Determination of Omega and Lambda

Inflation

Previous lectures: Inflation requires

$$\Omega = \Omega_{\mathrm{m}} + \Omega_{\Lambda} = 1 \tag{7.1}$$

Here,

Ω_{m}: Ω due to gravitating stuff,

Ω_{Λ}: Ω due to vacuum energy or other exotic stuff.

To decide whether that is true:

- need inventory of gravitating material in the universe,
- need to search for evidence of non-zero Λ

Also search for evidence in structure formation $\Longrightarrow$ Later...

Often, express Ω in terms of a mass to luminosity ratio.

Using canonical luminosity density of universe, one can show (Peacock, 1999, p. 368, for the B-band):

$$\left.\frac{M}{L}\right|_{\mathrm{crit}} = 1390\, h \frac{M_{\odot}}{L_{\odot}} \tag{7.2}$$

Introduction

Constituents of Ω_m:

- Radiation (CMBR)
- Neutrinos
- Baryons ("normal matter", Ω_b)
- Other, non-radiating, gravitating material ("dark matter")

Radiation: From temperature of CMBR, using $u = a_\mathrm{rad} T^4$:

$$\Omega_\gamma h^2 = 2.480 \times 10^{-5} \qquad (7.3)$$

for $h = 0.72$, $\Omega_\gamma = 4.8 \times 10^{-5}$

Massless Neutrinos have

$$\Omega_\nu = 3 \cdot \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} \Omega_\gamma = 0.68\, \Omega_\gamma \qquad (7.4)$$

Photons and massless neutrinos are unimportant for todays Ω.

Massive Neutrinos

Sudbury Neutrino Observator (SNO) and Super-Kamiokande: Neutrinos are not massless.

From neutrino decoupling and expansion:

Current neutrino density: 113 neutrinos/cm^3 per neutrino family.

In terms of Ω:

$$\Omega_\nu h^2 = \frac{\sum m_i}{93.5\,\mathrm{eV}} \tag{7.5}$$

$\Longrightarrow$ For $h = 0.75$, $m \sim 17\,\mathrm{eV}$ sufficient to close universe

Current mass limits:

ν_e: $m < 2.2\,\mathrm{eV}$

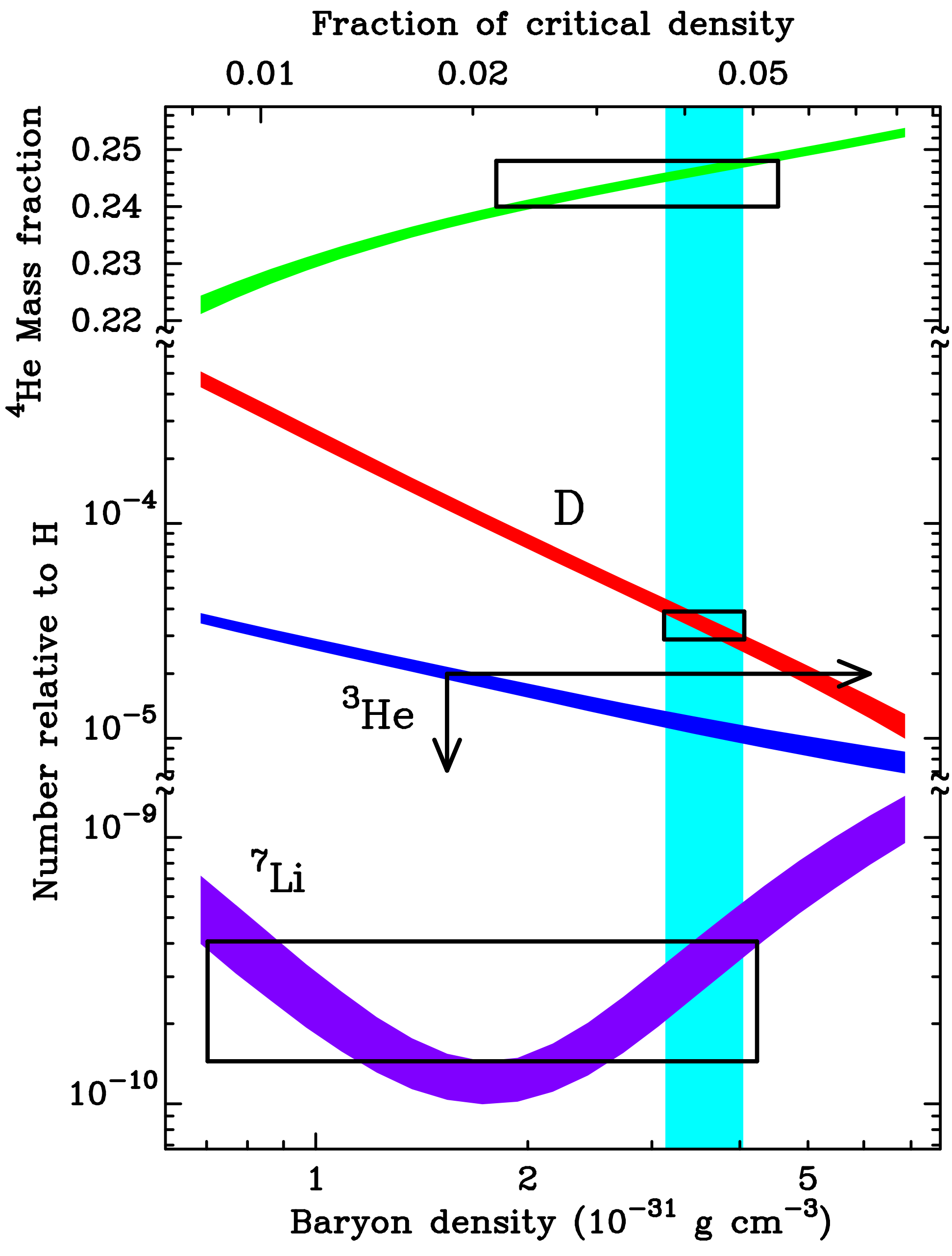
ν_μ: $m < 0.19\,\mathrm{MeV}$

ν_τ: $m < 18.2\,\mathrm{MeV}$

Source: `http://cupp.oulu.fi/neutrino/nd-mass.html` and Particle Physics Booklet 2000

Note that solar neutrino oscillations imply Δm between ν_e and ν_μ is $\sim 10^{-4}\,\mathrm{eV}$, i.e., most probable mass for ν_μ much smaller than direct experimental limit.

Baryons



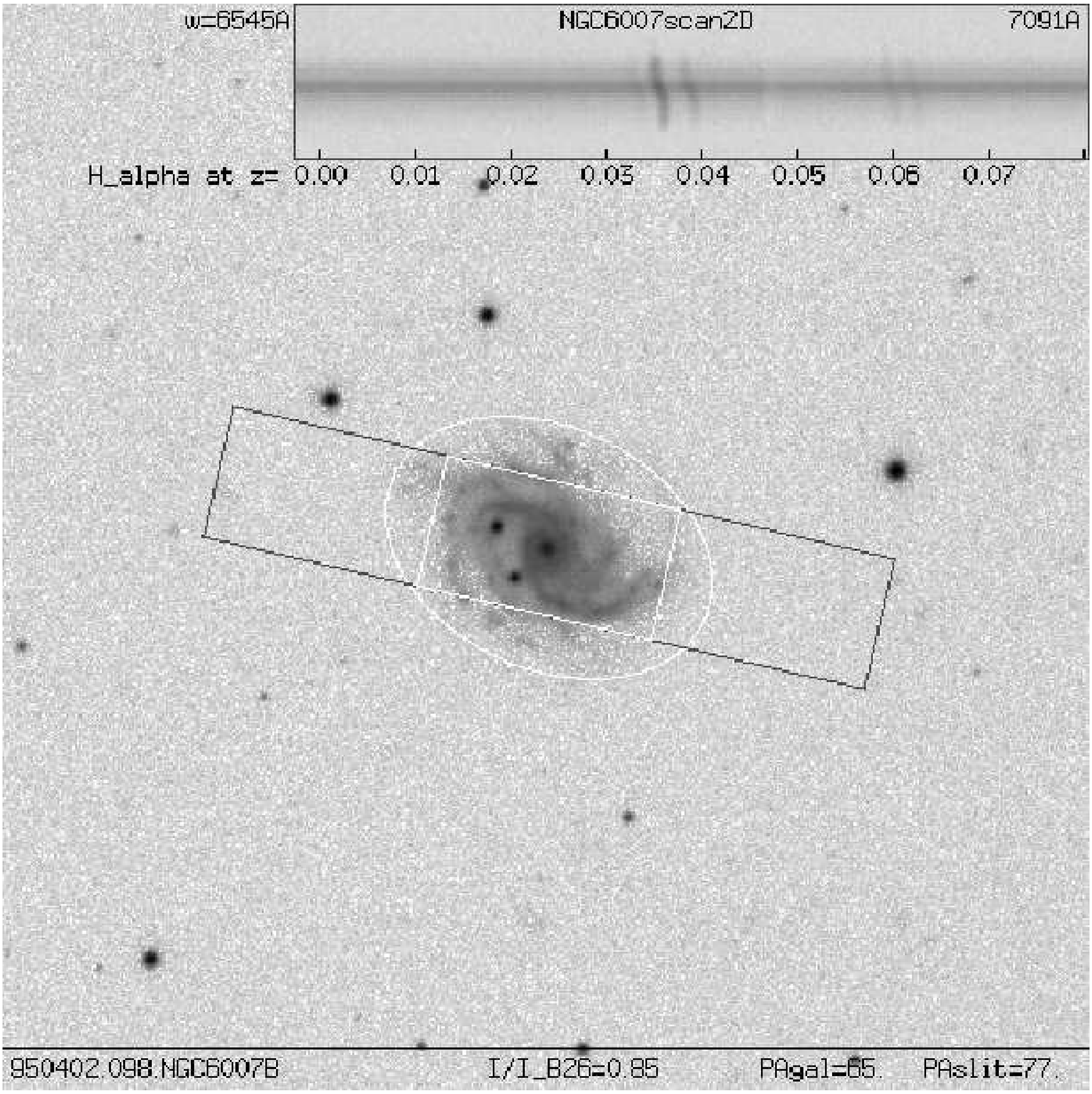
(Burles, Nollett & Turner, 1999, Fig. 1)

Best evidence for mass in baryons, Ω_b: primordial nucleosynthesis.

