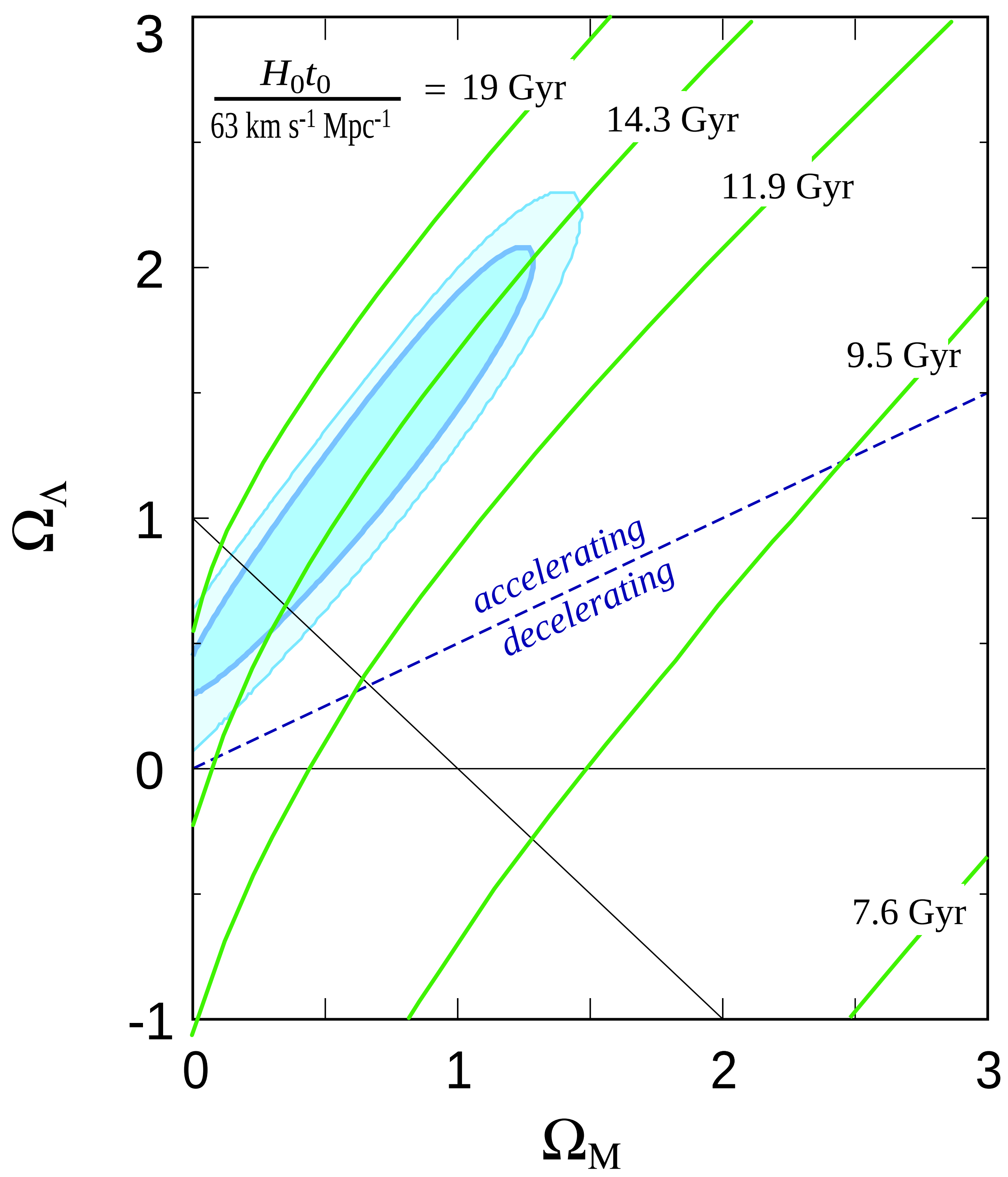
Sullivan et al., 2002



(68% and 90% confi dence contours for sources of systematic error, Perlmutter et al., 1999, Fig. 5)



Combined confi dence region (Perlmutter et al., 1999, Fig. 7; lower right: universes that are younger than oldest heavy elements)



Isochrones for age of universe for $H_0 = 63\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (for $h = 0.7$: age 10% smaller).
$\Longrightarrow$ Consistent with globular cluster ages!

(Perlmutter et al., 1999, Fig. 9)

Summary

For all practical purposes, currently the best values of $\Omega_{\rm m}$ and Ω_Λ are

$$\Omega_{\rm m} \sim 0.3 \qquad \text{and} \qquad \Omega_\Lambda = 0.7$$

Even if $\Omega \neq 1$:

$$\Omega_\Lambda \neq 0$$

And therefore

Baryons are an energetically unimportant constituent of the universe.

“The dark side of the force. . . ” :-)

Outlook

What is physical reason for $\Omega_\Lambda \neq 0$?

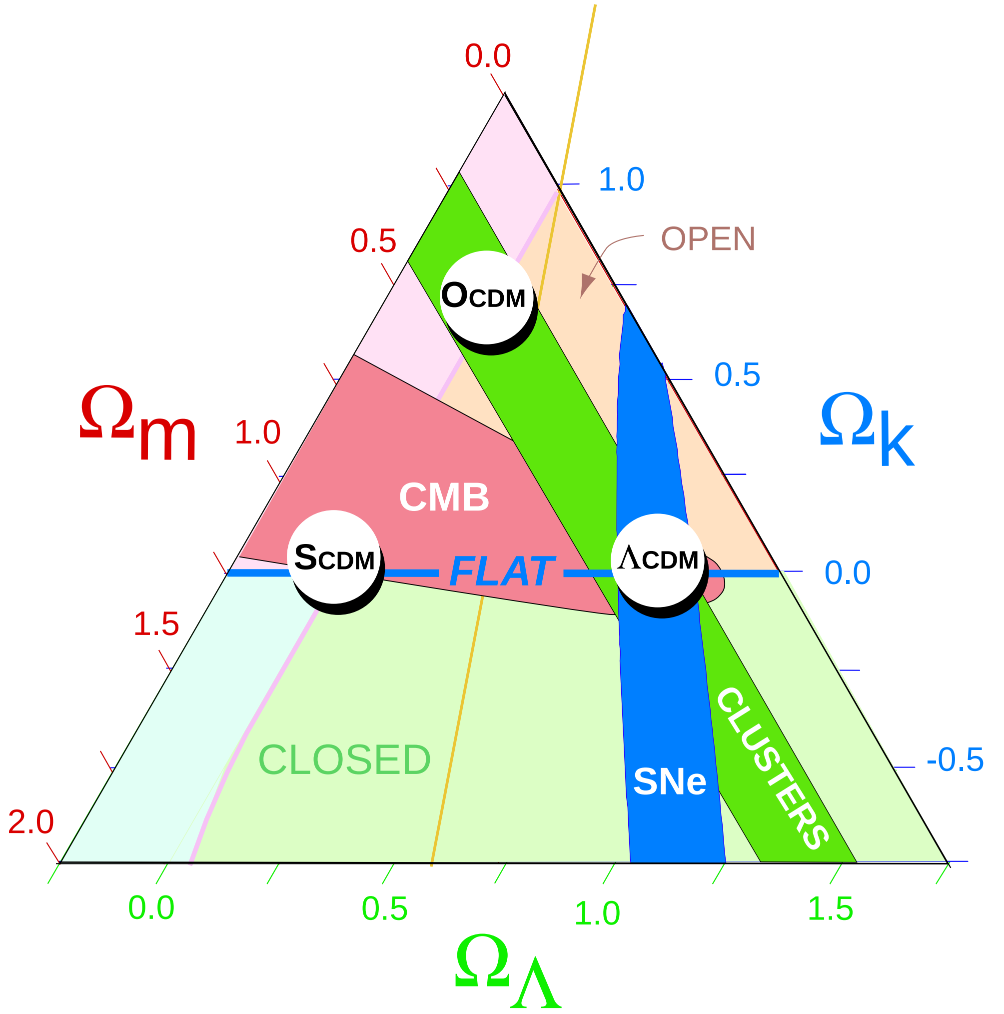
Currently discussed: quintessence: “rolling scalar fi eld”, corresponding to very lightweight particle ($\lambda_{\text{de Broglie}} \sim 1\,\text{Mpc}$), looks like time varying cosmological “constant”.

Why? $\Longrightarrow$ More naturally explains why Ω_Λ so close to 0 (i.e., why matter and vacuum have so similar energy densities)

Motivated by string theory and M theory...

Still VERY SPECULATIVE, decision Λ vs. quintessence should be possible in next 5...10 years when new instruments become available.

Outlook



Bahcall et al.

Even better constraints come from combination of SNe data with structure formation.

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Large Scale Structures and Structure Formation

The Lumpy Universe

So far: treated universe as smooth universe.

In reality:

Universe contains structures!

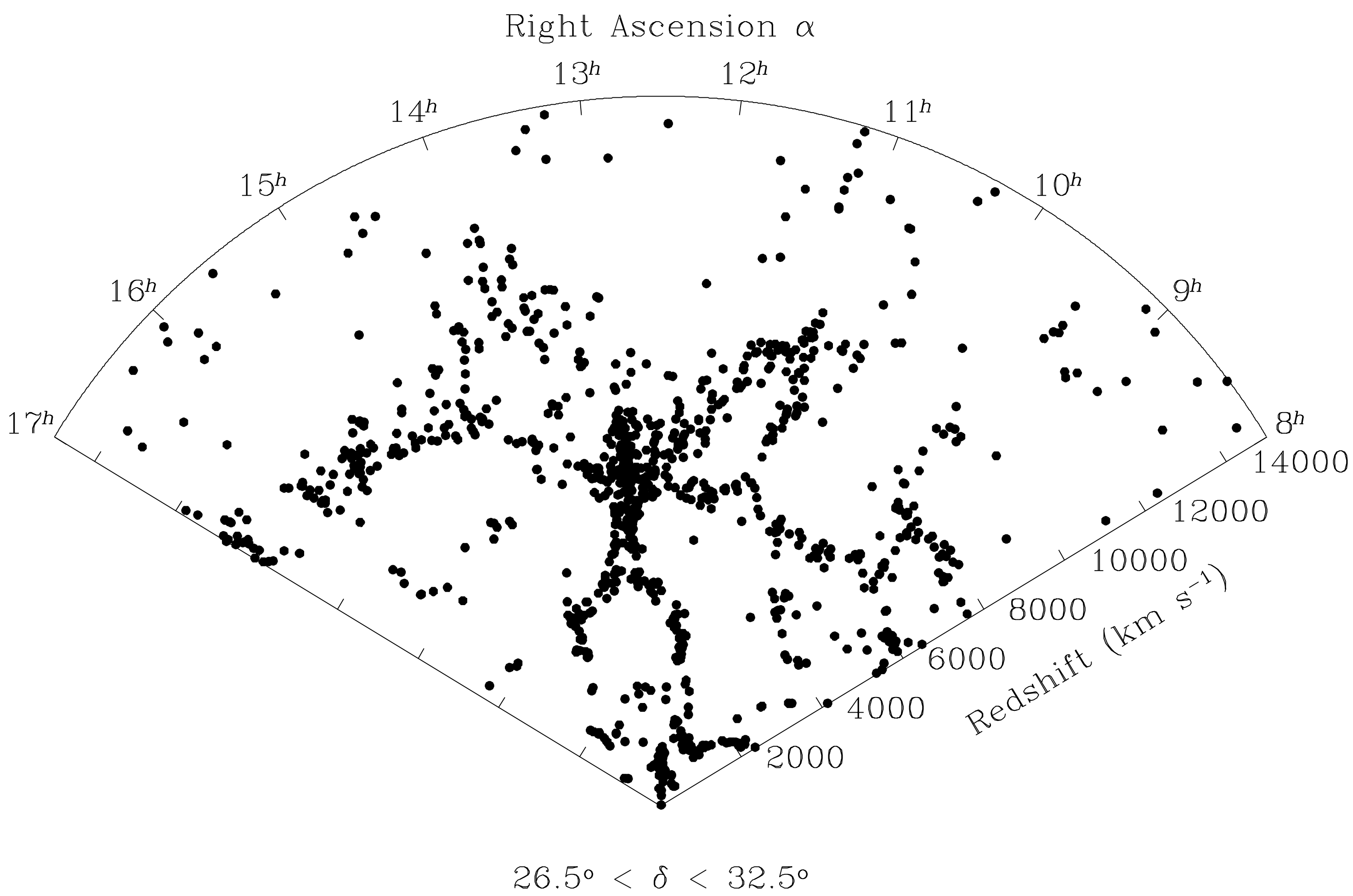
Last part of this class:

1. What are structures?
2. How can we quantify them?
3. How do structures form?
4. How do structures evolve?

Will see that all these questions are deeply connected with parameters of the universe seen so far:

1. H_0
2. Ω_0, Ω_b, Ω_m, Ω_Λ,...
3. Existence and Nature of Dark Matter

Introduction, I



(de Lapparent, Geller & Huchra, 1986, limiting mag $m_{\mathrm{B}} = 15.6$)

Lumpy universe: spatial distribution of galaxies and greater structures.

Introduction, II

How do we study the structure of the Universe?

$\Longrightarrow$ We need distance information for many ($10^4 \ldots 10^7$) objects

$\Longrightarrow$ Large redshift surveys

Review: Strauss & Willick (1995)

Redshift survey: Survey of (patch of) sky determining galaxy z and position to predefined magnitude or z.

First larger survey: de Lapparent, Geller & Huchra (1986)

Classification:

1D-surveys: very deep exposures of small patch of sky, e.g. HST Deep Field, Lockman Hole Survey, Marano Field.

