Introduction to Astronomy II

Maria Obst

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Contents

Part I.
Stars

1. End Stages of Stellar Evolution

There are several possibilities for a star to find its end.

1.1. White Dwarfs

Characteristics:
- end stages of evolution of stars born with $M \lesssim 8\,M_\odot$
- typically $M \sim 0.6\,M_\odot$ (Chandrasekhar limit: $1.4\,M_\odot$)
- mainly consist of C and O
- radius $\sim$ Earth
- typical density $\rho \sim 10^6\,\mathrm{g\,cm^{-3}}$

White dwarfs are mainly found in globular clusters (oldest building blocks of the Galaxy. They are stabilized by the pressure of the degenerate electron gas and cannot shrink. The cooling of the ionic gas takes a very long time and ends at low temperatures in a crystal structure similar to a diamond. A typical white dwarf consists of a C/O core ($r \sim 8500\,\mathrm{km}$), a Helium shell ($r \sim 200\,\mathrm{km}$) and mostly also a thin H layer ($r \sim 10\,\mathrm{km}$). Is the H present in the white dwarf spectrum, it is called DA, otherwise only He is visible and it is called DB.

1.2. Supernovae

1.2.1. Classification

Supernovae can be classified via their spectral characteristics as well as via their light curve (Fig. 1).
SNe 2, 1b and 1c are never seen in elliptical galaxies, which are void of gas and have no new star formation. They are generally associated with spiral arms and H II regions, i.e. star forming regions. The progenitors of such supernovae are massive stars ($\gtrsim 8\,M_\odot$). They are called "core collapse supernovae".
Supernovae 1a however are seen in all types of galaxies, coming from both young and old stellar populations. Therefore the progenitors cannot be massive stars, such stars do no longer exist in older populations. The common model to explain these supernovae are accreting carbon-oxygen white dwarfs undergoing thermonuclear runaway.

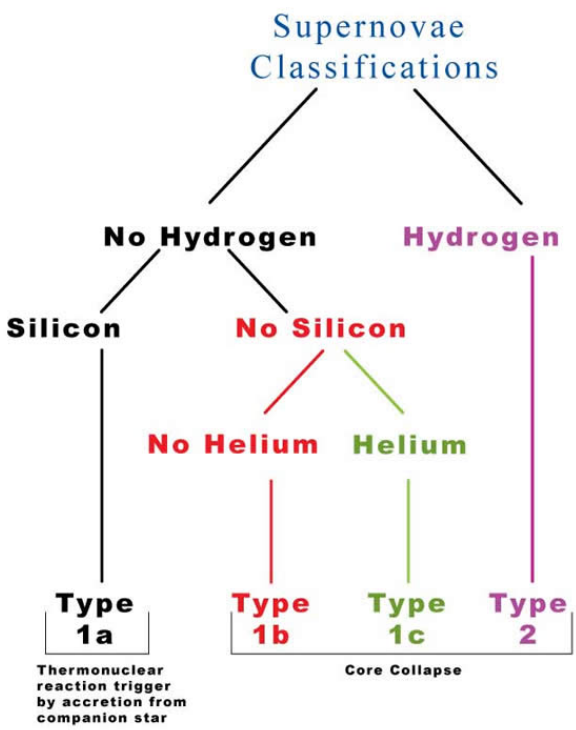


Figure 1: Supernova classification

1.2.2. Core Collapse Novae

The successive stages of nuclear burning in a massive star are
- H burning → He
- He burning → C, O, Ne, Mg
- C burning → Ne, Na, Mg
- Ne burning → O, Mg

- O burning → Si, P, S,...
- Si burning → Fe, Ni

The final state of such a massive ($> 10\,M_\odot$) star looks like an onion shell. As soon as there is a Fe core, fusion cannot produce energy any more and the core collapses:

- $t = 0\,\mathrm{s}$: Collapse of Fe core, triggered by electron capture and photodisintegration of Fe ($T \sim 10^{10}\,\mathrm{K}$, $\rho \sim 10^{10}\,\mathrm{g\,cm^{-3}}$)
  rebound: outer material rebounds off core, looses velocity because of photodisintegration and neutrino loss