$$\Omega_\mathrm{b} h^2 = 0.02 \pm 0.002 \qquad (7.6)$$

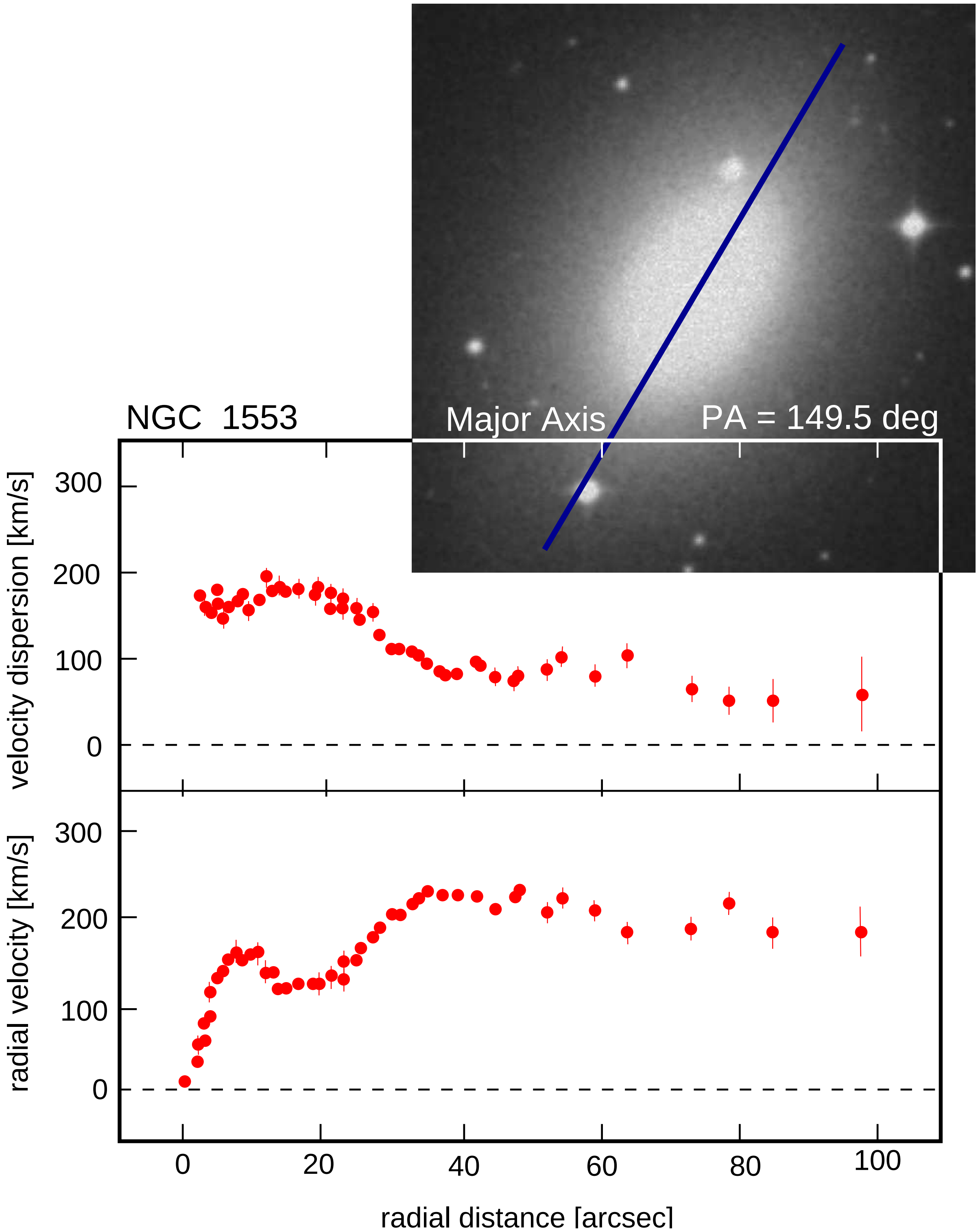
UWarwick

Galaxy Rotation Curves, I



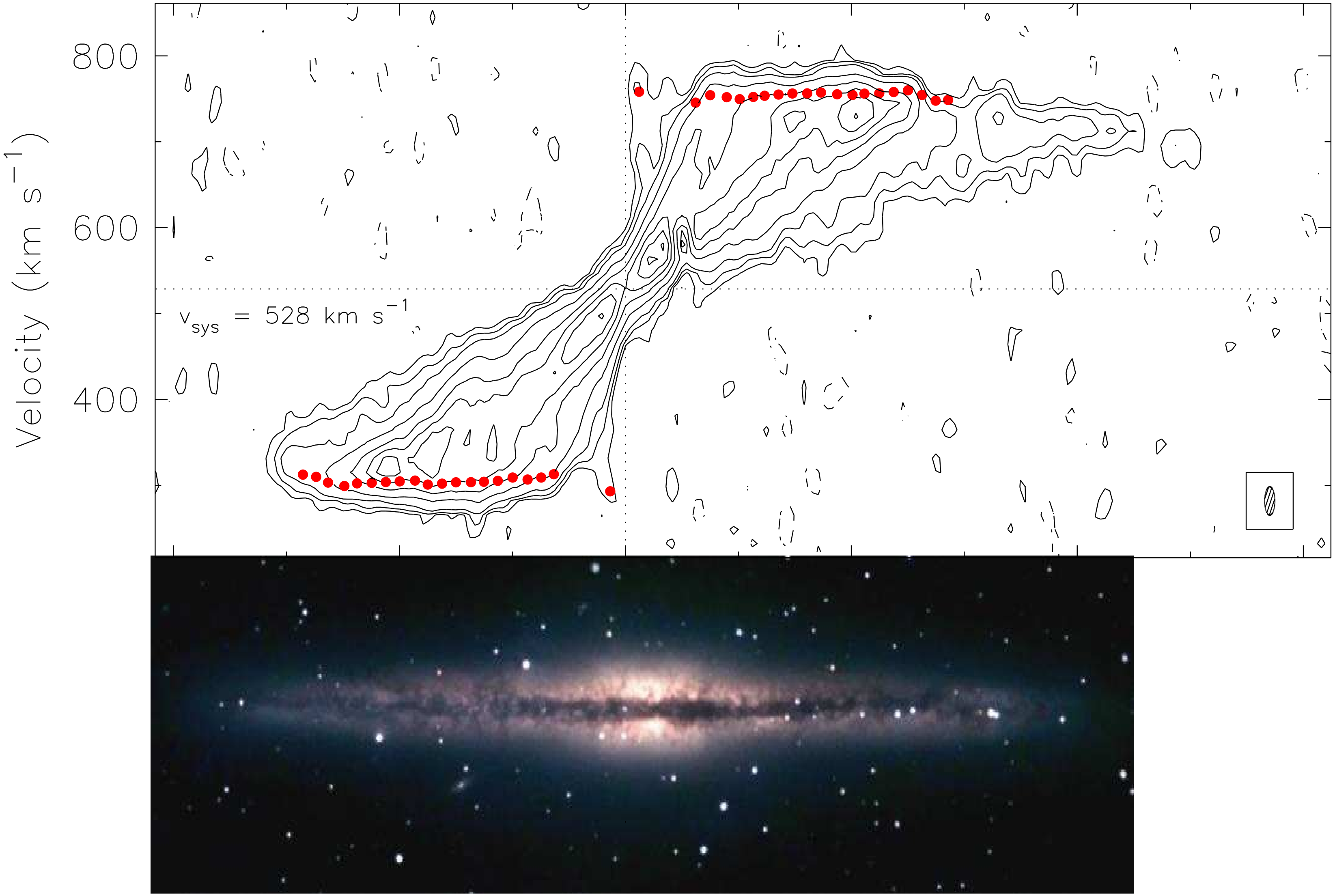
NGC 6007 (Jansen; `http://www.astro.rug.nl/~nfgs/`)

Galaxy Rotation Curves, II



NGC 1553 (S0) after Kormendy (1984, ApJ 286, 116)

Galaxy Rotation Curves, III



NGC 891 (Swaters et al., 1997, ApJ 491, 140 / Paul LeFevre, S&T Nov. 2002)

Galaxy Rotation Curves, IV

NGC 891, KPNO 1.3 m
Barentine & Esquerdo

Stellar motion due to mass within r:

$$\frac{GM(\leq r)}{r^2} = \frac{v_{\text{rot}}^2(r)}{r}$$
$$\Longrightarrow M(\leq r) = \frac{v_{\text{rot}}^2 r}{G}$$

therefore:

$v \sim$ const. $\Longrightarrow M(\leq r) \propto r$.

For disk in spiral galaxies, $I(r) = I_0 \exp(-r/h)$ such that

$$L(r < r_0) = I_0 \int_0^{r_0} 2\pi r \exp(-r/h)\,\mathrm{d}r$$
$$\propto h^2 - h(r+h)\exp(-r/h) \quad (7.7)$$

such that for $r \longrightarrow \infty$: $L(r < r_0) \to$ const..

If $M/L \sim$ const. $\Longrightarrow$ contradiction with observations!
(would expect $v \propto r^{-1/2}$)

Result for galaxies compared to stars

$$\left.\frac{M}{L}\right|_{\text{galaxies}} = 10\ldots20\,\frac{M_\odot}{L_\odot} \quad \text{vs.} \quad \left.\frac{M}{L}\right|_{\text{stars}} = 1\ldots3\,\frac{M_\odot}{L_\odot}$$

Only about 10% of the gravitating matter in universe radiates.

Galaxy Clusters, I

For mass of galaxy clusters, make use of the virial theorem:

$$E_{\text{kin}} = -E_{\text{pot}}/2 \tag{7.8}$$

in statistical equilibrium.

Measurement: assume isotropy, such that

$$\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle = 3 \langle v_\parallel^2 \rangle \tag{7.9}$$

assuming that velocity dispersion independent of m_i gives:

$$E_{\text{kin}} = \frac{1}{2} \sum_i m_i \mathbf{v}_i^2 = \frac{3}{2} M \langle v_\parallel^2 \rangle \tag{7.10}$$

where M total mass.

If cluster is spherically symmetric $\Longrightarrow$ Define weighted mean separation R_{cl}, such that

$$E_{\text{pot}} = \frac{GM^2}{R_{\text{cl}}} \tag{7.11}$$

From Eqs. (7.10) and (7.11):

$$M = \frac{3}{G} \langle v_\parallel^2 \rangle R_{\text{cl}} \tag{7.12}$$

Typical values: $v_\parallel \sim 1000\,\text{km}\,\text{s}^{-1}$, $R \sim 1\,\text{Mpc}$.

Derivation of the Virial Theorem

Assume system of particles, each with mass m_i. Acceleration on particle i:

$$\ddot{\mathbf{r}} = \sum_{j\neq i} \frac{Gm_j(\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \tag{7.13}$$

... scalar product with $m_i\mathbf{r}_i$

$$m_i\mathbf{r}_i \cdot \ddot{\mathbf{r}}_i = \sum_{j\neq i} \frac{Gm_im_j\mathbf{r}_i \cdot (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \tag{7.14}$$

... since

$$\frac{1}{2}\frac{\mathrm{d}^2\mathbf{r}_i^2}{\mathrm{d}t^2} = \frac{\mathrm{d}}{\mathrm{d}t}(\dot{\mathbf{r}}_i \cdot \mathbf{r}_i) = \ddot{\mathbf{r}}_i \cdot \mathbf{r}_i + \dot{\mathbf{r}}_i \cdot \mathbf{r}_i \tag{7.15}$$

... therefore Eq. (7.14)

$$\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}(m_i\mathbf{r}_i^2) - m_i\dot{\mathbf{r}}_i{}^2 = \sum_{j\neq i} \frac{Gm_im_j\mathbf{r}_i \cdot (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \tag{7.16}$$

Summing over all particles in the system gives

$$\frac{1}{2}\sum_i \frac{\mathrm{d}^2}{\mathrm{d}t^2}(m_i\mathbf{r}_i^2) - \sum_i m_i\dot{\mathbf{r}}_i{}^2 = \sum_i\sum_{j\neq i} \frac{Gm_im_j\mathbf{r}_i \cdot (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \tag{7.17}$$

$$= \frac{1}{2}\left(\sum_i\sum_{j\neq i} Gm_im_j\frac{\mathbf{r}_i \cdot (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_i - \mathbf{r}_j|^3} + \sum_j\sum_{i\neq j} Gm_jm_i\frac{\mathbf{r}_j \cdot (\mathbf{r}_i - \mathbf{r}_j)}{|\mathbf{r}_j - \mathbf{r}_i|^3}\right) \tag{7.18}$$

$$= \frac{1}{2}\left(\sum_i\sum_{j\neq i} Gm_im_j\frac{\mathbf{r}_i \cdot \mathbf{r}_j - \mathbf{r}_i^2}{|\mathbf{r}_i - \mathbf{r}_j|^3} + \sum_j\sum_{i\neq j} Gm_jm_i\frac{\mathbf{r}_j \cdot \mathbf{r}_i - \mathbf{r}_j^2}{|\mathbf{r}_j - \mathbf{r}_i|^3}\right) \tag{7.19}$$

$$= -\frac{1}{2}\sum_{\substack{i,j\\ i\neq j}} \frac{Gm_im_j}{|\mathbf{r}_i - \mathbf{r}_j|} \tag{7.20}$$

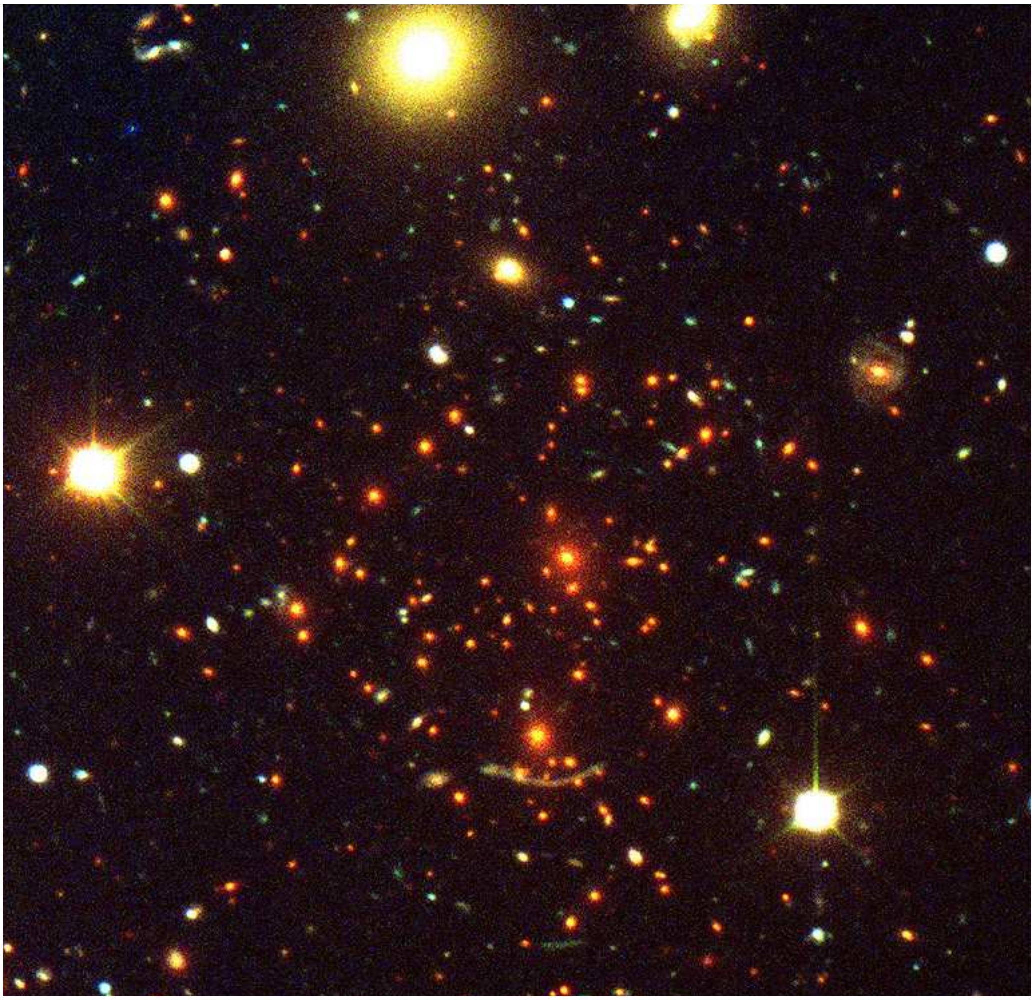
Thus, identifying the total kinetic energy, T, and the gravitational potential energy, U, gives

$$2T - U = \frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\sum_i m_i\mathbf{r}_i^2 = 0 \tag{7.21}$$

in statistical equilibrium.

Thus we find the virial theorem: $T = \frac{1}{2}|U|$

Galaxy Clusters, II



Abell 370 (VLT UT1+FORS)

More detailed analysis using more complicated mass models gives (Merritt, 1987):

$$\frac{M}{L} \sim 350h^{-1}\frac{M_\odot}{L_\odot} \tag{7.22}$$

would have expected $M/L = 10\ldots20$ as for galaxies

Dark matter is an important constituent in galaxy clusters

X-ray emission, I

X-ray emission from galaxy clusters gives mass to higher precision:

Assume gas in potential of galaxy cluster. Hydrostatic equilibrium:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM_r\rho}{r^2} \tag{7.23}$$

Pressure from equation of state:

$$P = nkT = \frac{\rho kT}{\mu m_\mathrm{H}} \tag{7.24}$$

where m_H: mass of H-atom, μ mean molecular weight of gas ($\mu = 0.6$ for fully ionized).
Eq. (7.24) gives

$$\frac{\mathrm{d}P}{\mathrm{d}r} = \frac{k}{\mu m_\mathrm{H}}\left(T\frac{\mathrm{d}\rho}{\mathrm{d}r} + \rho\frac{\mathrm{d}T}{\mathrm{d}r}\right) = \frac{\rho kT}{\mu m_\mathrm{H}}\left(\frac{\mathrm{d}\log\rho}{\mathrm{d}r} + \frac{\mathrm{d}\log T}{\mathrm{d}r}\right) \tag{7.25}$$

Inserting into Eq. (7.23) and solving gives

$$M_r = -\frac{kTr^2}{G\mu m_\mathrm{H}}\left(\frac{\mathrm{d}\log\rho}{\mathrm{d}r} + \frac{\mathrm{d}\log T}{\mathrm{d}r}\right) \tag{7.26}$$

Cluster gas mainly radiates by bremsstrahlung emission, with a spectrum

$$\epsilon(E) \propto \left(\frac{m_\mathrm{e}}{kT}\right)^{1/2} g(E,T)\,N\,N_\mathrm{e}\,\exp\left(-\frac{E}{kT}\right) \tag{7.27}$$

where N: number density of nuclei, $g(E,T)$: Gaunt factor (roughly constant).
$\Longrightarrow T$ from X-ray spectrum, N from measured flux $\Longrightarrow M_r$.