2D-surveys: cover long strip of sky, e.g., CfA-Survey ($1.5 \times 100^\circ$), 2dF-Survey (“2 degree Field”).

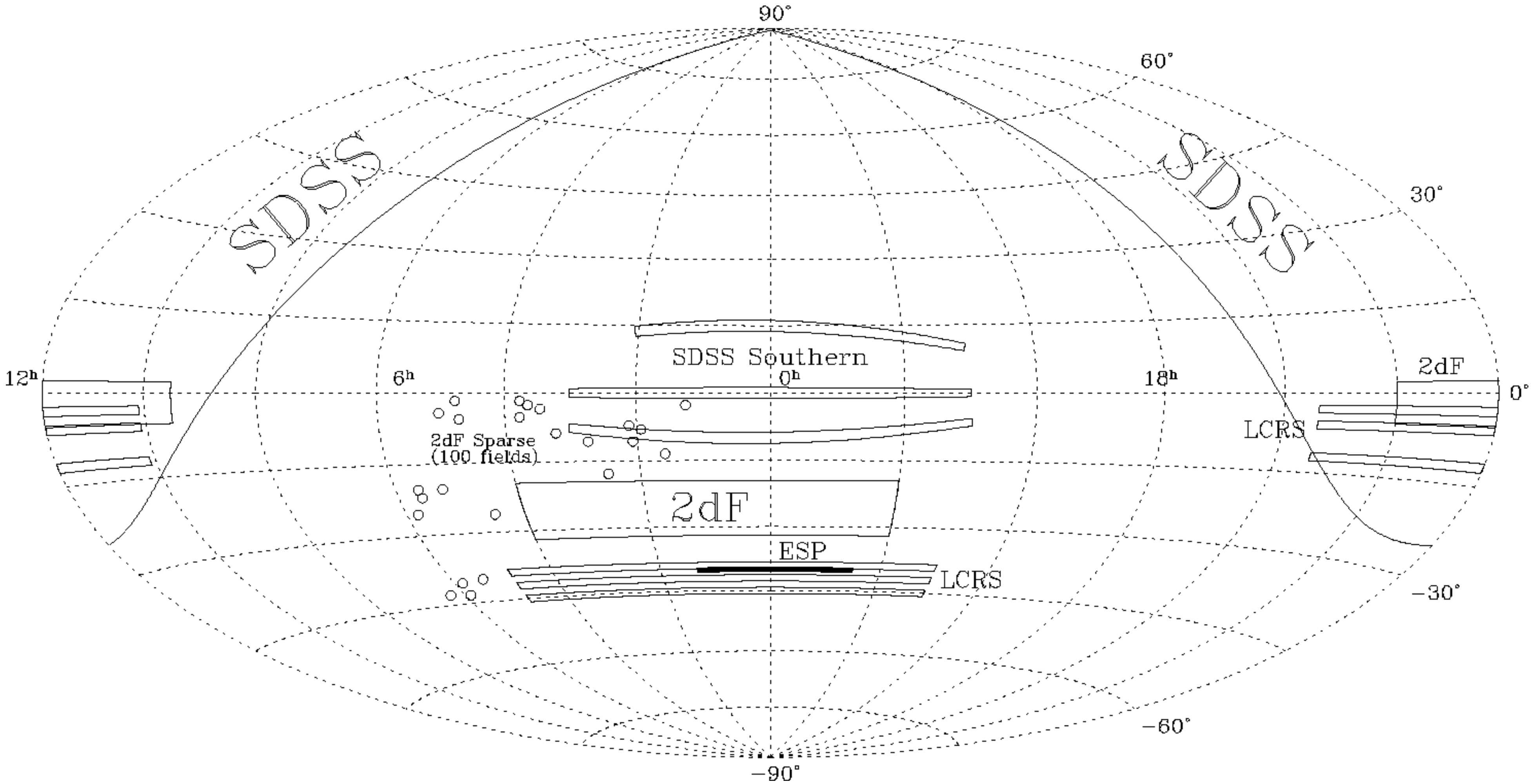
3D-surveys: cover part of the sky, e.g., Sloan Digital Sky Survey.

These surveys attempt to go to certain limit in z or m.

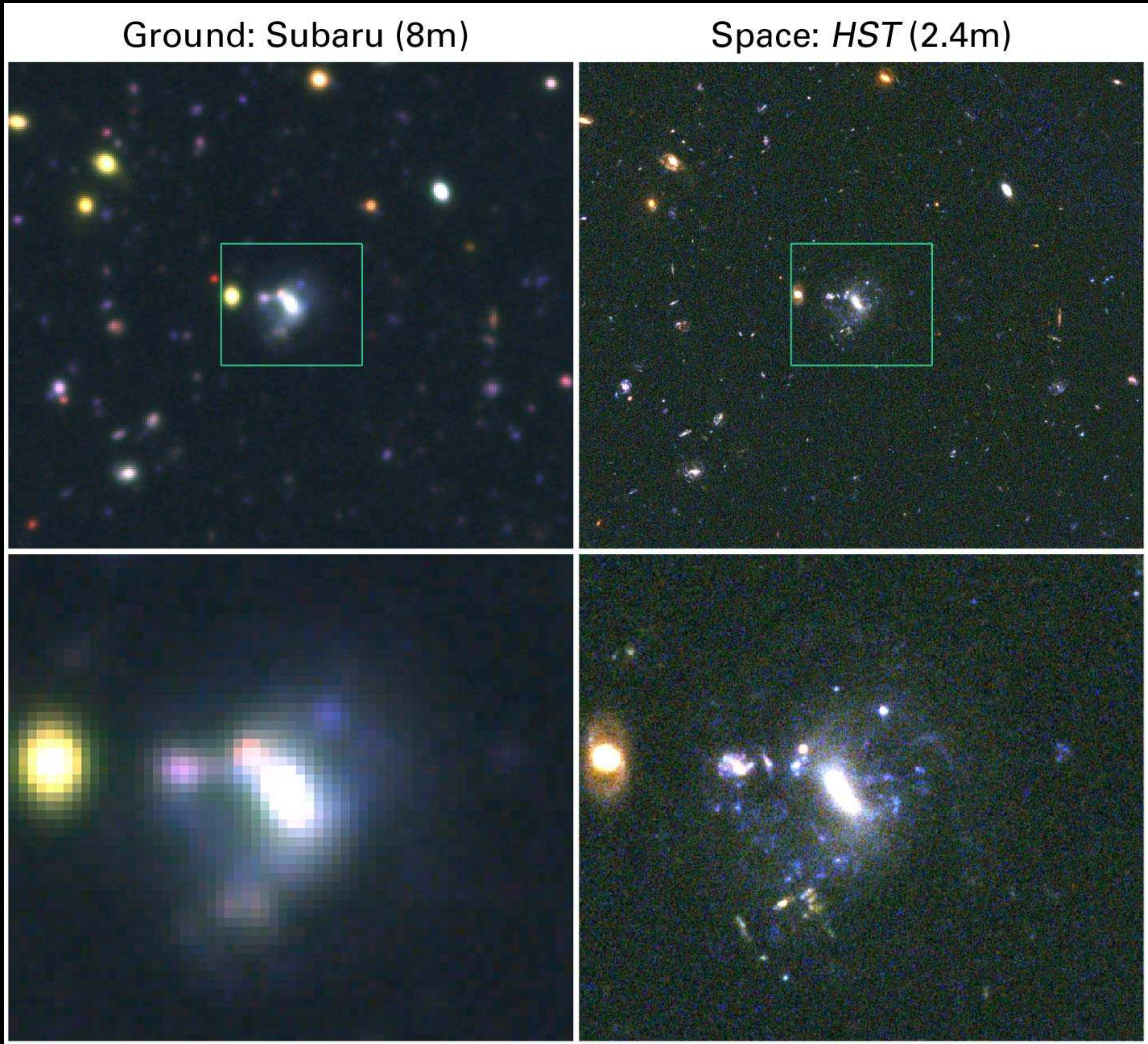
Other approaches: use pre-existing galaxy catalogues (e.g., QDOT Survey [IRAS galaxies], APM survey,...).

We will concentrate here on the larger surveys based on no other catalogue.

Introduction, III



(Strauss, 1999)



To go deep one needs to go to space

STScI

Hubble Space Telescope, II

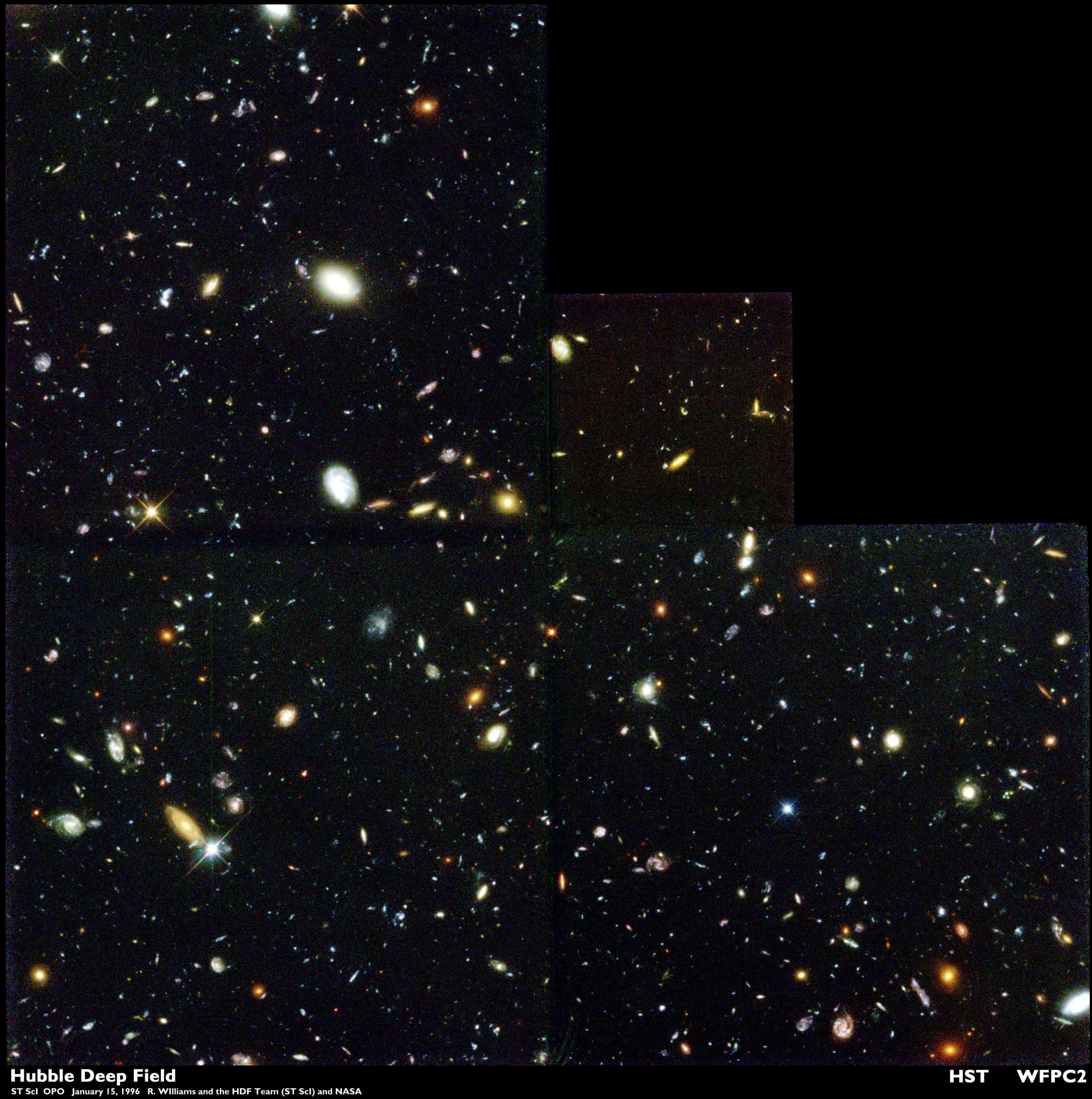
The Hubble Space Telescope has a large set of instruments well suited for cosmological observations:

Current HST Instruments :

- ACS: Advanced Camera for Surveys (03.2002–)
- NICMOS: Near Infrared Camera and Multi Object Spectrometer (02.1997–)
- STIS: Space Telescope Imaging Spectrograph (02.1997–2004)
- WFPC2 The Wide Field Planetary Camera 2 (12.1993–)
- FGS: The Fine Guidance Sensors

Former Generation Instruments :

- FOC: The Faint Object Camera (04.1990–03.2002)
- FOS: The Faint Object Spectrograph (04.1990–02.1997)
- GHRS: The Goddard High Resolution Spectrograph (04.1990–02.1997)
- HSP: The High Speed Photometer (04.1990–10.1993)
- WF/PC-1: Wide Field Planetary Camera 1 (04.1990–10.1993)