- $t = 0.1\,\mathrm{s}$: proto-neutron star formed with $R \sim 30\,\mathrm{km}$, $M = 1.4\,M_\odot$, standing shock $\sim 150\,\mathrm{km}$ above neutron star

- $t = 0.1\,\mathrm{s}$ until $t = 0.2\,\mathrm{s}$: start to radiate $\sim 10^{53}\,\mathrm{erg\,s^{-1}}$ as neutrinos, triggers convection, heats material by depositing $10^{51}\,\mathrm{erg}$ (→ convection)

- $t = 0.2\,\mathrm{s}$: SN explosion is triggered

In this supernova explosion, there is so much energy that also heavier elements than Fe can be formed:
- **Neutron capture** on Fe → neutron-rich isotopes → β-decay
- **Slow process** (s-process): time for β-decay larger than for neutron capture (during AGB evolution)
- **Rapid process** (r-process): time for β-decay shorter than for neutron capture (during supernova explosion)

With SN calculations and fusion mechanism in stars, it is possible to explain the observed element abundance pattern of the solar system ($\mathrm{B^2FH}$).
The expanding envelope of the progenitor is powered by radioactive decay of short lived isotopes created in the core collapse. As this envelope is illuminated from inside and explosions are not necessarily symmetric, all kinds of filaments and structures can be seen of supernova remnants. Even layers that were cast of earlier can sometimes be seen after the supernova explosion.

1.3. Neutron Stars

Neutron stars form after the core collapse of massive stars. During the supernova, densitites get so high that neutronization sets in:

$$\mathrm{p} + \mathrm{e}^- \quad \rightarrow \quad \mathrm{n} + \nu_\mathrm{e} \tag{1}$$

1.3.1. Properties

- Pressure mainly through degenerate neutrons (similar to degenerate electrons for WD)
- Typical density: $\rho \sim 10^{14}\,\mathrm{g\,cm^{-3}}$ (nuclear densities)
- Typical radius: $10 \ldots 15\,\mathrm{km}$
- surface gravity: $\sim 10^{11}\times$ Earth
- Detailed structure not yet fully understood (supraconducting matter, suprafluidity, central composition unknown)

During the collapse, angular momentum is conserved (under the assumption that the explosion is symmetric). The total angular momentum of a homogeneous sphere is

$$J = I\omega \tag{2}$$

where

$$I = \frac{2}{5}MR^2 \tag{3}$$

The much lower radius of the newly formed neutron star put in this equation shows that neutron stars are extremely fast rotators (close to break-up speed).

1.3.2. Pulsars

Is the radiation of a rapidly rotating neutron star focused into a certain direction, it is called a pulsar. Light is emitted mostly in the direction of the magnetical field axis. The rotation axis however is not necessarily aligned to the magnetic field. If now in each rotation the lightbeam touches the Earth once (lighthouse effect), we observe pulses with the rotation period of the neutron star

$$P = \frac{2\pi R}{v_{\mathrm{rot}}} \tag{4}$$

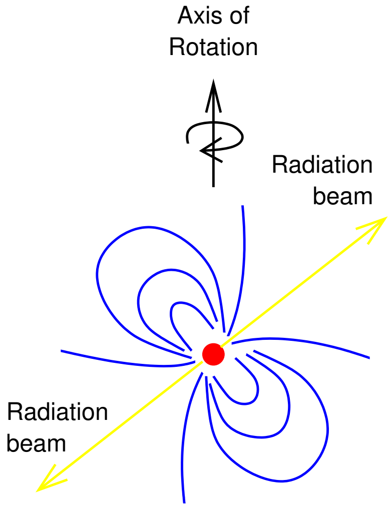


Figure 2: Pulsar: lighthouse effect

The shortest periods observed are about 1 millisecond, implying a radius of less than 50 km.
What is also conserved is the magnetic flux

$$\Phi = BR^2 \tag{5}$$

The magnetic field after the SN is

$$B_{\mathrm{NS}} = \left(\frac{R_{\mathrm{before}}}{R_{\mathrm{NS}}}\right)^2 B_{\mathrm{before}} \tag{6}$$

Therefore, neutron stars have strong magnetic fields ($\sim 10^{6\ldots8}$ T).