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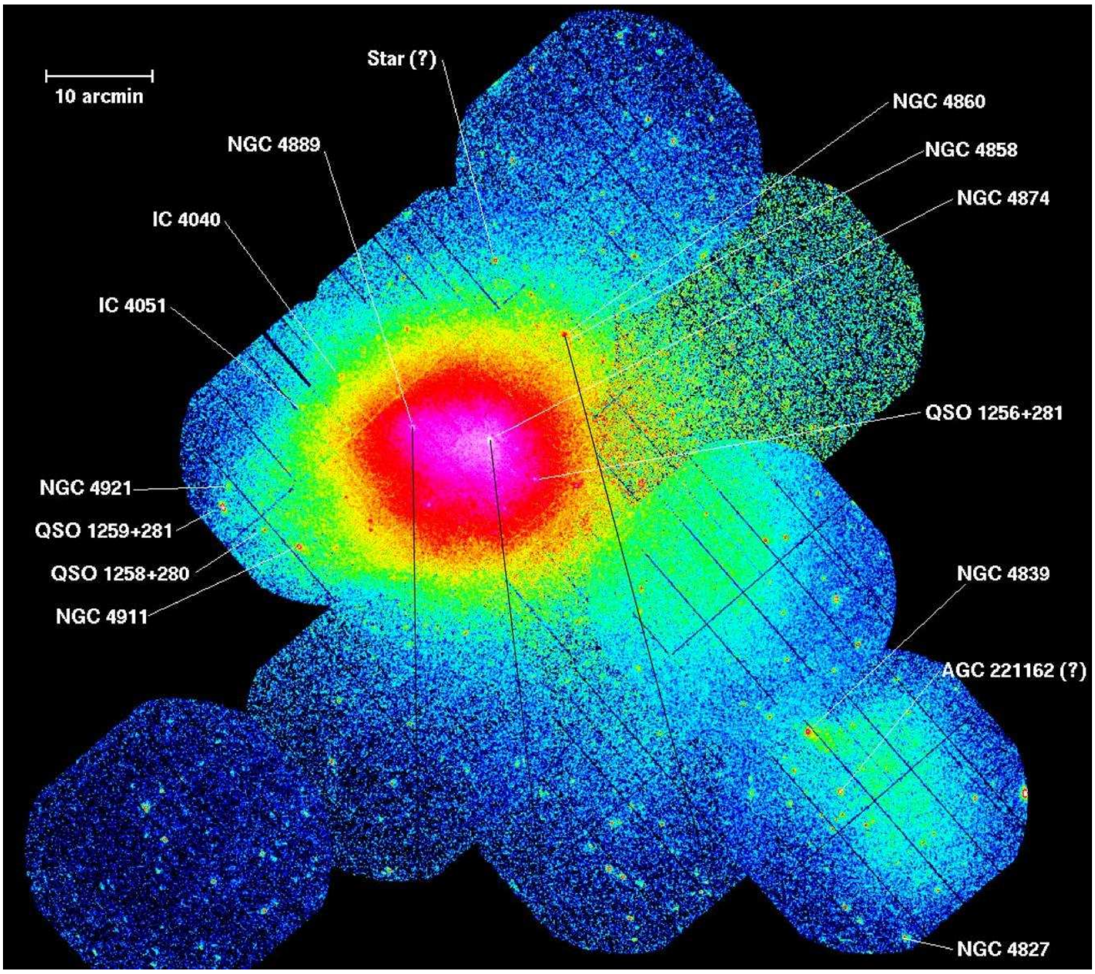
X-ray emission, II



XMM-Newton, EPIC-pn

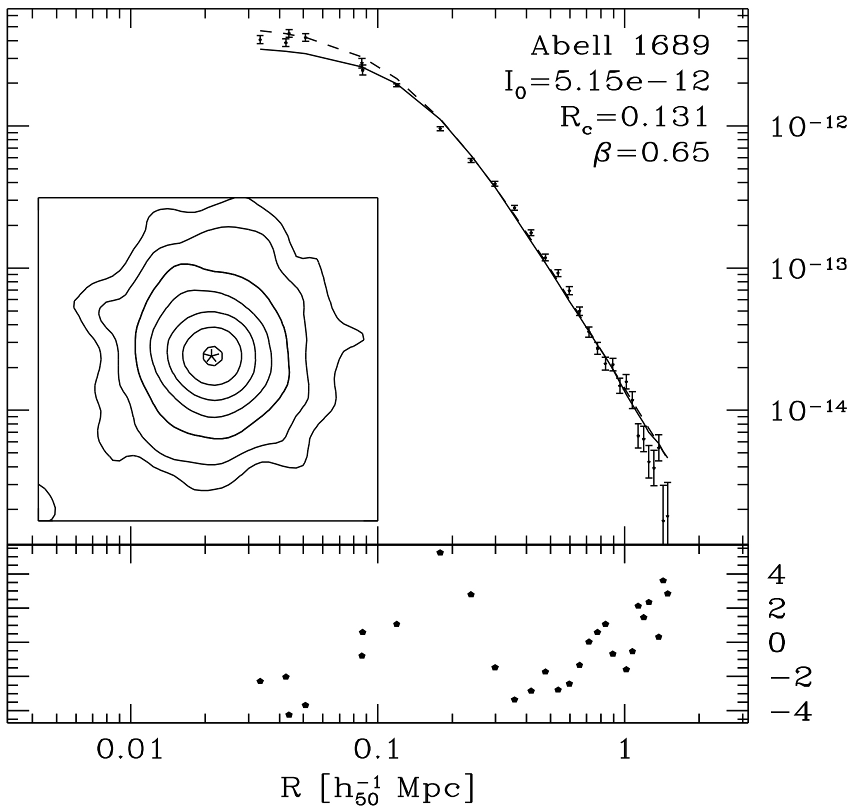
Result for Coma:

$$\frac{M_{\mathrm{B}}}{M_{\mathrm{tot}}} = 0.01 + 0.05\, h^{-3/2} \tag{7.28}$$

Technical problems:

- see through cluster $\Longrightarrow$ integrate over line of sight, assuming spherical geometry.
- spherical geometry is assumed
- Gas cools by radiating was wrong ("cooling flow")

X-ray emission, III



(Mohr, Mathiesen & Evrard, 1999)

Generally: assume intensity profile from β-model,

$$\frac{I(r)}{I_0} = \left(1 + \left(\frac{r}{R_c}\right)^2\right)^{-3\beta+\frac{1}{2}} \tag{7.29}$$

and obtain T from fitting X-ray spectra to "shells" $\Longrightarrow$ technically complicated...

Summary for X-ray mass determination for 45 clusters (Mohr, Mathiesen & Evrard, 1999):

$$f_{\mathrm{gas}} = (0.07 \pm 0.002)h^{-3/2} \tag{7.30}$$

resulting in

$$\Omega_{\mathrm{m}} = \Omega_{\mathrm{b}}/f_{\mathrm{gas}} = (0.3 \pm 0.05)\, h^{-1/2} \tag{7.31}$$

Sunyaev-Zeldovich, I

Gas in cooling flow influences CMBR by Compton upscattering $\Longrightarrow$ Sunyaev-Zeldovich effect.

Derivation of following formulae follows from Fokker-Planck equation and Kompaneets equation, see, e.g., Peacock (1999, p. 375ff.).

Compton y-parameter (=optical depth for Compton scattering):

$$y = \int \left(\frac{kT_\mathrm{e}}{m_\mathrm{e}c^2}\right) \sigma_\mathrm{T} N_\mathrm{e}\,\mathrm{d}l \qquad (7.32)$$

Intensity change in Rayleigh-Jeans regime due to Compton upscattering:

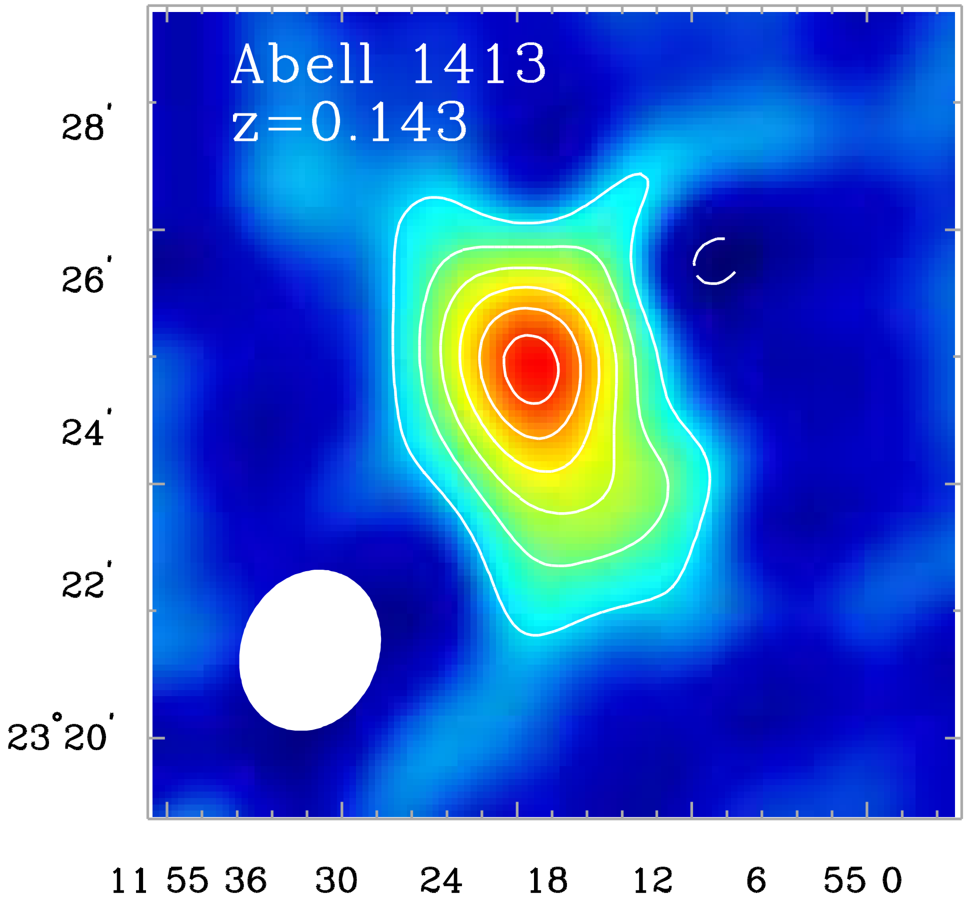
$$\frac{\Delta I}{I} = -2y \sim 10^{-4} \qquad (7.33)$$

(for typical parameters).

$\Longrightarrow$ Measure of $\int N_\mathrm{e} T_\mathrm{e}\,\mathrm{d}l \Longrightarrow$ Mass!

T is known from X-ray spectrum.

Sunyaev-Zeldovich, II



(decrement from 3 K background, Carlstrom et al., 2000, Fig. 3)

SZ analysis gives gas fraction for 27 clusters

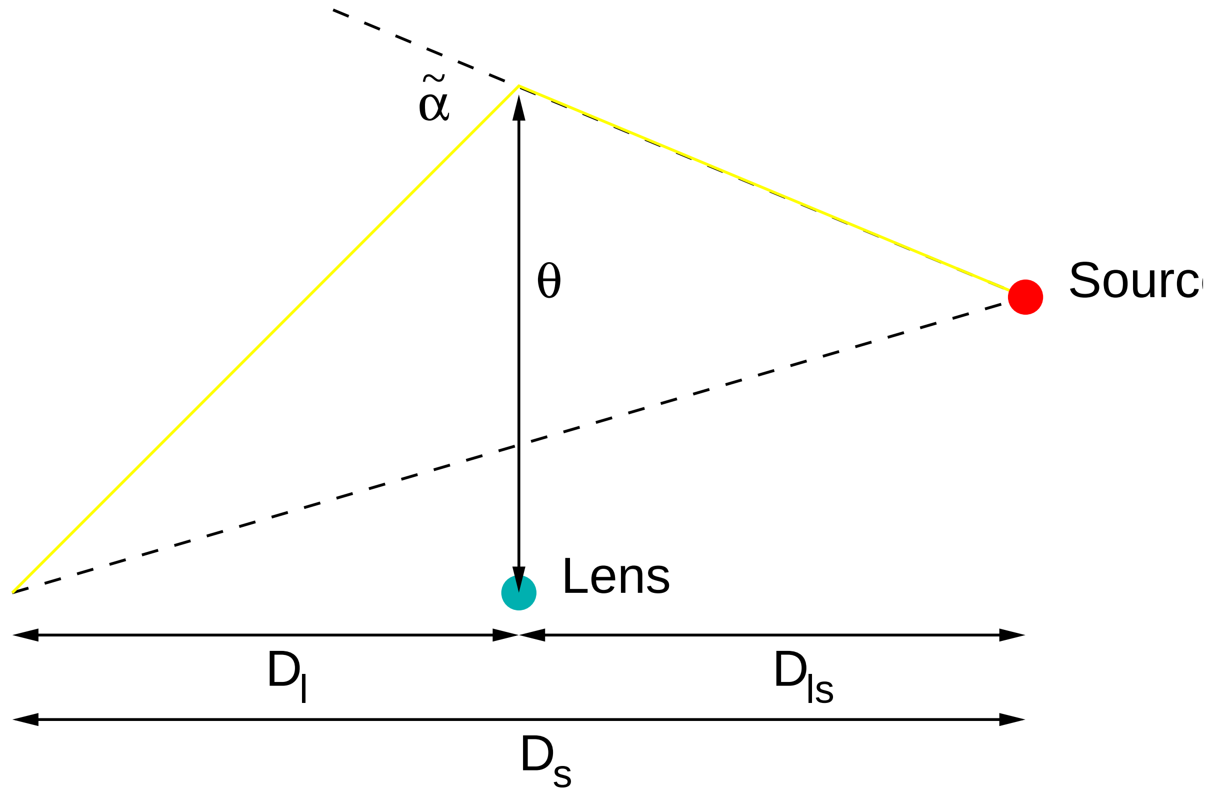
$$f_{\mathrm{gas}} = (0.06 \pm 0.006)h^{-3/2} \tag{7.34}$$

remarkably similar to X-ray result $\Longrightarrow$ clumping of gas does not influence results! (SZ only traces real gas…)

f_{gas} translates to

$$\Omega_{\mathrm{m}} = (0.25 \pm 0.04)\, h^{-1} \tag{7.35}$$

Gravitational Lenses, I



(after Longair, 1998, Fig. 4.8a)