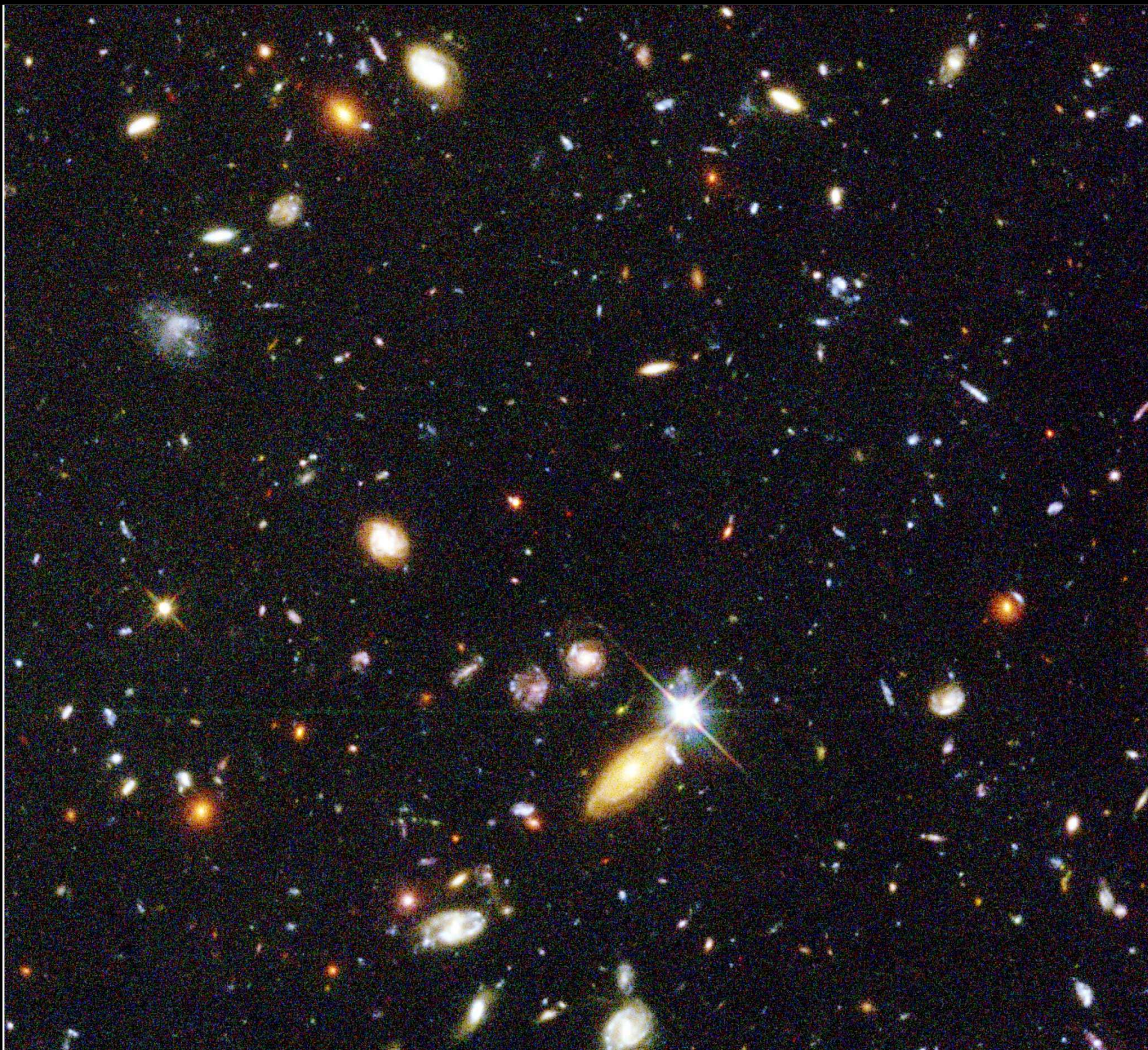


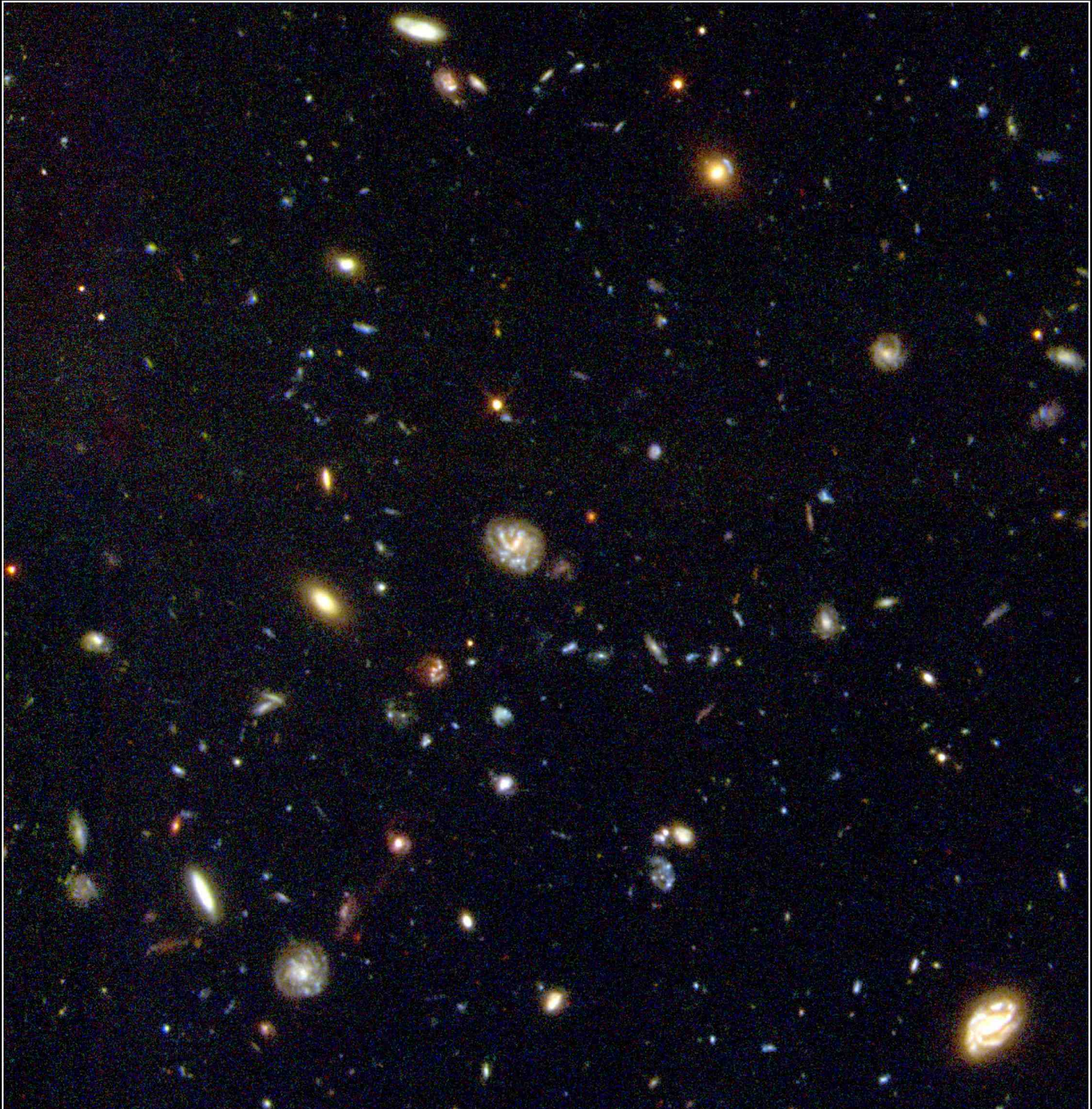
1995 December: Hubble Deep Field:

$\sim$ 150 ksec/Filter for four HST Filters

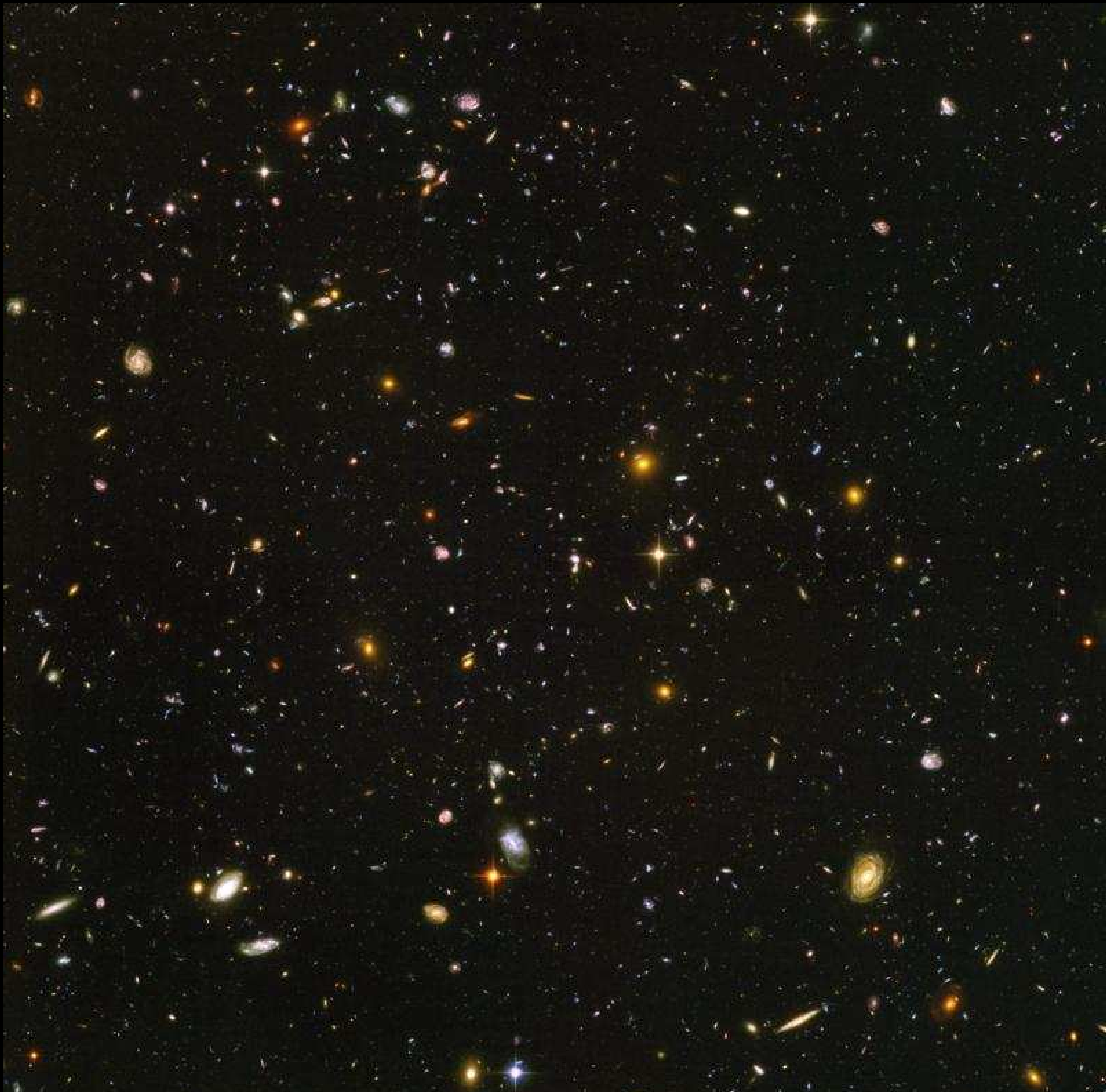
Many galaxies with weird shapes $\Longrightarrow$ protogalaxies!

Redshifts: $z \in [0.5, 5.3]$ (Fernández-Soto et al., 1999)





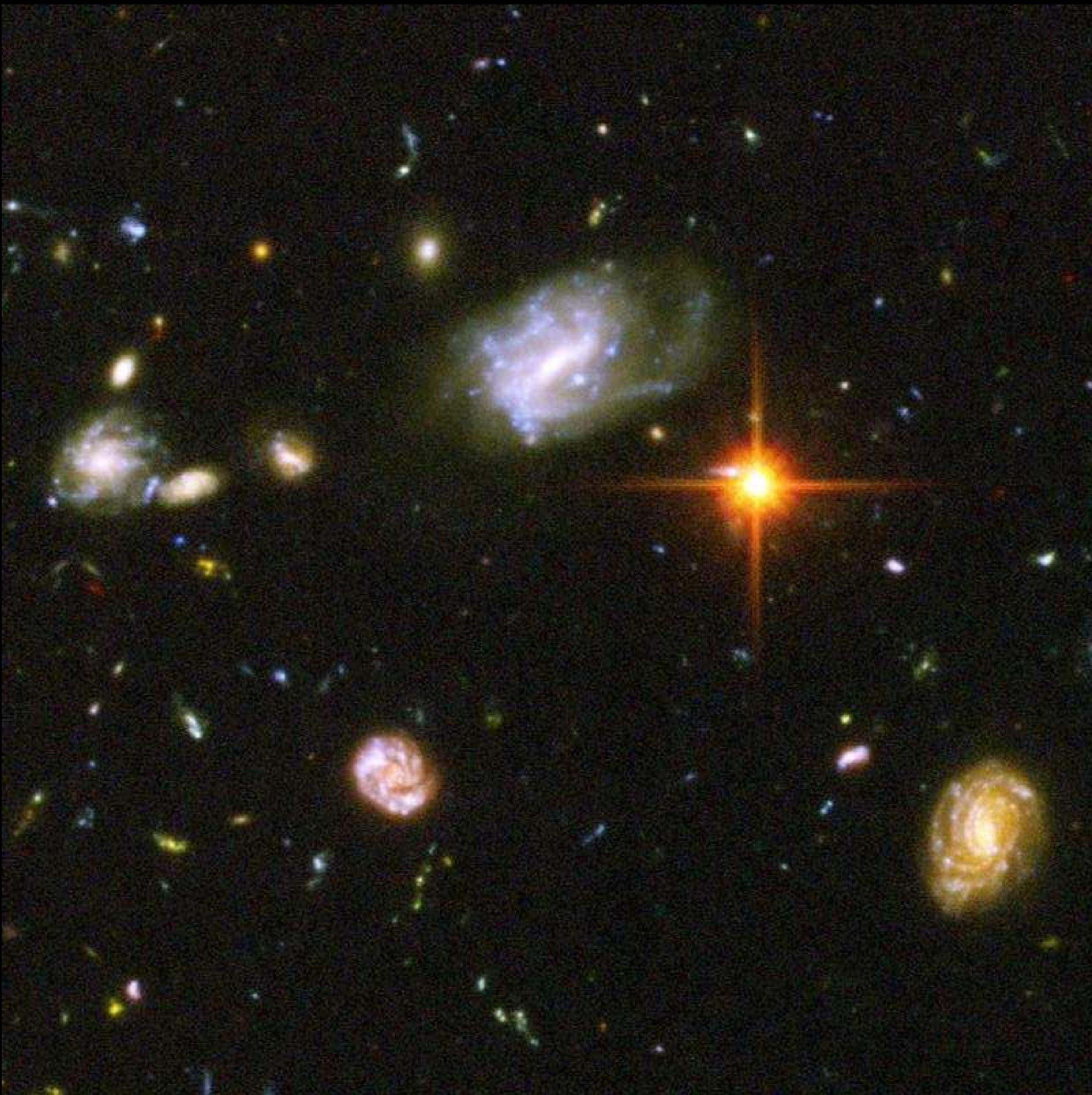
1998: Hubble Deep Field South, 10 d of total observing time!