1.4. Black Holes

The upper mass limit for neutron stars is the Oppenheimer Volkoff limit of about $3\,M_\odot$. Objects with masses above this limit are called black holes.
A test mass needs a certain velocity to be able to escape the gravitational influence of a massive object:

$$v \geq v_{\mathrm{escape}} = \sqrt{\frac{2GM}{R}} \tag{7}$$

For a black hole, $v_{\mathrm{escape}} > c$, not even light can escape from its gravitational attraction. This can only be the case if the radius is

$$R \leq R_{\mathrm{S}} = \frac{2GM}{c^2} \sim 3\,\mathrm{km}\,\frac{M}{M_\odot} \tag{8}$$

where R_{S} is called the Schwarzschild-Radius.
General Relativity tells us that mass curves space and light moves through the curved space. The so called "Schwarzschild Metric" describes the "shape of space" in the vicinity of a mass M. Relativistic calculations show the same result for the Schwarzschild radius as the Newtonian approach. In general relativity, black holes are objects that are smaller than their Schwarzschild radius.
The characteristics of a black hole can be fully described by its mass, charge and angular momentum.

1.5. Gamma-Ray Bursts

Gamma-Ray Bursts are extremely bright flares with a duration of a few seconds. During the burst, GRBs are the brightest gamma-ray objects in the sky, brighter than the sun.
The short duration however made them difficult to study at first. In the 1990s however, new satellites showed that they are very common events. A wide variety of burst lightcurves both in duration and morphology is observed, where shorter bursts tend to have harder spectra.
The bursts are distributed isotropically on the sky and show X-ray afterglows that allow for localization. They occur at cosmological distances. Gamma-ray bursts are possibly related to supernova explosions. The relativistic fireball model tries to give a physical model to explain the observations.

2. Binary Stars

One of the few possiblities to detect a compact object is its location in a binary systems:

- In detached systems: Doppler motion of the visible star (as for Exoplanets)

- In interacting binaries (semi-detached): Mass transfer from the normal star and accretion onto the compact object

But how to distinguish between white dwarfs, neutron stars and black holes?

- If the lower limit to the mass of the compact object is less than the Chandrasekhar mass, no distinction is possible
- If the lower limit to the mass of the compact object is larger than the Chandrasekhar limit but less than the Oppenheimer Volkoff limit, it is a neutron star or a black hole
- If the lower limit to the mass of the compact object is larger than the Oppenheimer Volkoff limlit, it is a black hole
- If the compact object is a pulsar, it is a neutron star

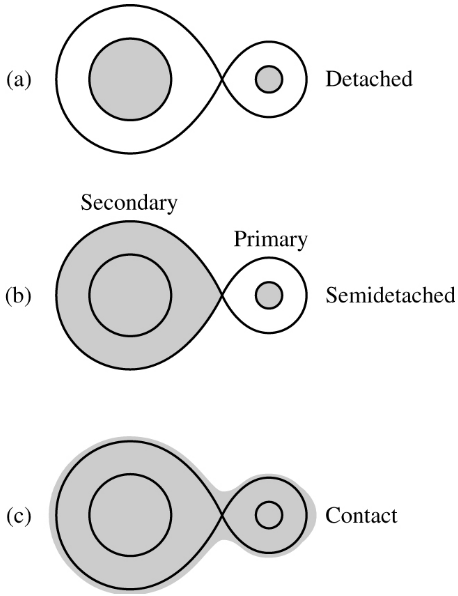


Figure 3: Realization of binary systems (Carroll, Ostlie, Fig. 18.4)

In semi-detached binary systems, the secondary star is a normal star. If the primary is a white dwarf, it is called a cataclysmic variable. If th primary is a neutron star or a black hole, we speak of X-ray binaries. Material accretes from the normal star onto the compact object over the inner Lagrange point L_1. The material forms an accretion disk with a temperature of about 10^7 K, and radiates in the X-rays.