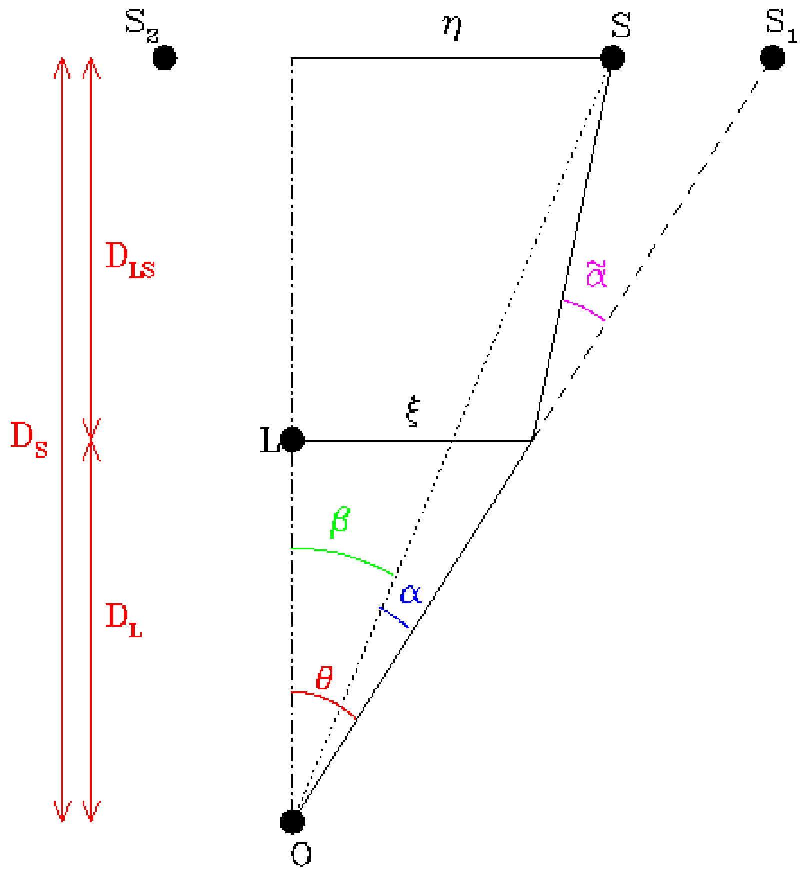
GR: Angular deflection due to mass M:

$$\tilde{\alpha} = \frac{4GM}{\theta c^2} = \frac{2}{c^2} \cdot \frac{2GM}{\theta} \tag{7.36}$$

where θ distance of closest approach (*twice* classical result).

Measurement of deflection from solar eclipse 1919: most convincing observational evidence for reality of GR.

Gravitational Lenses, II



Wambsganss, 1998, Fig. 3

In the small angle approximation:

$$\theta D_\mathrm{s} = \beta D_\mathrm{s} + \tilde{\alpha} D_\mathrm{ls} \tag{7.37}$$

defining the reduced deflection angle,

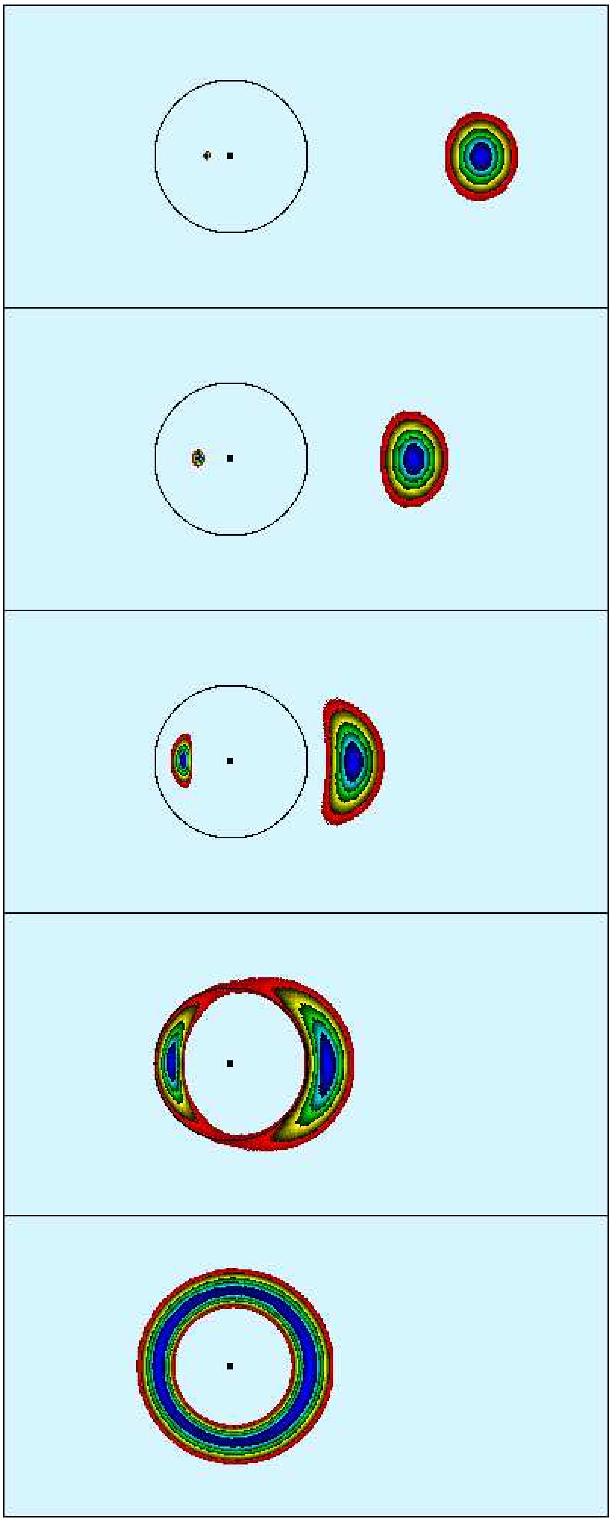
$$\alpha = \frac{D_\mathrm{ls}}{D_\mathrm{s}} \tilde{\alpha} \tag{7.38}$$

gives the lens equation

$$\beta = \theta - \alpha = \theta - \frac{D_\mathrm{ls}}{D_\mathrm{l} D_\mathrm{s}} \cdot \frac{4GM}{c^2 \theta} = \theta - \frac{1}{D} \cdot \frac{4GM}{c^2 \theta} \tag{7.39}$$

(last expression valid for a point-mass)

Gravitational Lenses, III



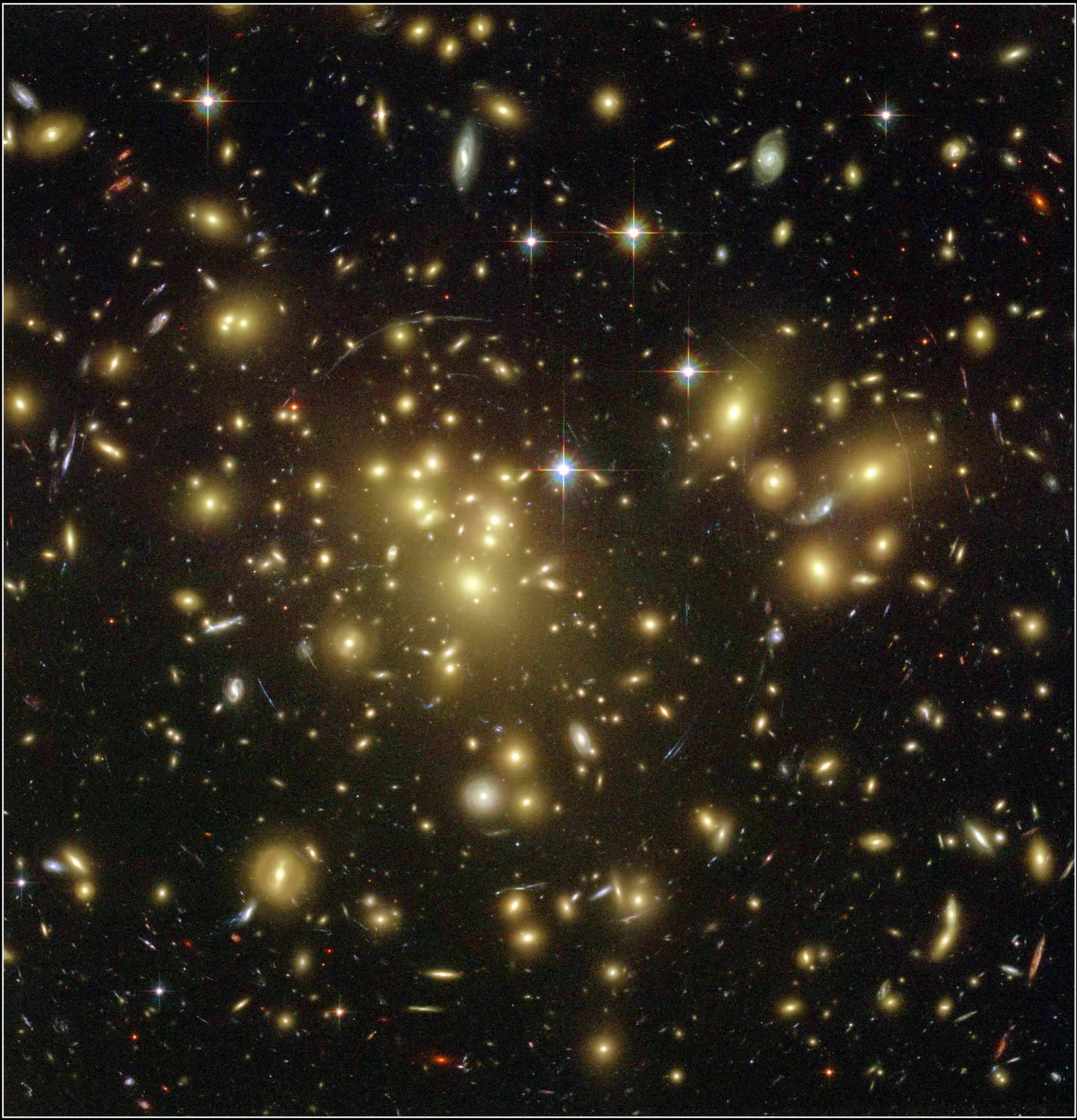
Einstein ring: source directly behind lens,
Obtain radius by setting $\beta = 0$ in lens-equation Eq. (7.39):

$$\theta_\mathrm{E}^2 = \frac{4GM}{c^2}\frac{1}{D} \qquad (7.40)$$

i.e.,

$$\theta_\mathrm{E} = 98.9'' \left(\frac{M}{10^{15}\,M_\odot}\right)^{1/2} \frac{1}{(D/1\,\mathrm{Gpc})^{1/2}} \qquad (7.41)$$

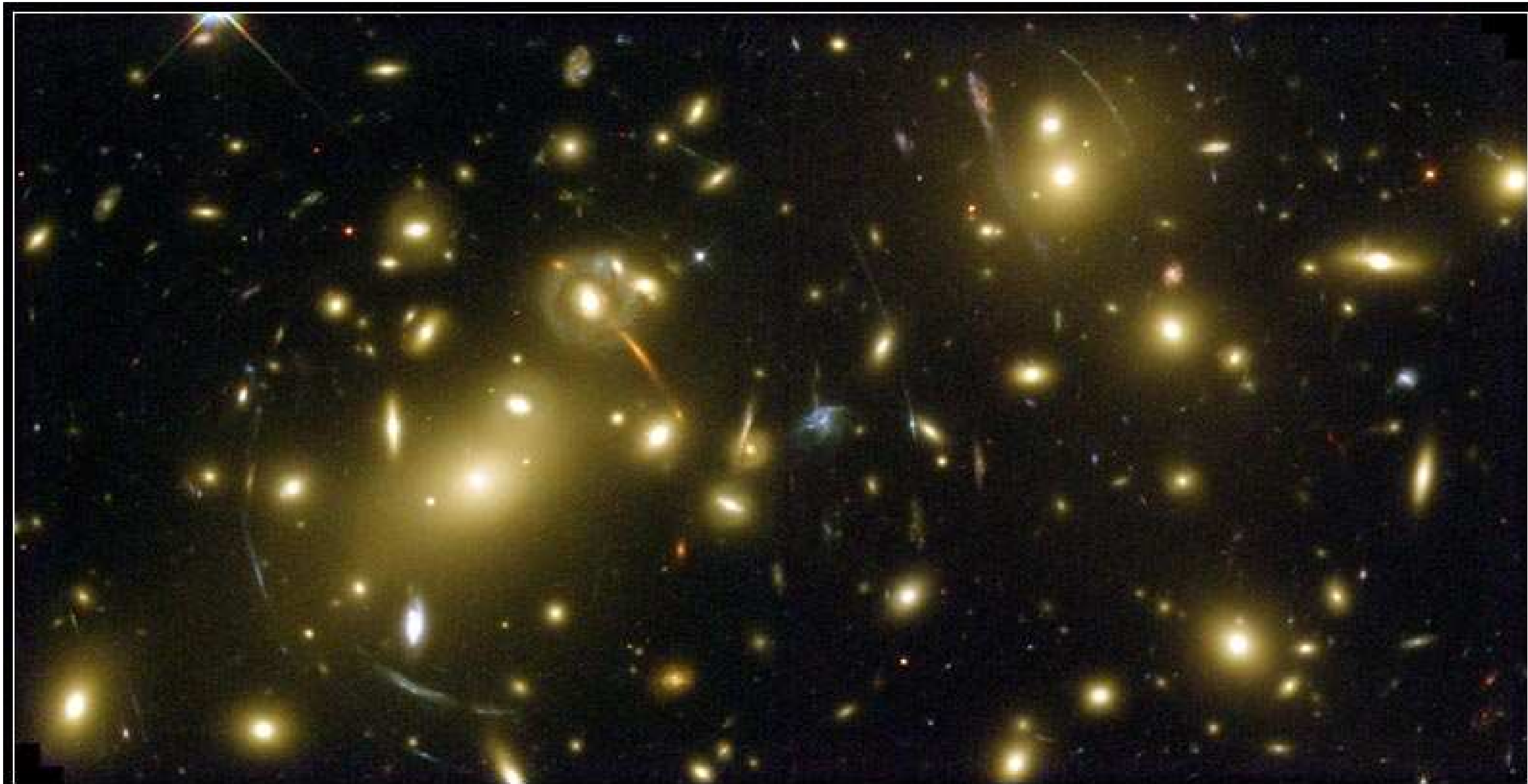
Mass measurements possible by observing "giant luminous arcs" and Einstein rings.