2004: Hubble Ultra Deep Field, 1 Msec long exposure of field in Fornax. Uses updated HST with Advanced Camera for Surveys (ACS) and Near Infrared Camera and Multi-Object Spectrometer (NICMOS); diameter: 3′ (2× older HDF)

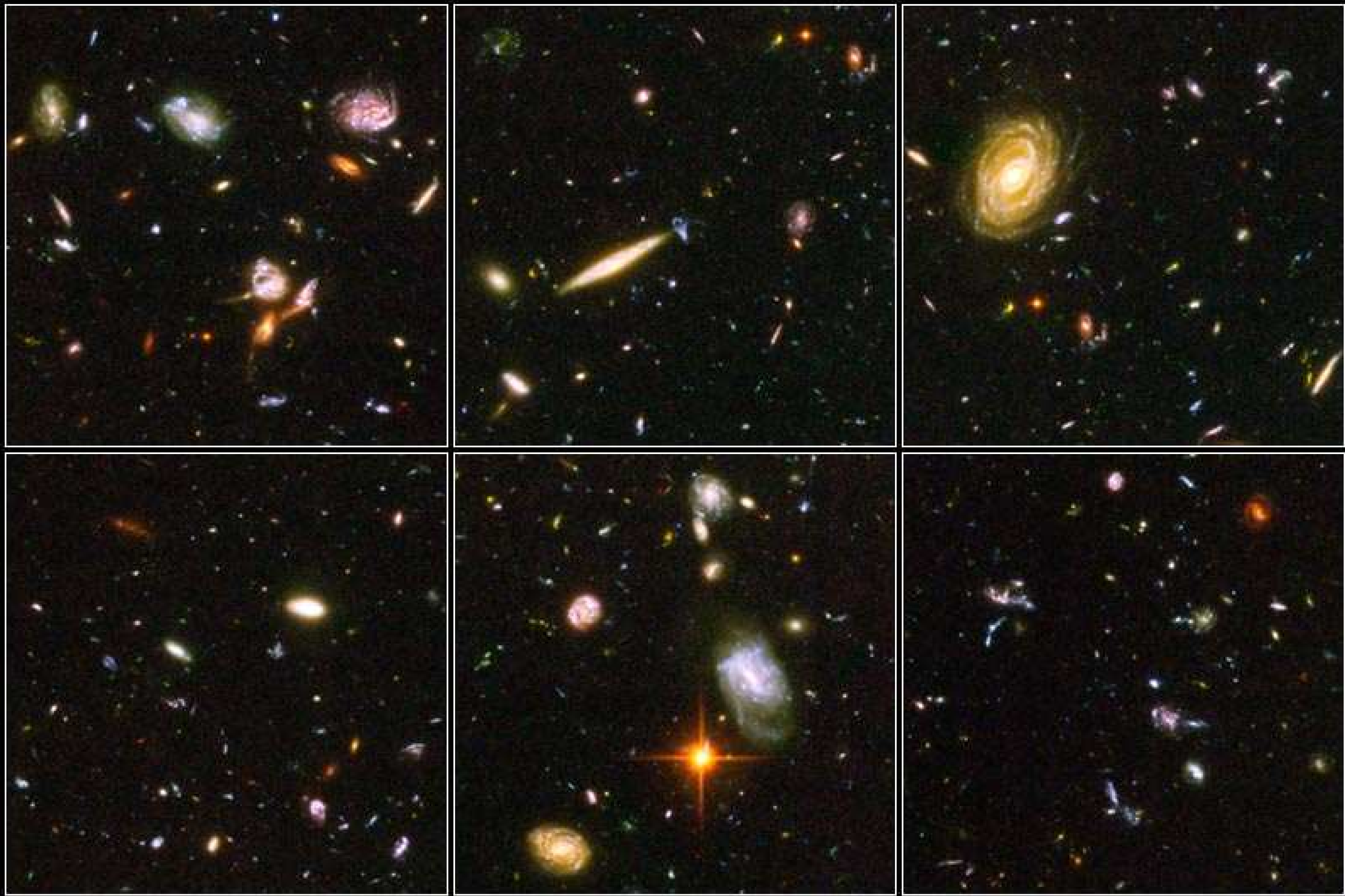
Limiting magnitude: 30 mag, ∼10000 galaxies visible, up to $z \gtrsim 7$

IR reveals many reddened objects



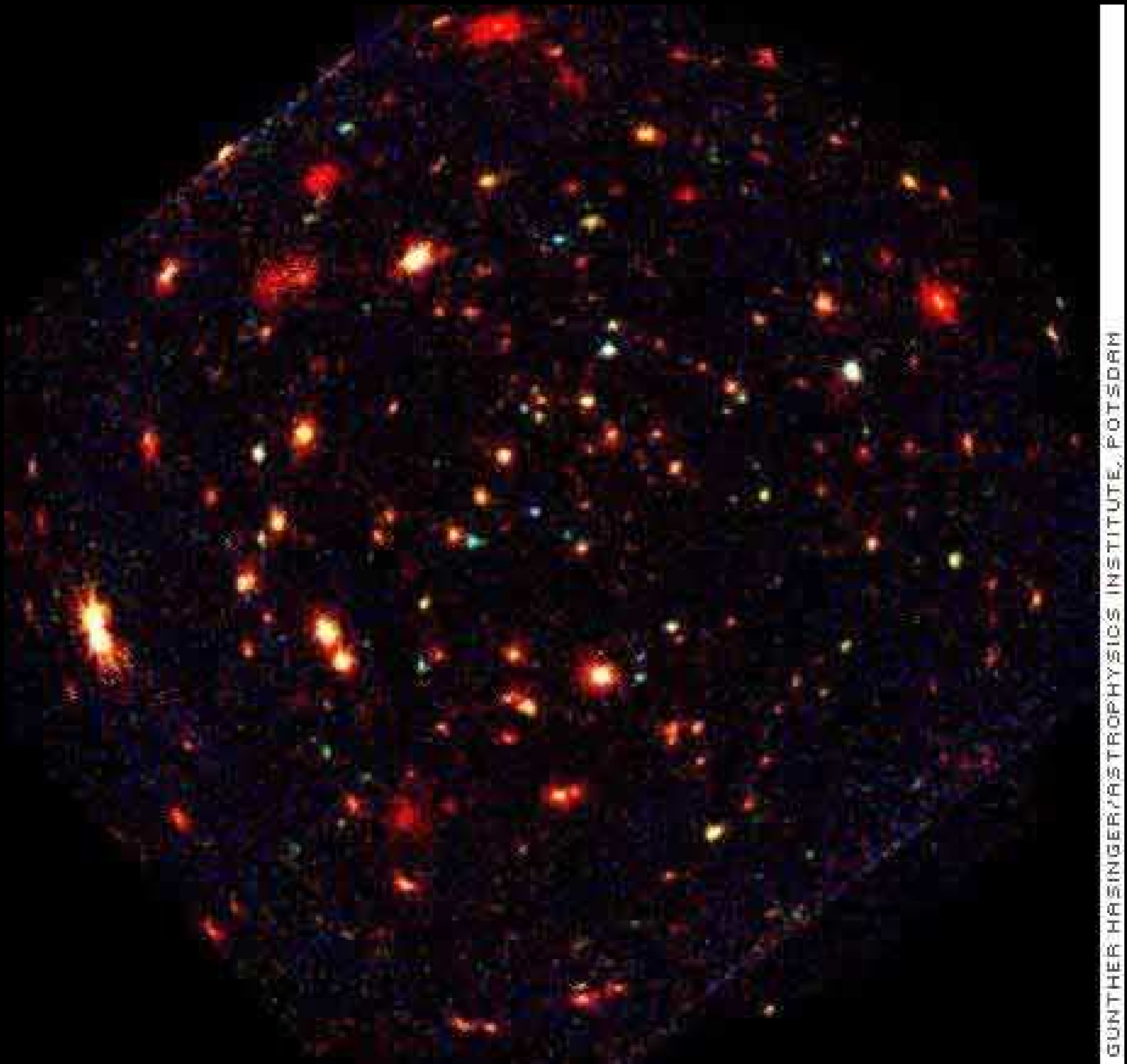
Hubble Ultra Deep Field Details

HST ▪ ACS



NASA, ESA, S. Beckwith (STScI) and The HUDF Team

STScI-PRC04-07c



Lockman Hole: Northern Sky region with very low N_{H}

$\Longrightarrow$ low interstellar absorption

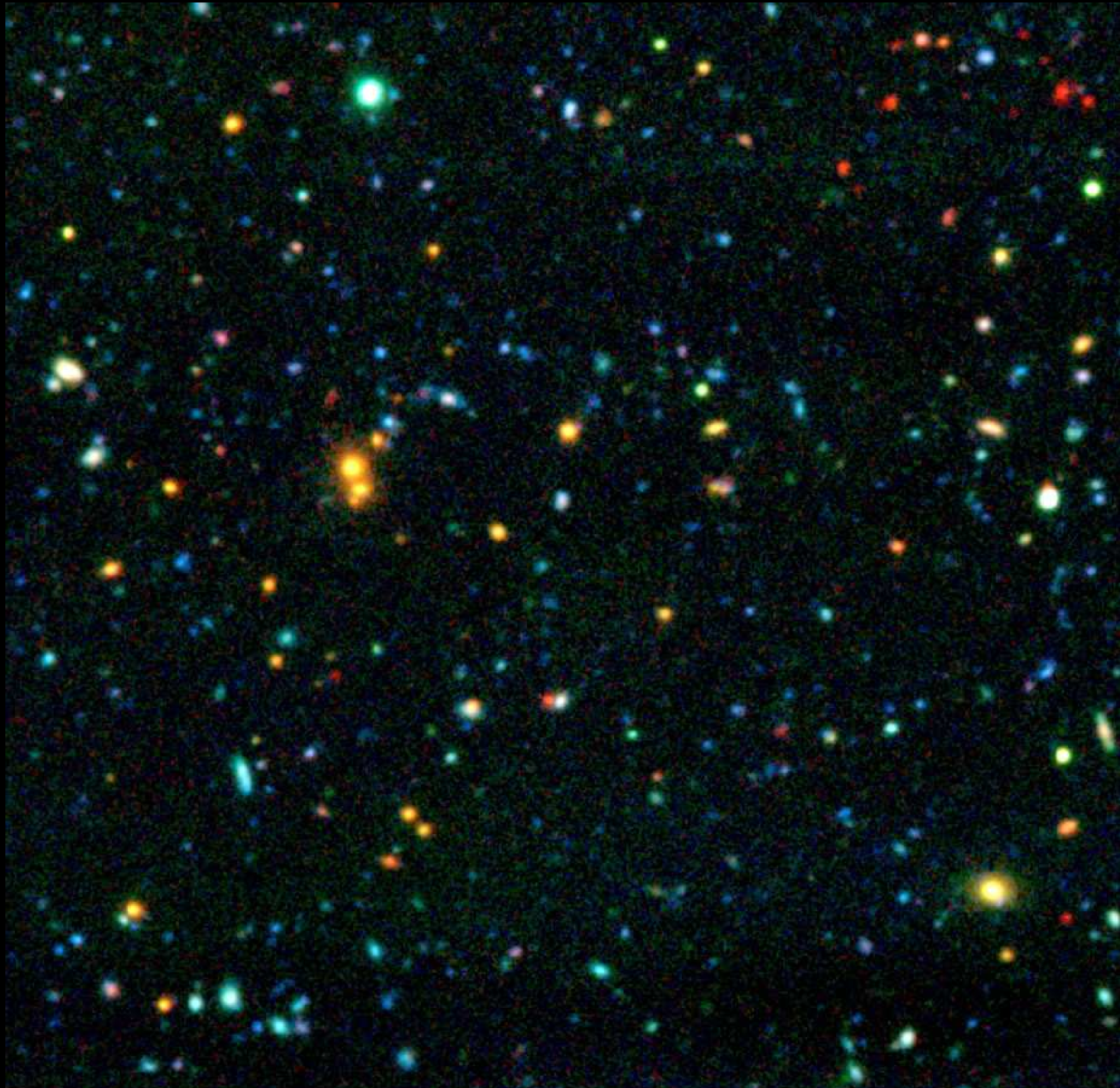
$\Longrightarrow$ “Window in the sky”

$\Longrightarrow$ X-rays: evolution of active galaxies with z!

XMM-Newton, Hasinger et al., 2001,
blue: hard X-ray spectrum,
red: soft X-ray spectrum

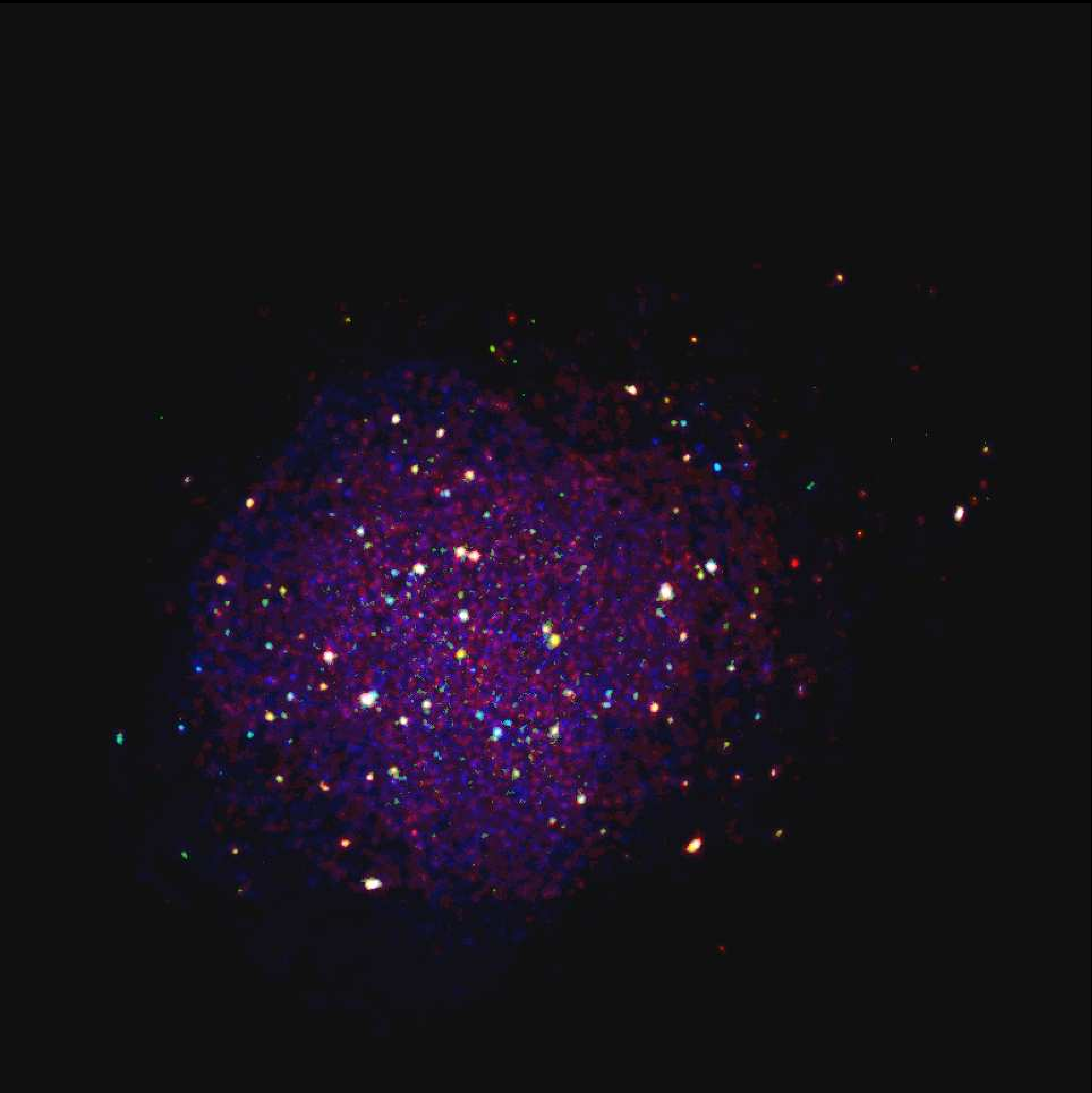
Chandra Deep Field South:
1 Msec (10.8 days) on one region in Fornax
($\alpha_{\mathrm{J2000.0}} = 3^{\mathrm{h}}32^{\mathrm{m}}28.0^{\mathrm{s}}$,
$\delta_{\mathrm{J2000.0}} = -27^{\circ}48'30''$,
coaligned with HDF-S
Deepest X-ray field ever
color code: spectral hardness

scale: $15' \times 15'$; courtesy NASA/JHU/AUI/R.Giacconi et al.



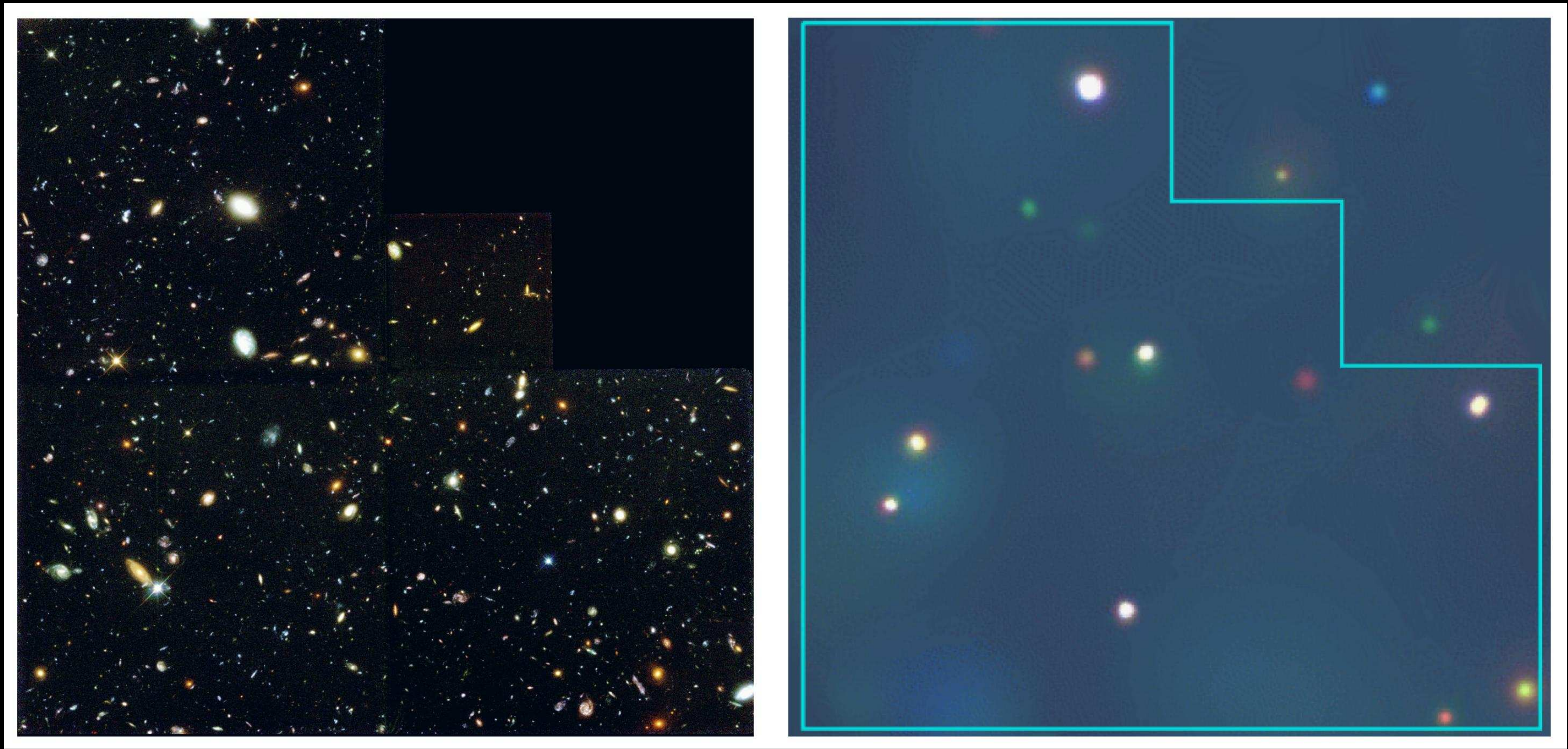
Distant Galaxies in "AXAF Deep Field" (VLT ANTU / ISAAC + NTT / SUSI-2)

ESO PR Photo 06b/00 (17 February 2000)



1D Surveys ("Deep Exposures") give snapshot of evolution of galaxies over large z.

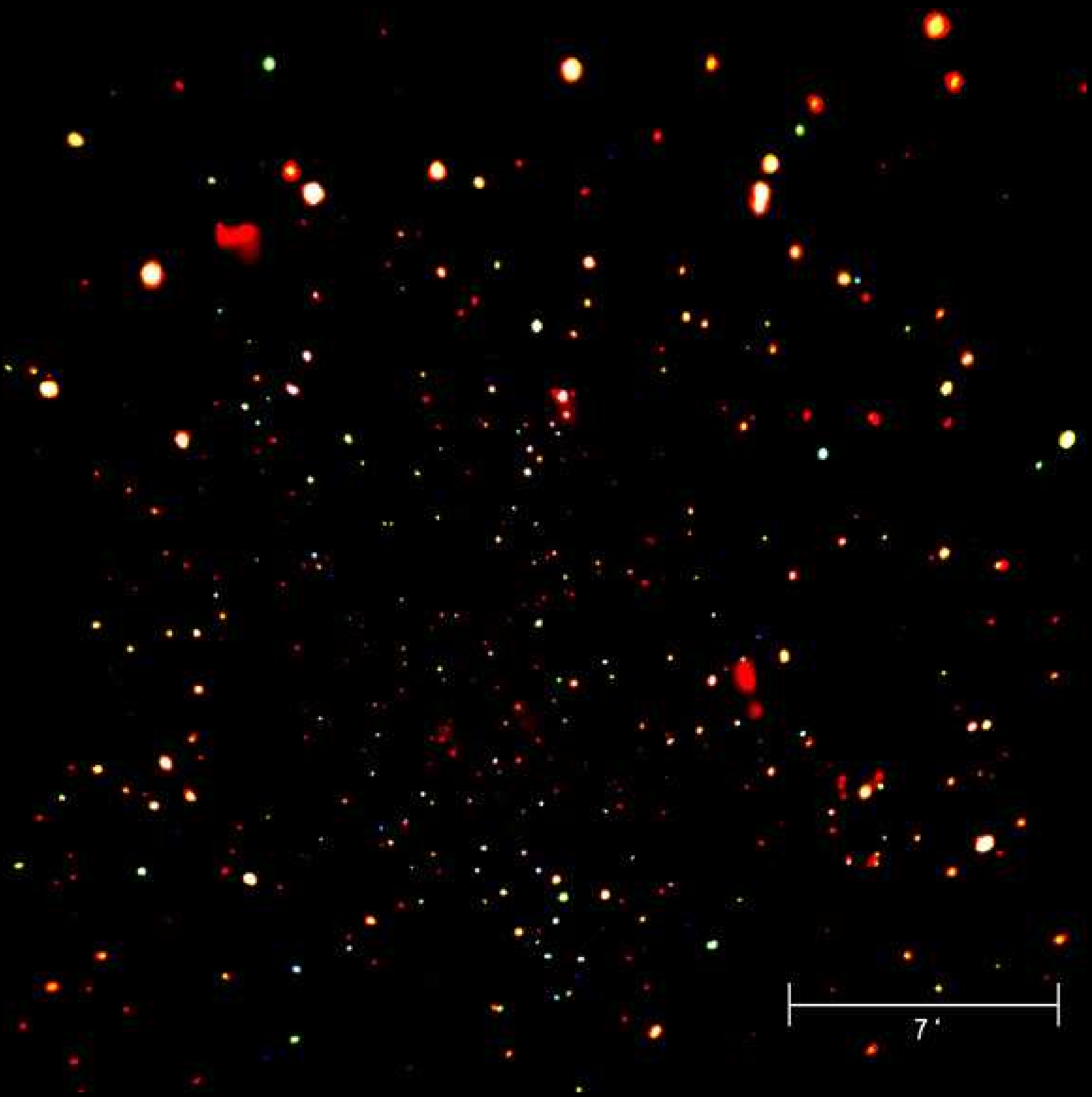
Deep *XMM-Newton* image of the Marano Field (IAAT/AIP/MPE)