Galaxy Cluster Abell 1689
Hubble Space Telescope • Advanced Camera for Surveys

NASA, N. Benitez (JHU), T. Broadhurst (The Hebrew University), H. Ford (JHU), M. Clampin(STScI), G. Hartig (STScI), G. Illingworth (UCO/Lick Observatory), the ACS Science Team and ESA
STScI-PRC03-01a

General results of mass determinations from lensing agree with other methods.



Galaxy Cluster Abell 2218 **HST • WFPC2**

NASA, A. Fruchter and the ERO Team (STScI) • STScI-PRC00-08

Summary

So far, we have seen:

Photons:

$$\Omega_\gamma h^2 = 2.480 \times 10^{-5} \tag{7.42}$$

Neutrinos:

$$\Omega_\nu h^2 = 1.69 \times 10^{-5} \tag{7.43}$$

Baryons: (from nucleosynthesis)

$$\Omega_\mathrm{b} h^2 = 0.02 \tag{7.44}$$

where stars:

$$\Omega_\mathrm{stars} \sim 0.005 \ldots 0.01 \tag{7.45}$$

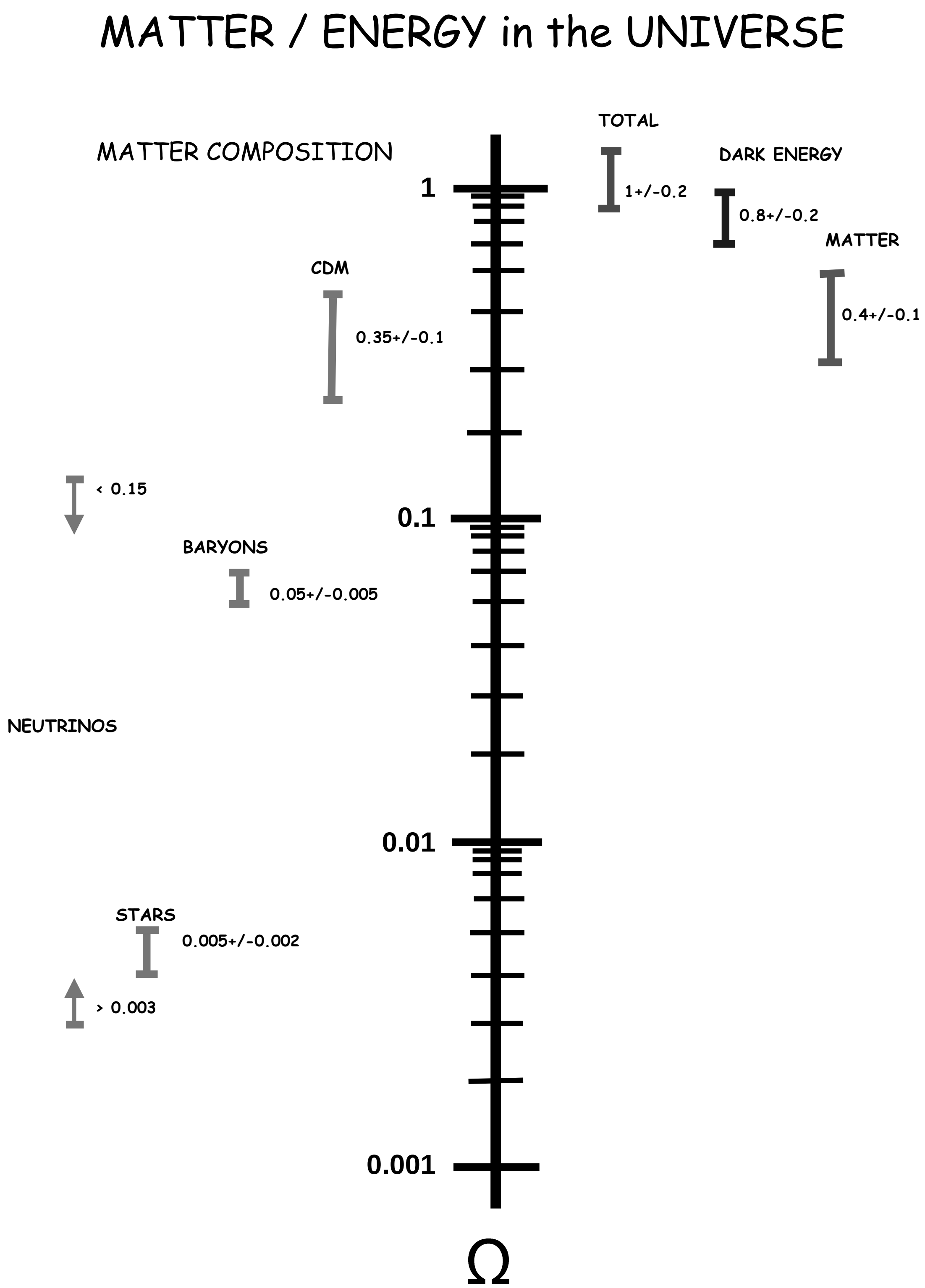
Baryons+dark matter: (from clusters)

$$\Omega_\mathrm{m} \sim 0.25 \tag{7.46}$$

(of which $\sim 10\%$ in baryons)

If we believe in $\Omega_\mathrm{total} \equiv 1 \Longrightarrow \Omega_\Lambda \sim 0.7$.

Summary



(Turner, 1999, Fig. 1, numbers slightly different to ours. . .)

Introduction

Clusters and galaxies: $\Omega_{\mathrm{m}} \sim 0.3$, but for baryons $\Omega_{\mathrm{b}} \sim 0.02 \Longrightarrow$ Rest of gravitating material is dark matter.

$\Longrightarrow$ Two dark matter problems:

$$\Omega_{\mathrm{m}} \xleftarrow{\text{nonbaryonic dark matter}} \Omega_{\mathrm{b}} \xleftarrow{\text{baryonic dark matter}} \Omega_{\mathrm{stars}}$$

baryonic dark matter= undetected baryons:

- diffuse hot gas?
- MACHOs (Massive compact halo objects; white dwarfs, neutron stars, black holes, brown dwarfs, jupiters,. . .)

nonbaryonic dark matter= exotic stuff:

- massive neutrinos
- axions
- neutralinos

Baryonic Dark Matter, I

Intra Cluster Gas:

Pro:

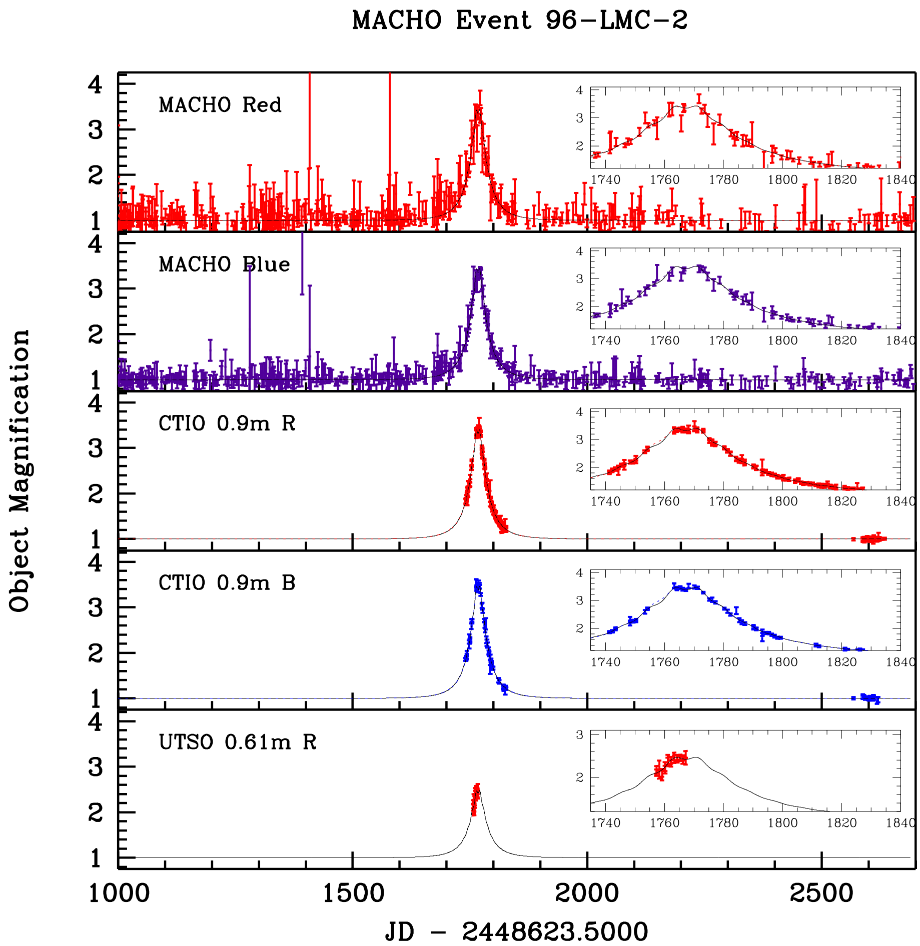
1. same location where the hot gas in clusters also found,
2. structure formation suggests most baryons are *not* in structures today

Contra:

1. 90% of the universe is *not* in clusters...
2. gas has not been detected at any wavelength

If gas cold enough, would not expect it to be detectable, so point 2 is not really valid.

Baryonic Dark Matter, II



(Alcock et al., 2001, Fig. 2)

MACHOS:

Pro:

1. detected by microlensing towards SMC and LMC (see figure) $\Longrightarrow$ MW halo consists of 50% WD

Contra:

1. possible "self-lensing" (by stars in MW or SMC/LMC; confirmed for a few cases)
2. where are white dwarfs?
3. WD formation rate too high (100 year^{-1} Mpc^{-3})

Nonbaryonic Dark Matter

Nonbaryonic dark matter:

Requirements:

- gravitating
- non-interacting with baryons

$\Longrightarrow$ Grab-box of elementary particle physics:

1. Neutrinos with non-zero mass

   Pro: It exists, mass limits are a few eV, need only $\langle m_\nu \rangle \sim 10\,\mathrm{eV}$

   Contra: ν are relativistic $\Longrightarrow$ Hot dark matter $\Longrightarrow$ Forces top down structure formation, contrary to what is believed to have happened.

2. Axion

   (=Goldstone boson from QCD, invented to prevent strong CP violation in QCD; $m \sim 10^{-5...-2}\,\mathrm{eV}$)

   Pro: It *could* exist, would be in Bose-Einstein condensate due to inflation ($\Longrightarrow$ Cold dark matter!), might be detectable in the next 10 years

   Contra: We do not know it exists...

3. Neutralino or other WIMPs (weakly interacting massive particles; masses $m \sim \mathrm{GeV}$)

   Pro: Also is CDM

   Contra: We do not know they exist...

Friedmann with $\Lambda \neq 0$, I

$\Longrightarrow$ Need to study cosmology with $\Lambda \neq 0$.

Reviews: Carroll, Press & Turner (1992), Carroll (2000)

Friedmann equation with $\Lambda \neq 0$:

$$H^2(t) = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{R^2} + \frac{\Lambda}{3} \qquad (6.136)$$

And define the Ω's:

$$\Omega_\mathrm{m} = \frac{8\pi G\rho_\mathrm{m}}{3H_0^2} \qquad (7.47)$$

$$\Omega_\Lambda = \frac{\Lambda c^4}{3H_0^2} \qquad (6.120)$$

$$\Omega_k = -\frac{k}{R_0^2 H_0^2} \qquad (7.48)$$

Because of Eq. (6.136),

$$\Omega_\mathrm{m} + \Omega_\Lambda + \Omega_k = \Omega + \Omega_k = 1 \qquad (7.49)$$

Friedmann with $\Lambda \neq 0$, II

It is easier to work with the dimensionless scale factor,

$$a = \frac{R(t)}{R_0} \tag{4.30}$$

$\Longrightarrow$ Friedmann:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\frac{\rho_{\mathrm{m},0}}{a^3} - \frac{k}{a^2 R_0^2} + \frac{\Lambda}{3} \tag{7.50}$$

since $\rho_\mathrm{m} = \rho_{\mathrm{m},0} a^{-3}$ (Eq. 4.67).
Inserting the Ω's

$$\left(\frac{\dot{a}/H_0}{a}\right)^2 = \frac{\Omega_\mathrm{m}}{a^3} + \frac{1-\Omega_\mathrm{m}-\Omega_\Lambda}{a^2} + \Omega_\Lambda \tag{7.51}$$