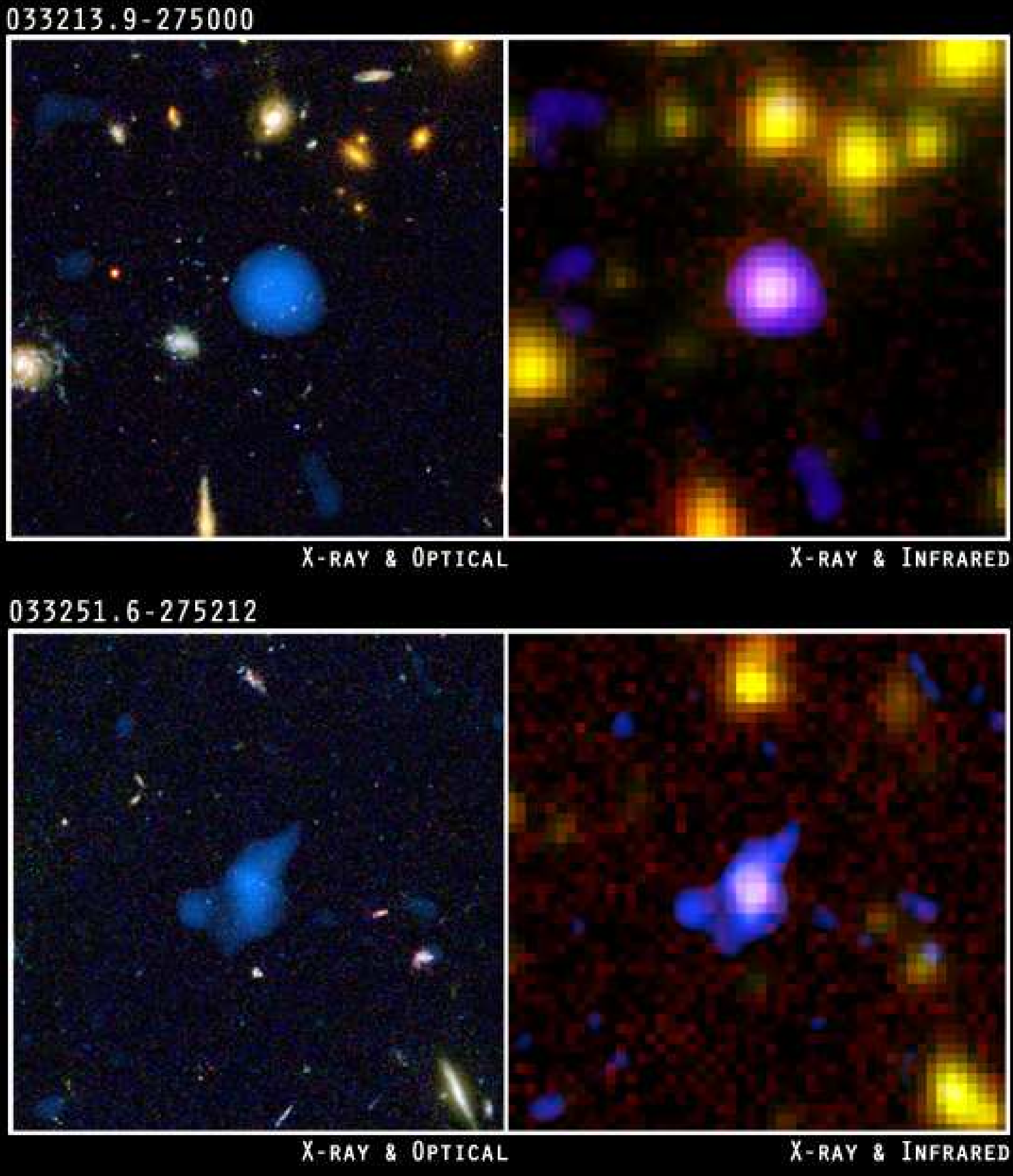
HST   Chandra

Chandra/HST Image of Hubble Deep Field North; 500 ksec

Joint multi-wavelength campaigns allow the measurement of broad-band spectra of sources in the early universe!

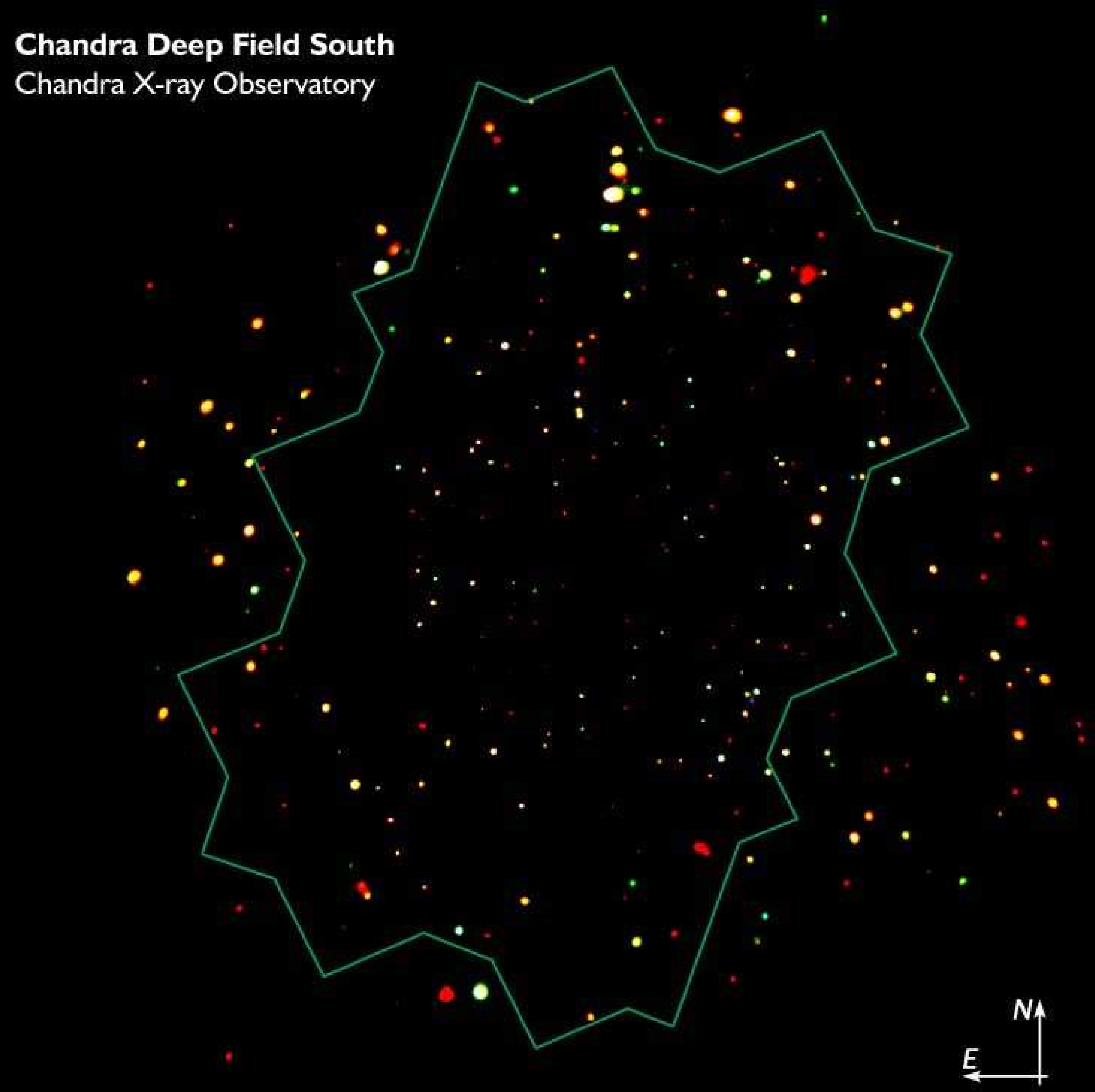


⟹ GOODS-Survey (Great Observatories Origins Deep Survey), centered on CDF-S
(same image as before, this time smoothed)

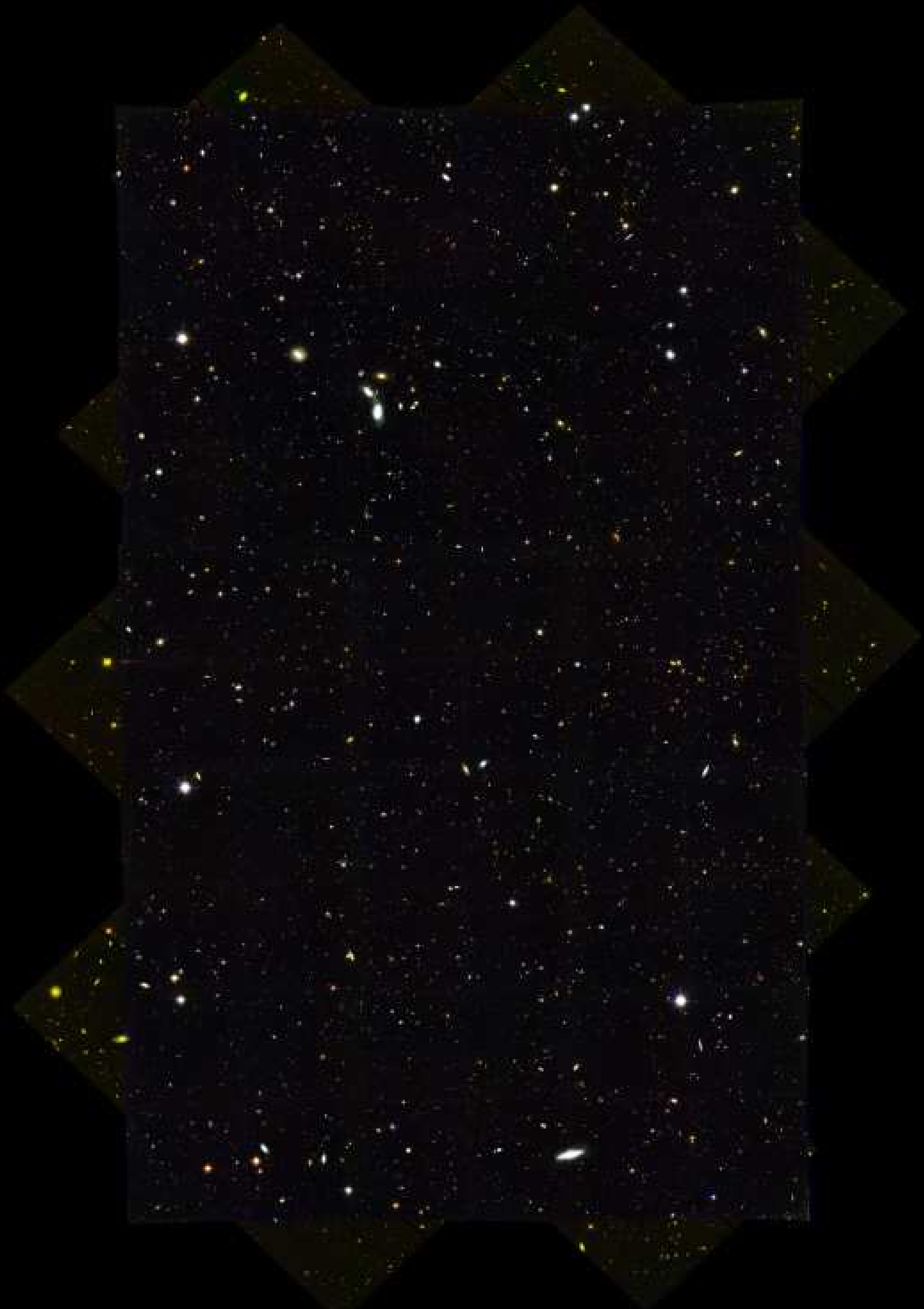


IR, optical, and X-ray image of small fraction of GOODS

CXC/NASA

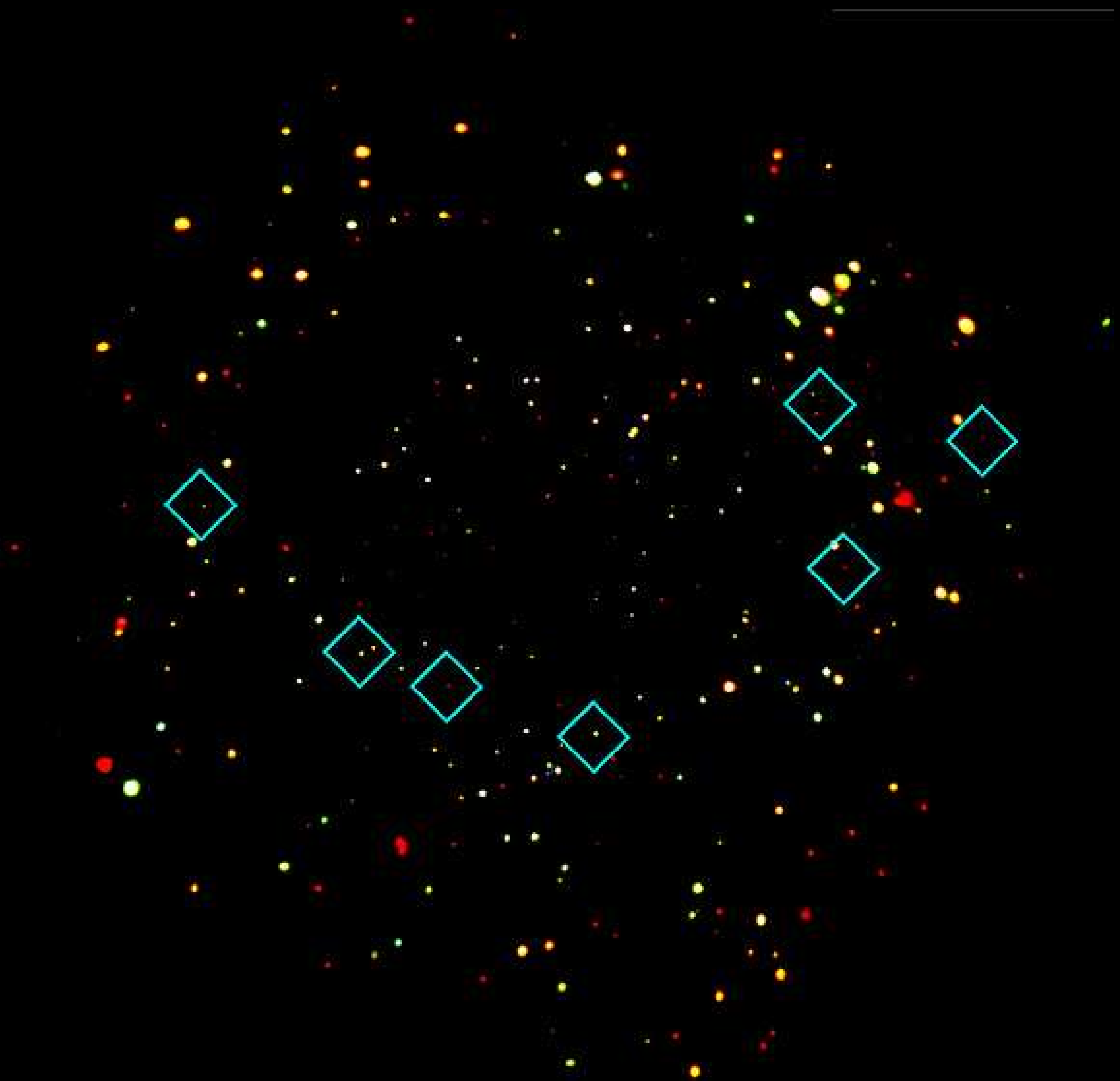


Chandra and HST fi elds aligned



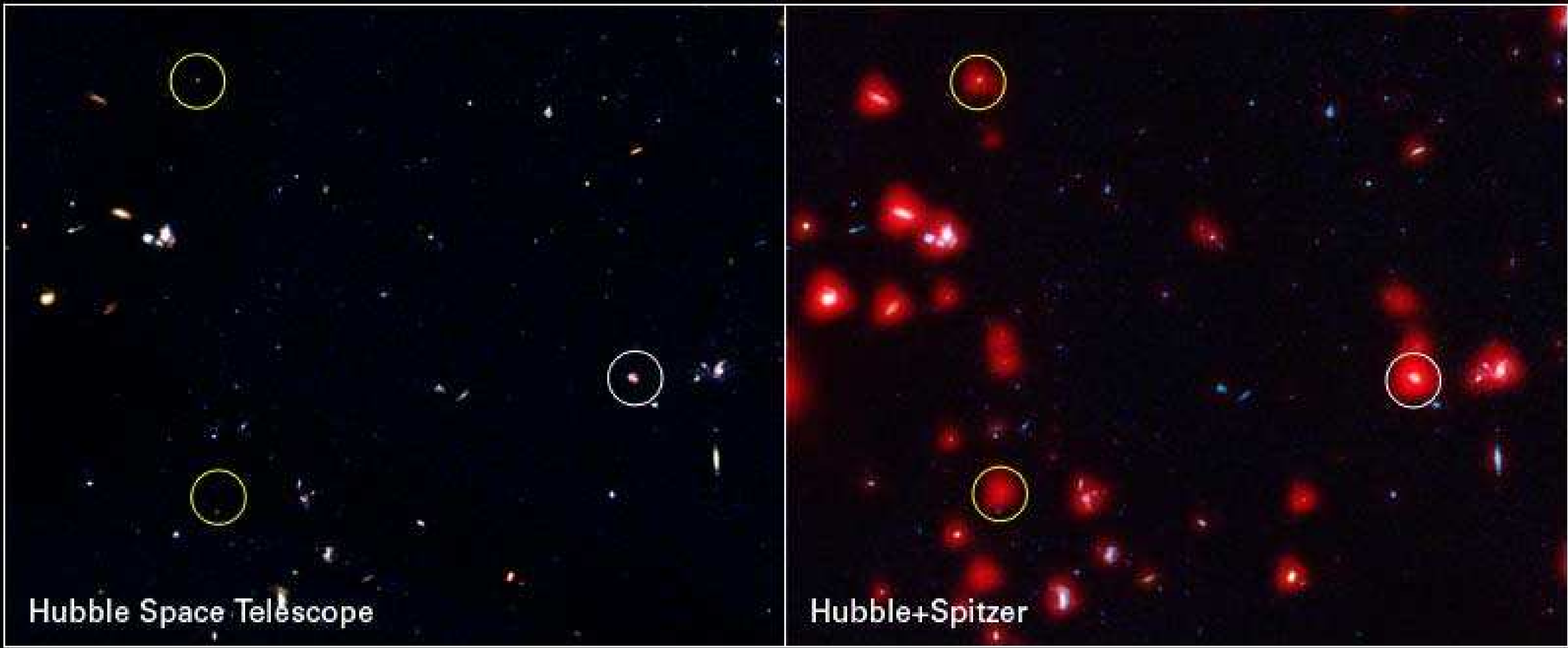
HST ACS observations of
whole area of CDF-S

CXC



CDFS: blue boxes contain objects not visible in HST

$\Longrightarrow$ farthest black holes known



STScI/Caltech

1/200th of the whole GOODS fi eld in optical and IR

2D/3D Surveys: Technology

Future for Large Scale Structure: 2D and 3D Surveys observing large part of sky with dedicated instruments.

Currently largest surveys:

Las Campanas Redshift Survey (LCRS): 26418 redshifts in six $1.5 \times 80^\circ$ slices around NGP and SGP, out to $z = 0.2$.

CfA Redshift Survey: 30000 galaxies

APM: (Oxford University) $2 \sim 10^6$ galaxies, 10^7 stars around SGP, 10% of sky, through $B = 21$ mag.

2MASS: IR Survey of complete sky (Mt. Hopkins/CTIO) completed 2000 October 25), 3 bands, $\sim 2 \times 10^6$ galaxies, accompanying redshift survey (8dF, CfA)

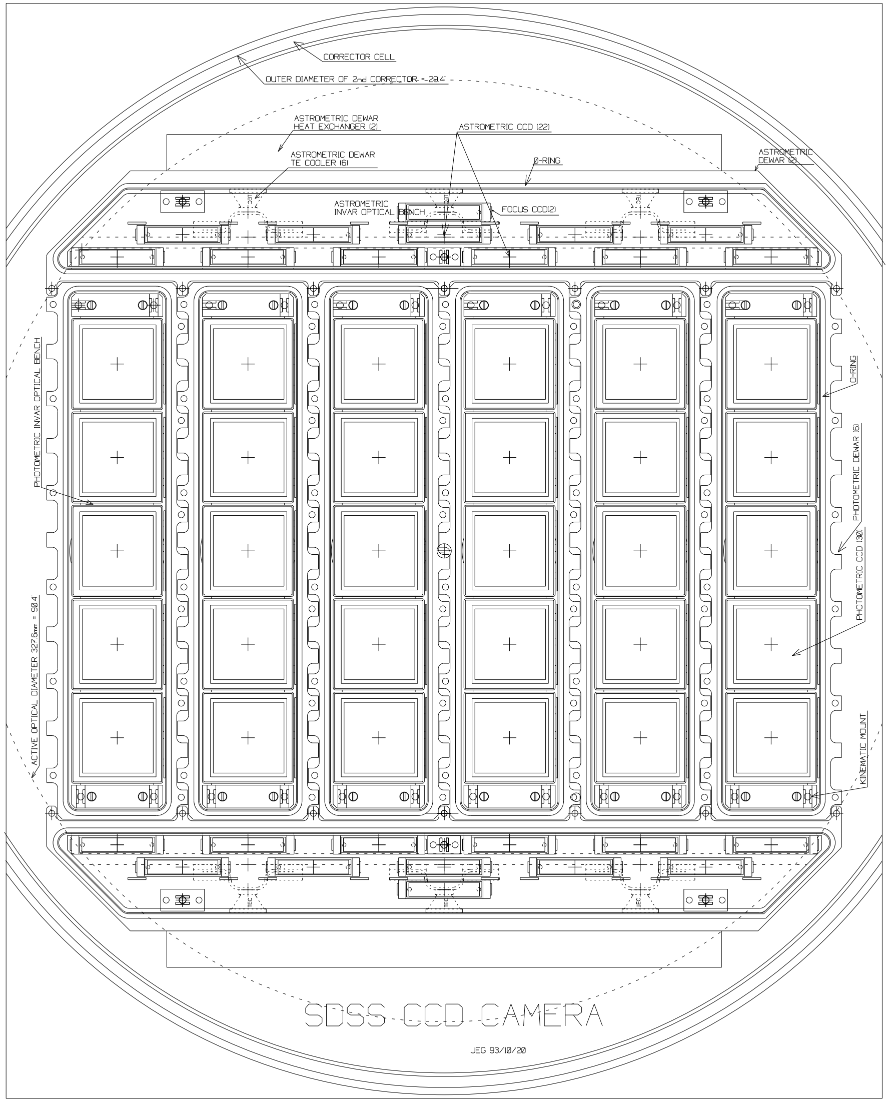
Sloan Digital Sky Survey (SDSS): dedicated 2000 October 5, Apache Point Obs., NM, 25% of whole sky, $\sim 10^8$ objects, now in Google Earth

And many more (e.g., Keck, ESO, LSST,...).

SDSS 2.5 m telescope at Apache Point Observatory

courtesy SDSS

2D/3D Surveys: Technology



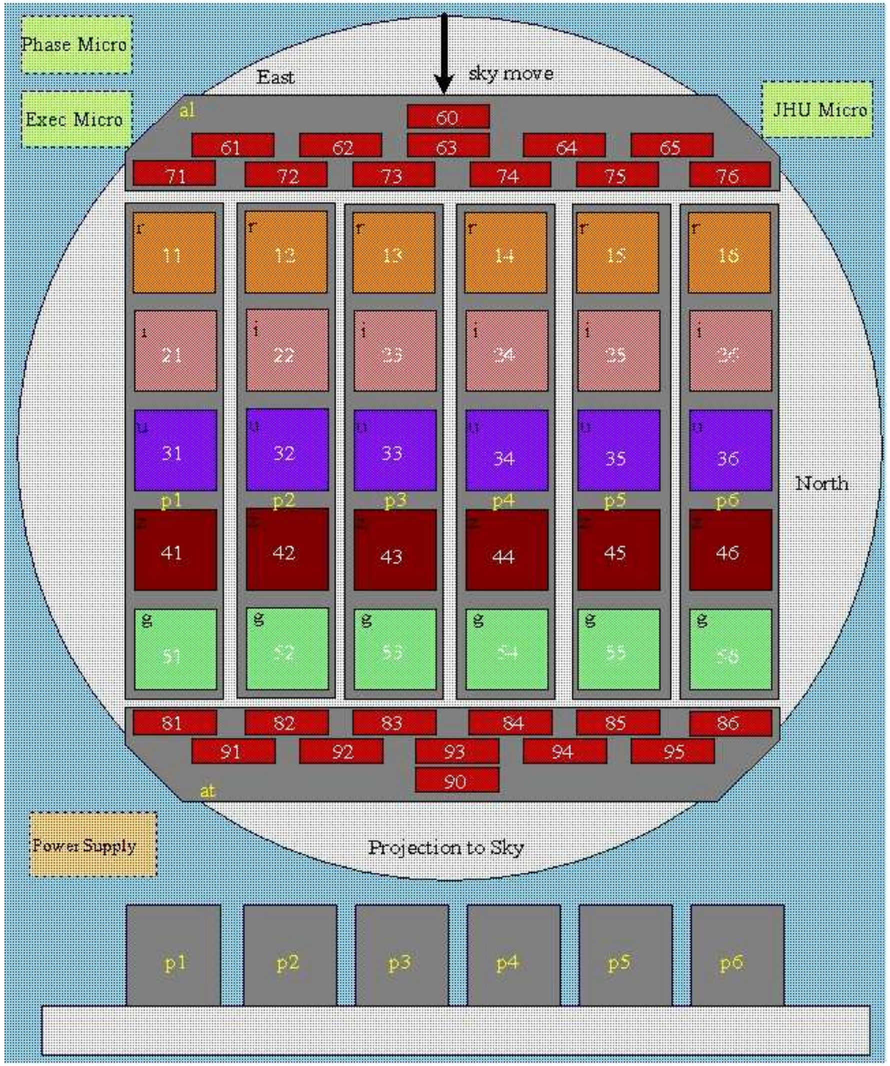
CCD alignment of SDSS:

- focal plane: 2.5°,
- 5 rows of 2048 × 2048 CCDs with r, i, u, z, g filters, saturation at $r = 14$
- 22 2048 × 400 CCD, saturation at $r = 6.6$ for astrometry

Imaging by slewing over CCD Array

(Strauss, 1999, Fig. 5)

2D/3D Surveys: Technology



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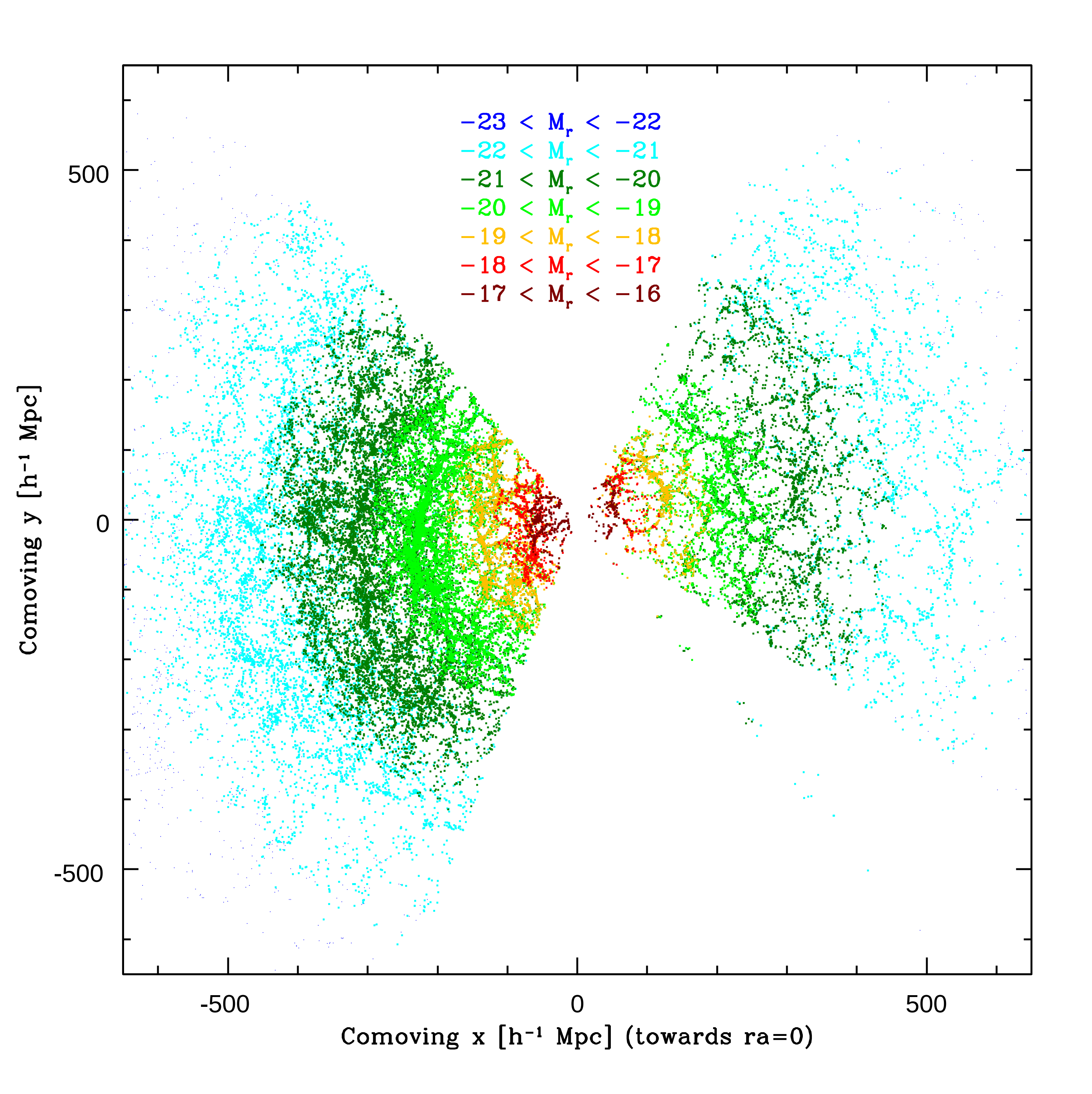
Imaging by slewing over CCD Array