Substituting the time in units of todays Hubble time,

$$\tau = H_0 \cdot t \tag{7.52}$$

results in

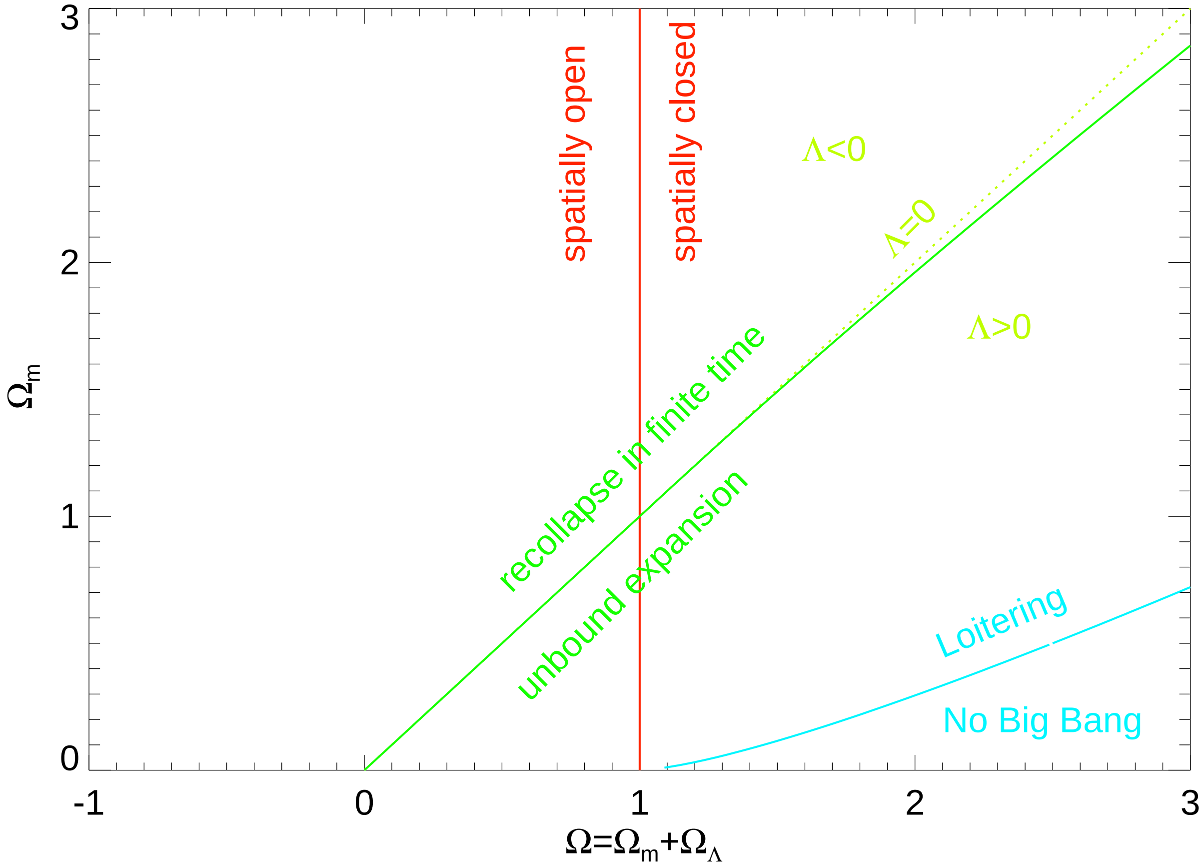
$$\left(\frac{\mathrm{d}a}{\mathrm{d}\tau}\right)^2 = 1 + \Omega_\mathrm{m}\left(\frac{1}{a} - 1\right) + \Omega_\Lambda(a^2 - 1) \tag{7.53}$$

with the boundary conditions

$$a(\tau = 0) = 1 \quad \text{and} \quad \left.\frac{\mathrm{d}a}{\mathrm{d}\tau}\right|_{\tau=0} = 1 \tag{7.54}$$

For most combinations of Ω_m and Ω_Λ, need to solve numerically.

Friedmann with $\Lambda \neq 0$, III

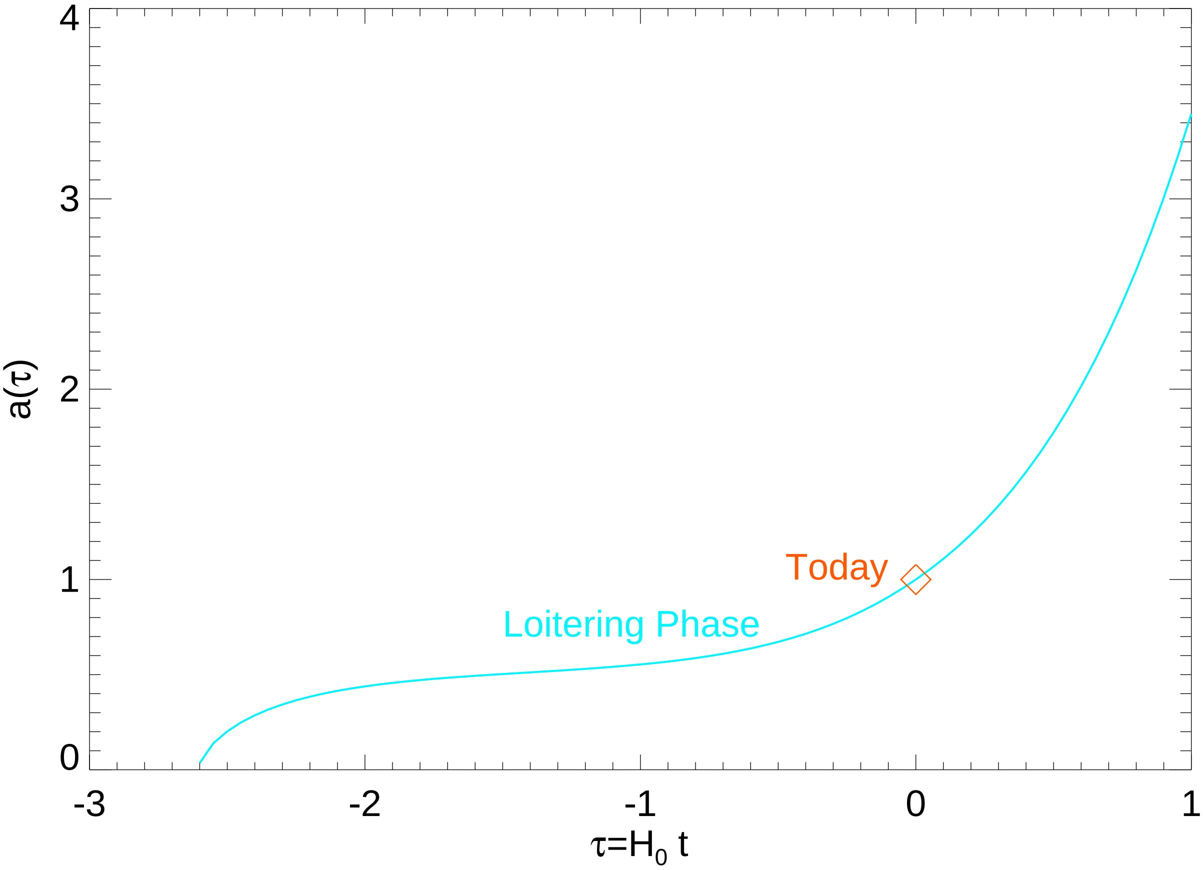


(after Carroll, Press & Turner, 1992, Fig. 1)

With Λ, evolution of universe is more complicated than without:

- unbound expansion possible for $\Omega < 1$,
- For Ω_Λ large: no big bang!
- For Ω_Λ large: possible "loitering phase"

$\Omega_\Lambda > 1$, I



"Loitering universe" with $\Omega_\mathrm{m} = 0.55$, $\Omega_\Lambda = 2.055$

For large Ω_Λ: contraction from $+\infty$ and reexpansion

$\Longrightarrow$ no big bang.

For slightly smaller Ω_Λ: phase where $\dot{a} \sim 0$ in the past

$\Longrightarrow$ loitering universe.

Threshold for presence of turning-point (Carroll, Press & Turner, 1992, Eq. 12):

$$\Omega_\Lambda \geq \Omega_{\Lambda,\mathrm{thresh}} = 4\Omega_\mathrm{m} \left\{ C_\kappa \left[\frac{1}{3} C_\kappa^{-1} \left(\frac{1 - \Omega_\mathrm{m}}{\Omega_\mathrm{m}} \right) \right] \right\}^3 \quad (7.55)$$

where $\kappa = \mathrm{sgn}(0.5 - \Omega_\mathrm{m})$ and $C_\kappa(\theta)$ was defined in Eq. (4.25).

$\Omega_\Lambda > 1$, II

QSO at $z = 5.82$, courtesy SDSS

For $\Omega_\Lambda = \Omega_{\Lambda,\text{thresh}}$: turning-point, i.e., there is a minimal a.

Since

$$1 + z = \frac{1}{a} \tag{4.43}$$

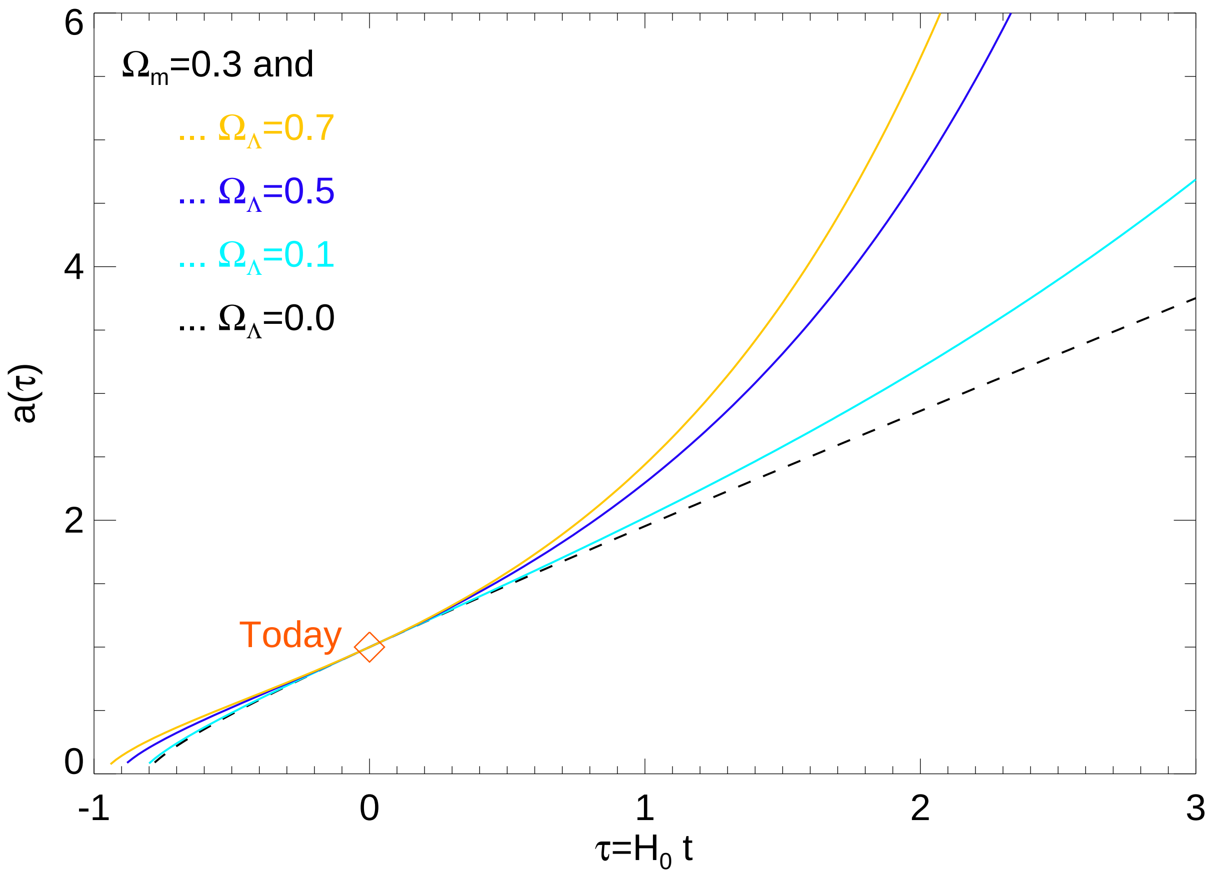
existence of turning-point $\Longrightarrow$ maximal possible z:

$$z \le 2C_\kappa \left(\frac{1}{3} C_\kappa^{-1} \left\{ \frac{1 - \Omega_\text{m}}{\Omega_\text{m}} \right\} \right) - 1 \tag{7.56}$$

(Carroll, Press & Turner, 1992, Eq. 14).

Since quasars observed with $z = 5.82$, this means that $\Omega_\text{m} < 0.007$, clearly not what is observed $\Longrightarrow$ $\Omega_\Lambda < 1$.

$\Omega_\Lambda < 1$



For $\Omega_\Lambda < 1$ evolution has two parts:

- matter domination, similar to earlier results
- Λ domination, exponential rise.

Exponential rise called by some workers the "second inflationary phase"...

Note accelerating effect of Ω_Λ!

$\Omega_\Lambda < 1$

Computation of age similar to $\Omega_\Lambda = 0$ case (see, e.g., Eq. 4.86), but generally only possible numerically.

Result:

Universes with $\Omega_\Lambda > 0$ are *older* than those with $\Omega_\Lambda = 0$.

This solves the age problem, that some globular clusters have age comparable to age of universe if $\Omega_\Lambda = 0$.

Analytical formula for age (Carroll, Press & Turner, 1992, Eq. 17):

$$t = \frac{2}{3H_0} \frac{\sinh^{-1}\left(\sqrt{(1-\Omega_a)/\Omega_a}\right)}{\sqrt{1-\Omega_a}} \tag{7.57}$$

for $\Omega_a < 1$, where

$$\Omega_a = 0.7\Omega_\mathrm{m} + 0.3(1-\Omega_\Lambda) \tag{7.58}$$

For $\Omega_\mathrm{m} = 0.3$, $\Omega_\Lambda = 0.7$, $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$: $t = 13.5\,\mathrm{Gyr}$.

Remember that for $\Omega_\mathrm{m} = 1$, $t = 3/2H_0$!

Luminosity Distance

Influence of Λ most prominent at large distances!
$\Longrightarrow$ Expect influence on Hubble Diagram.
$\Longrightarrow$ Need to find relation between measured flux, emitted luminosity, and redshift.

Assume source with luminosity L at comoving coordinate r, emitting isotropically into 4π sr.

At time of detection today, photons are

- on sphere with proper radius $R_0 r$,
- redshifted by factor $1 + z$,
- spread in time by factor $1 + z$.

$\Longrightarrow$ observed flux is

$$F = \frac{L}{4\pi R_0^2 r^2 (1+z)^2} \tag{7.59}$$

The luminosity distance is defined as

$$d_\mathrm{L} = R_0 \cdot r \cdot (1+z) \tag{7.60}$$

The computation of d_L is somewhat technical, one can show that (Carroll, Press & Turner, 1992):

$$d_\mathrm{L} = \frac{c}{H_0} |\Omega_k|^{-1/2} \cdot S_{-\mathrm{sgn}(\Omega_k)} \left\{ |\Omega_k|^{1/2} \int_0^z \left[(1+z)^2 (1+\Omega_\mathrm{m} z) - z(2+z)\Omega_\Lambda \right]^{1/2} \mathrm{d}z \right\} \tag{7.61}$$

Supernovae

Best way to determine Ω_Λ:

Type Ia supernovae

Remember: SN Ia = CO WD collapse. . . (Hoyle, Fowler, Colgate, Wheeler,. . .

The distance modulus is

$$m - M = 5\log\left(\frac{d_\mathrm{L}}{1\,\mathrm{Mpc}}\right) + 25 \qquad (7.62)$$

Use SNe as standard candles $\Longrightarrow$ Deviations from $d_\mathrm{L} \propto z$ indicative of Λ.

Two projects:

- High-z Supernova Team (STSCI, Riess et al.)
- Supernova Cosmology Project (LBNL, Perlmutter et al.)

Both find SNe out to $z \sim 1$.

Present mainly Perlmutter et al. results here, Riess et al. (1998) are similar.

Supernovae

Basic observations: easy:

- Detect SN in rise $\Longrightarrow$ CTIO 4 m
- Follow SN for $\sim$ 2–3 months with 2–4 m class telescopes, HST, Keck. . .

More technical problems in data analysis:

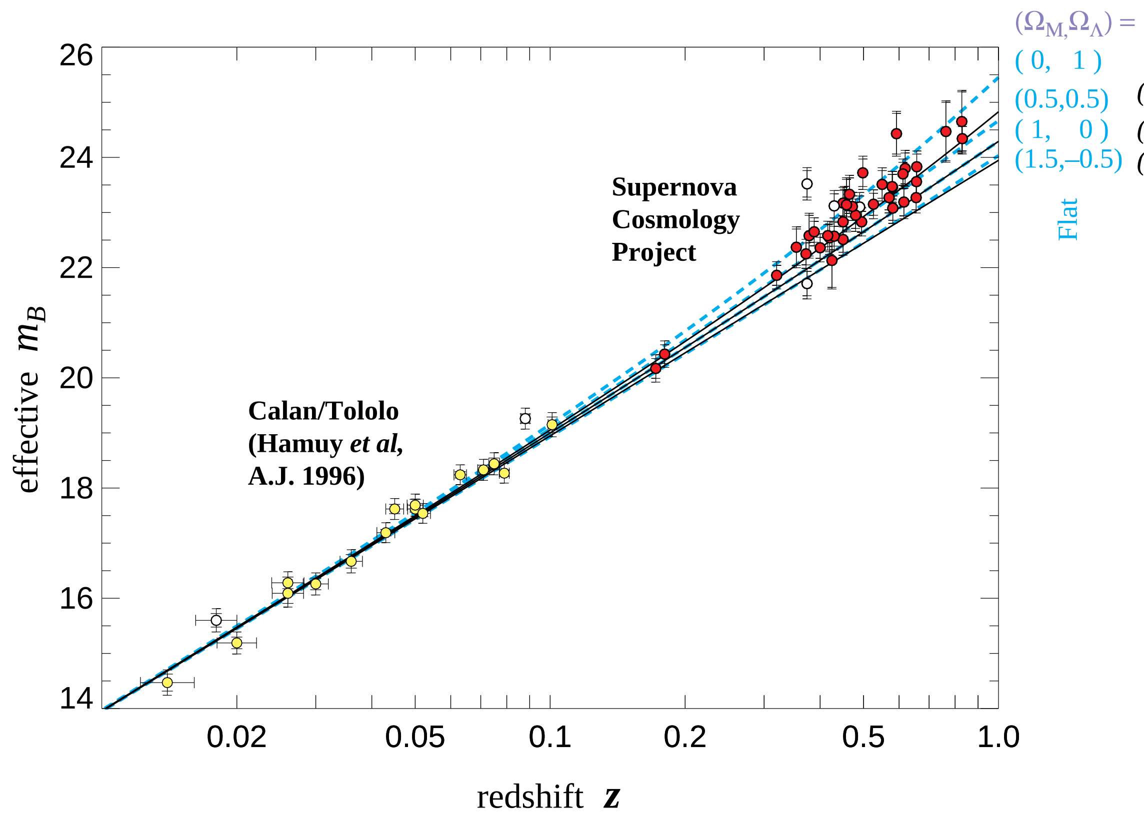
Conversion into source frame:

- Correction of photometric flux for redshift: "K-correction"
- Correct for time dilatation in SN light curve

Further things to check

- SN internal extinction
- Galactic extinction
- Galactic reddening
- Photometric cross calibration
- Peculiar motion of SN

Supernovae



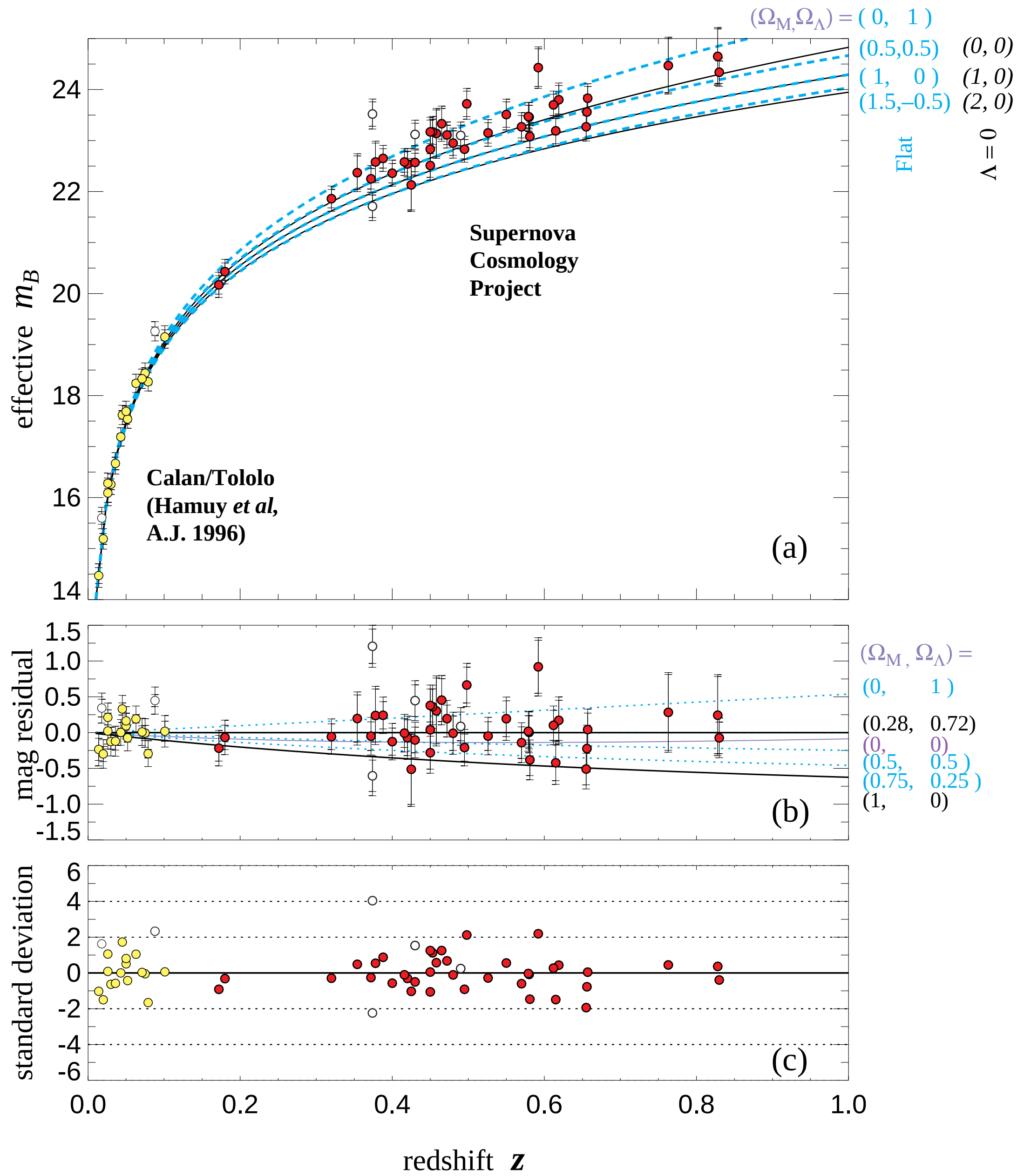
(Perlmutter et al., 1999, Fig. 1)

42 SNe from SCP, 18 low redshift from Calán/Tololo SN Survey

Vertical error bars: measurement uncertainty *plus* 0.17 mag intrinsic mag. dispersion

Horizontal error bars: 300 km s^{-1} peculiar velocity uncertainty

Supernovae

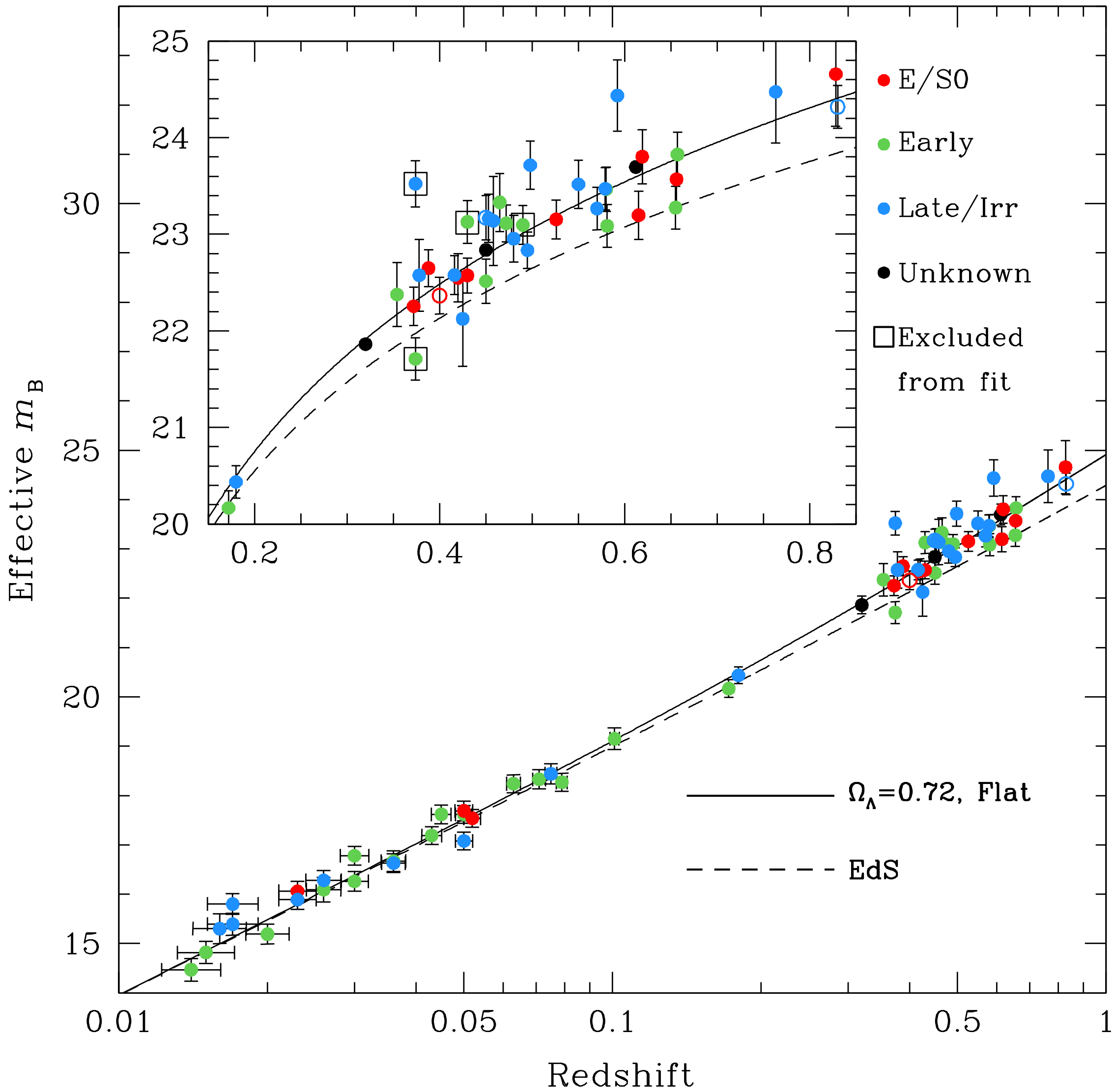


(Perlmutter et al., 1999, Fig. 2)

Best fit: $\Omega_{\text{m, flat}} = 0.28^{+0.09}_{-0.08}$, $\chi^2/\text{DOF} = 56/50$

corresponding best free fit: $(\Omega_\text{m}, \Omega_\Lambda) = (0.73, 1.32)$.