SDSS

2D/3D Surveys: Technology

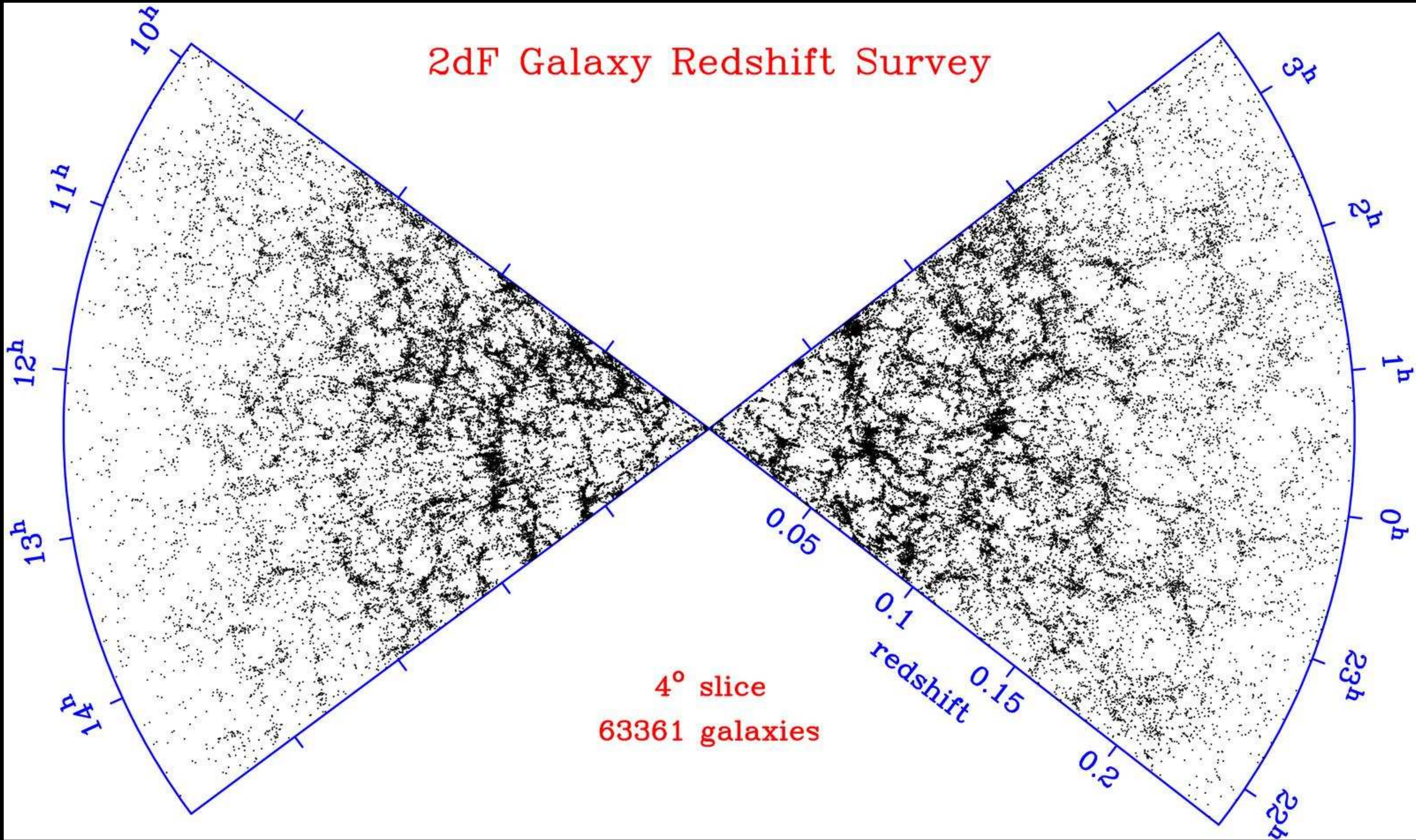
courtesy SDSS

Spectroscopy with grism (combination of prism and grating), light from objects via optical fi bers and plug plate.

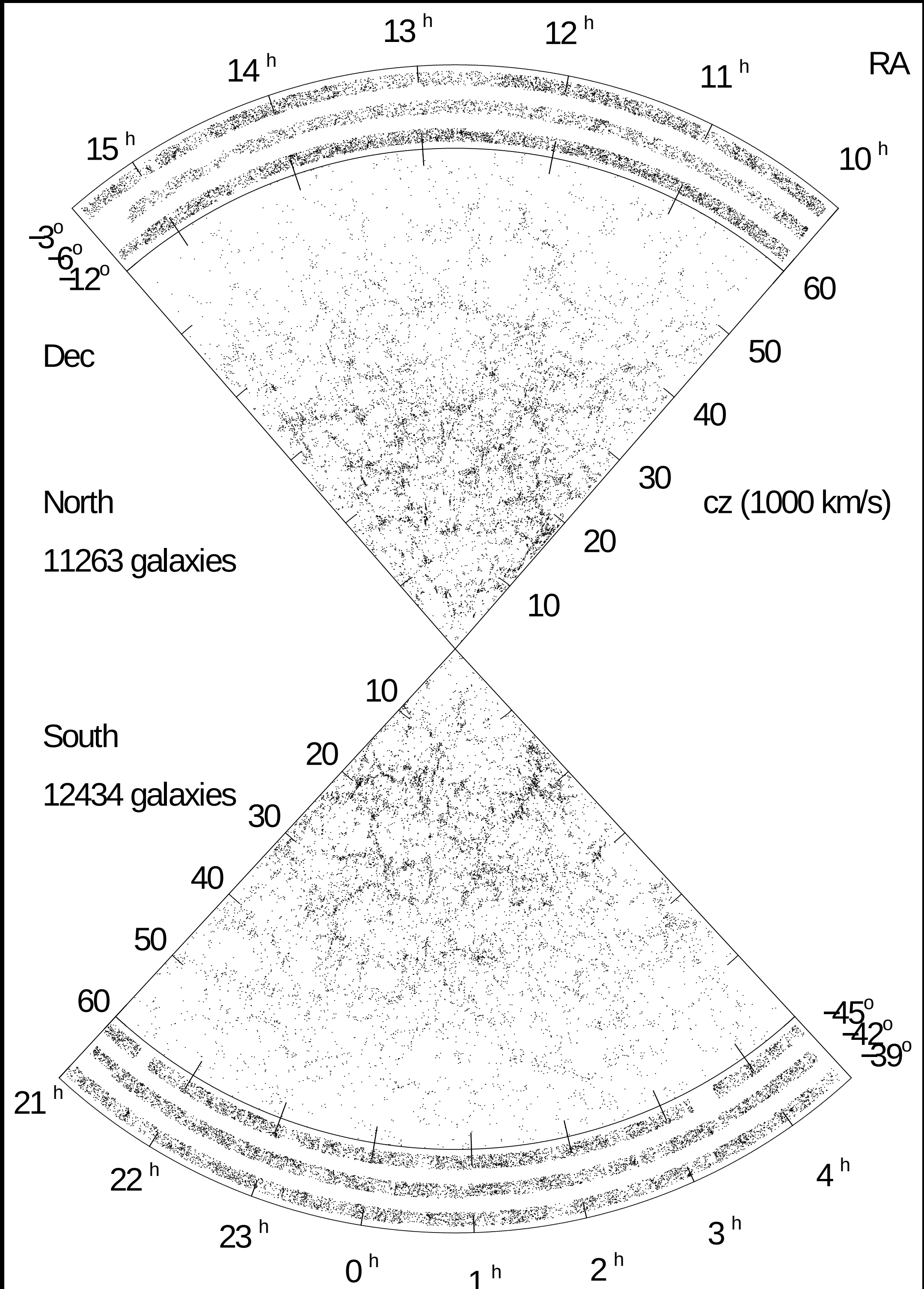


Galaxy distribution from the SDSS

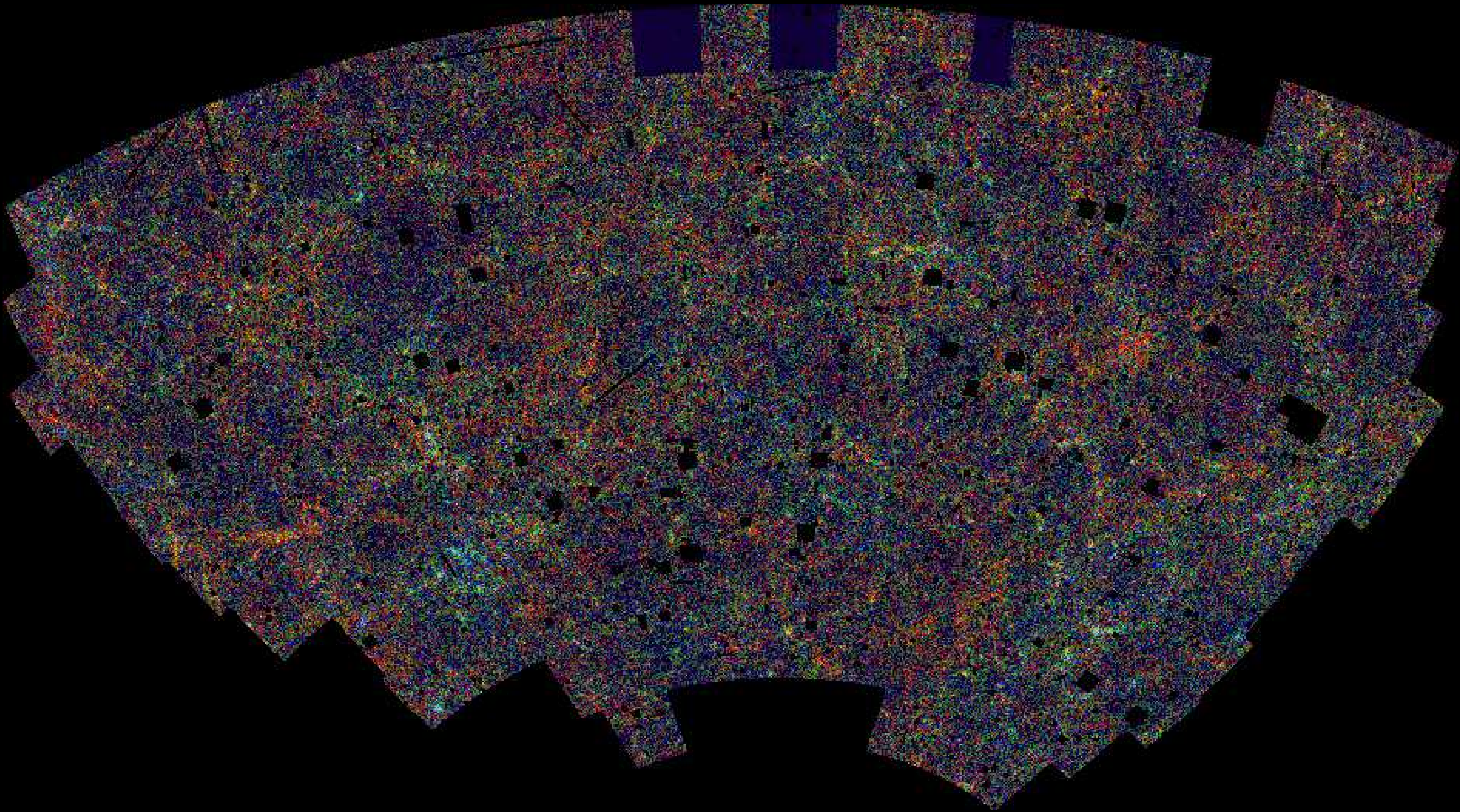
(Tegmark et al., 2004, Fig. 4)



courtesy 2dF collaboration



The complete LCRS survey (at cz large: reach mag. limit)



Galaxies in APM catalogue, color: avg. B in pixel: blue (18) – green (19) – red (20)