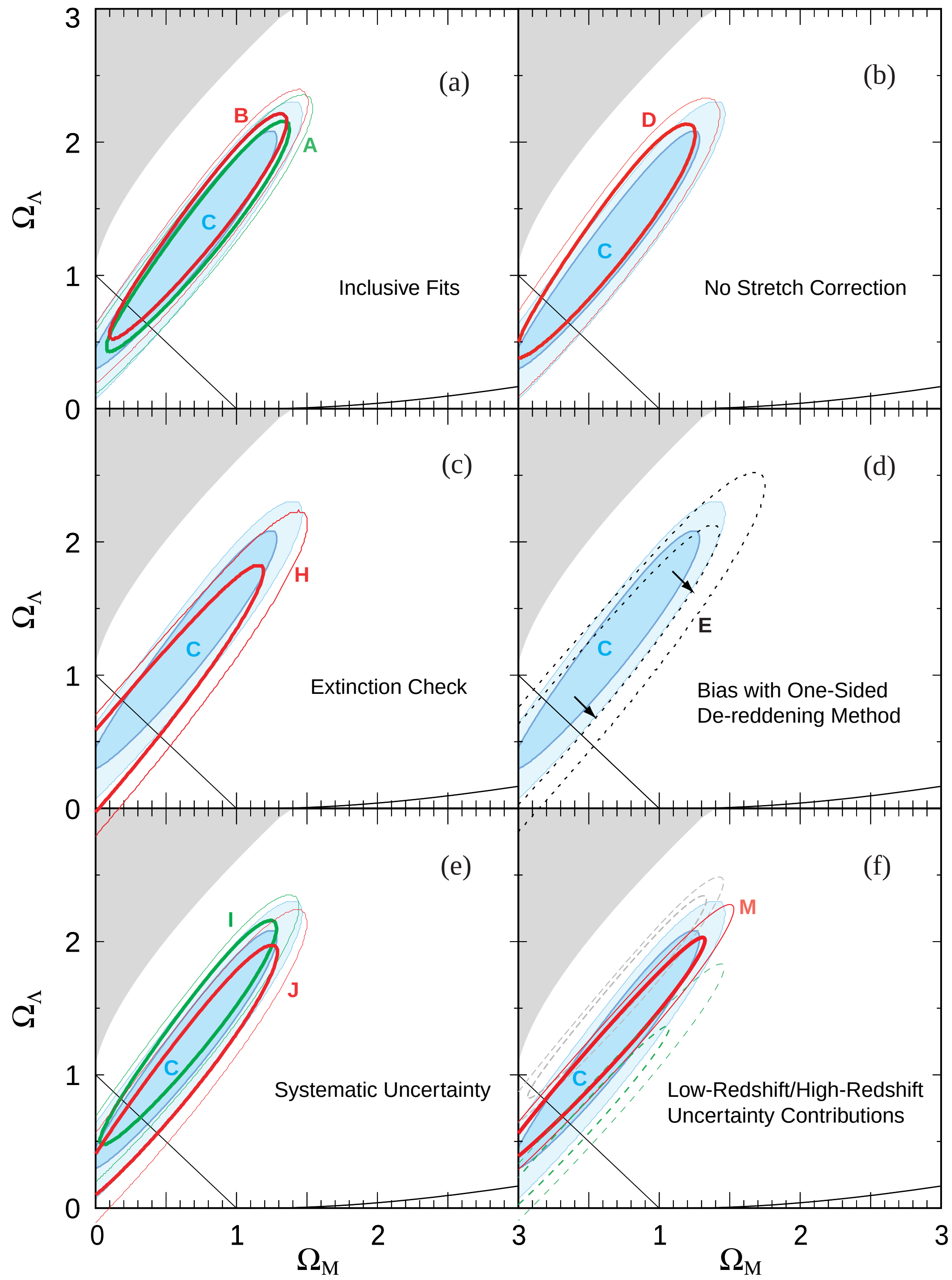
Supernovae



Sullivan et al., 2002

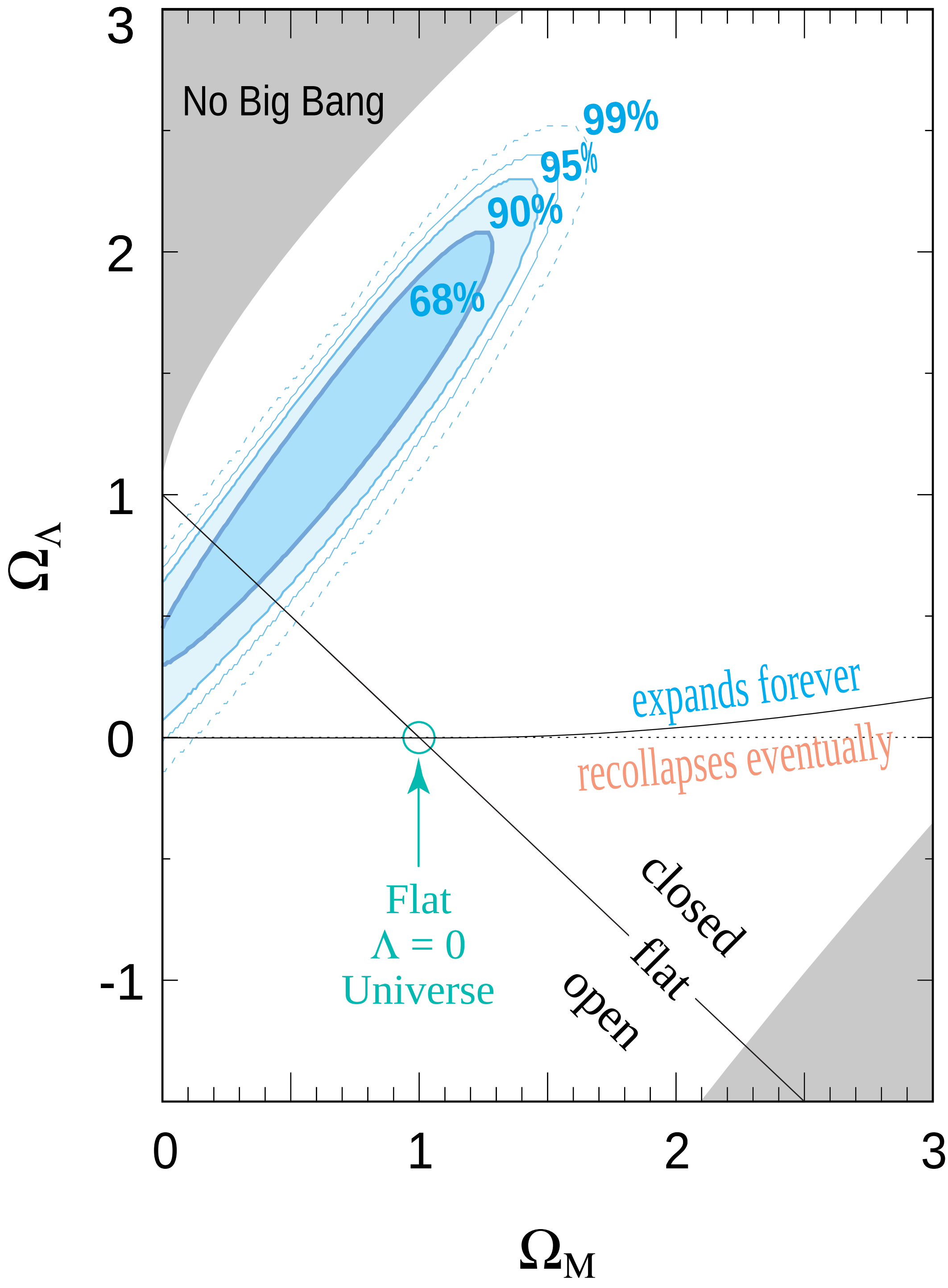
Updated 2002 Hubble diagram for SN Iae confirms Perlmutter 1999.

Supernovae



(68% and 90% confidence regions for sources of systematic error, Perlmutter et al., 1999, Fig. 5)

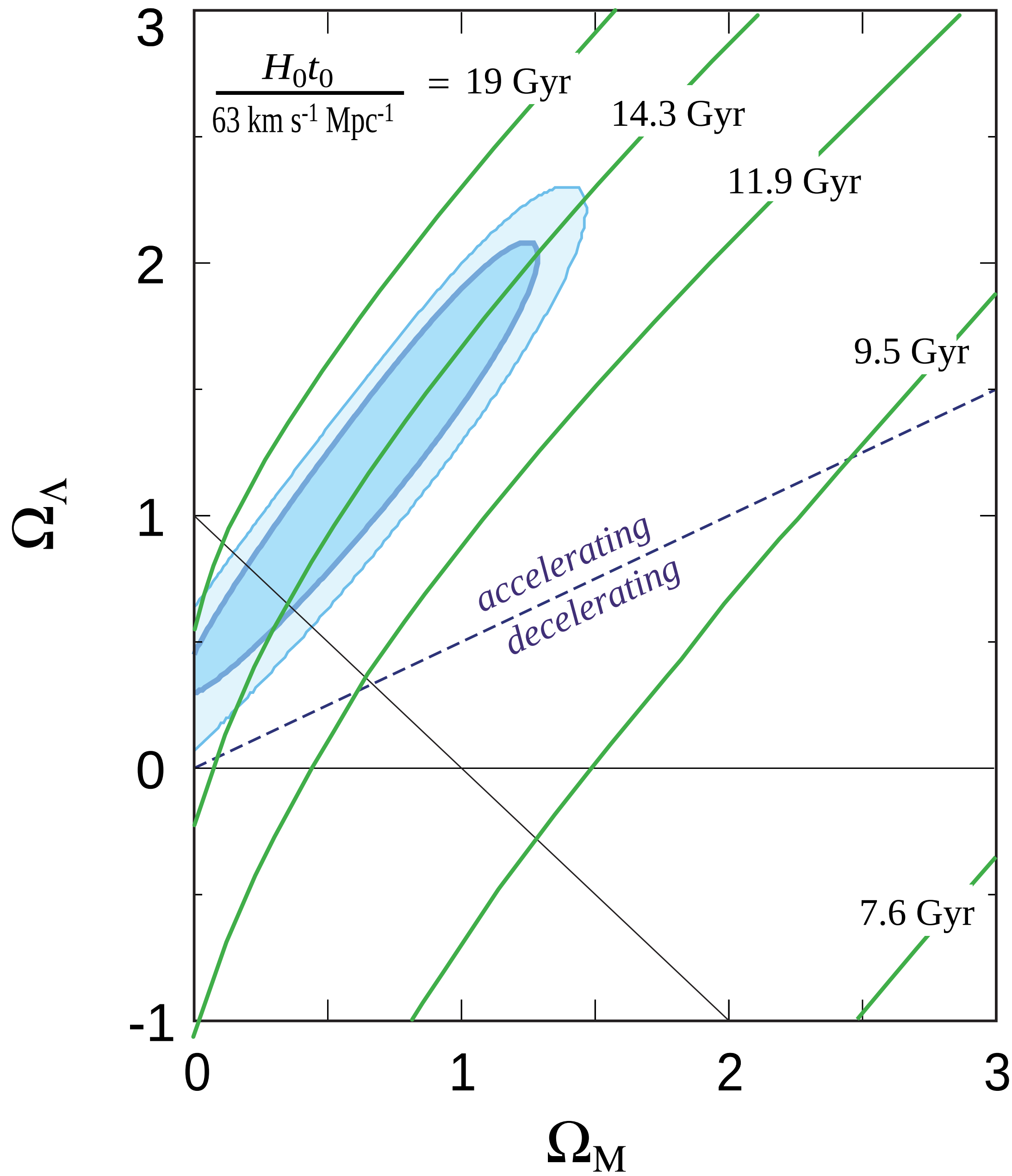
Supernovae



(Combined confidence region Perlmutter et al., 1999, Fig. 7 (lower right: universes that are younger than oldest heavy elements.))

UWarwick

Supernovae



(Perlmutter et al., 1999, Fig. 9)

Isochrones for age of universe for $H_0 = 63\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (for $h = 0.7$: age 10% smaller).

$\Longrightarrow$ Consistent with globular cluster ages!

Summary

For all practical purposes, the currently best values are

$$\Omega_{\mathrm{m}} \sim 0.3 \qquad \Omega_\Lambda = 0.7$$

Even if $\Omega \neq 1$:

$$\Omega_\Lambda \neq 0$$

And therefore

Baryons are an energetically unimportant constituent of the universe.

“The dark side of the force. . . ” :-)

Small print: Influences of

- Metallicity evolution
- Dust
- Malmquist bias
- ???

. . . these are believed to be small, however, see Drell, Loredo & Wasserman (2000) for a critique

Outlook

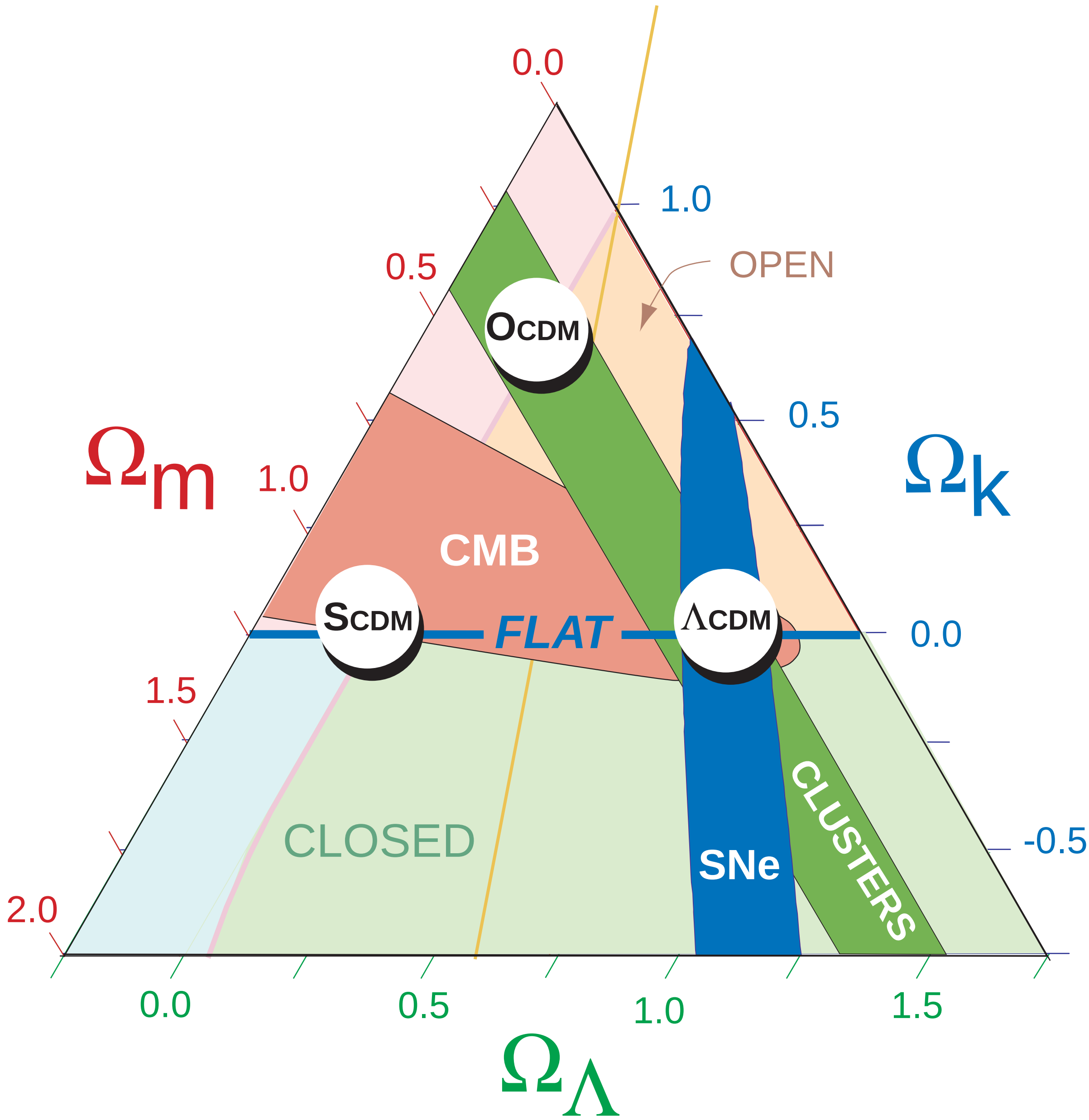
What is physical reason for $\Omega_\Lambda \neq 0$?
Currently discussed: quintessence: "rolling scalar field", corresponding to very lightweight particle ($\lambda_{\text{de Broglie}} \sim 1\,\text{Mpc}$), looks like time varying cosmological "constant".

Why? $\Longrightarrow$ More naturally explains why Ω_Λ so close to 0 (i.e., why matter and vacuum have so similar energy densities)

Motivated by string theory and M theory...

Still VERY SPECULATIVE, decision Λ vs. quintessence should be possible in next 5...10 years when new instruments become available.

Outlook



Bahcall et al.

Even better constraints come from combination of SNe data with structure formation.

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