2.1. Accretion

The energy gained by nuclear fusion (e.g. four protons to Helium) is about $6{\cdot}10^{11}\,\mathrm{J\,g^{-1}}$. Accretion of material of the mass m from ∞ to R_S onto a black hole with the mass M however gives

$$\Delta E_\mathrm{acc} = \frac{GMm}{R_\mathrm{S}} \tag{9}$$

which produces an energy of about $10^{13}\,\mathrm{J\,g^{-1}}$. Therefore, accretion is the most efficient astrophysical energy source and accreting objects are the most luminous sources in the whole universe.

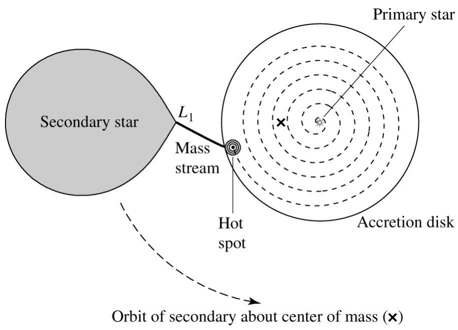


Figure 4: Accretion in binary systems (Carroll, Ostlie, Fig. 18.6)

2.2. Explosions

2.2.1. Novae

Is the primary object in a binary system a white dwarf, it can accumulate Hydrogen by accretion. However, with the degenerate electron gas being at temperatures of a few million K, hydrogen burning in the CNO cycle starts. A thermal run-away as in the core helium flash at the tip of the RGB occures. Explosions expell almost all accreted material.

2.2.2. Type 1a Supernovae

Supernovae of the type 1a are a result of the thermonuclear runaway in carbon-oxygen white dwarfs close to the Chandrasekhar limit.

- orbital shrinkage by common envelope ejection
- single degenerate scenario: continuous mass transfer to the white dwarf
  - white dwarf & normal star
  - white dwarf & helium star
- double degenerate scenario: two white dwarfs, orbital shrinkage by gravitational wave radiation $\rightarrow$ white dwarf merger

Part II.
Galaxies

3. The Milky Way

The Milky way is a barred spiral galaxy ($L \sim 2 \cdot 10^{10}\, L_\odot$) with a radiating mass of about $10^{10}\, M_\odot$ and a total mass of about $10^{12}\, M_\odot$. The stellar density is about $0.3\, M_\odot\, \mathrm{pc}^{-3}$.

3.1. Build-Up

The components of the Milky Way are

- Galactic disk
  - rotating
  - young and old stars, open star clusters
  - gas and dust
- Galactic halo
  - non-rotating
  - old stars only, globular clusters
  - no gas, no dust
- Galactic bulge: rigid rotation

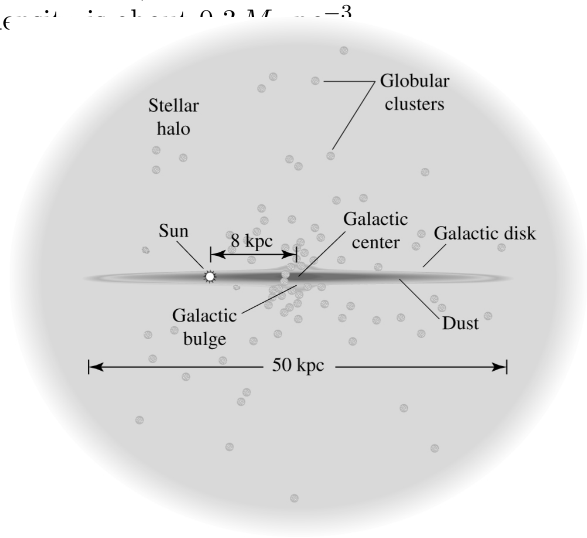


Figure 5: The Milky Way (Carroll, Ostlie, Fig. 24.6)

3.2. Structure

3.2.1. Local Standard of Rest

To describe the motion of stars in the Milky Way, we use a cylindrical coordinate system R, θ, z.
As all observations of the Galaxy are made from the position of the Sun which moves through space, we define a local coordinate system centered on the Sun, which moves on a circular orbit around the Galactic Center: The Local Standard of Rest (LSR). Note that the Sun also moves with respect to the LSR.

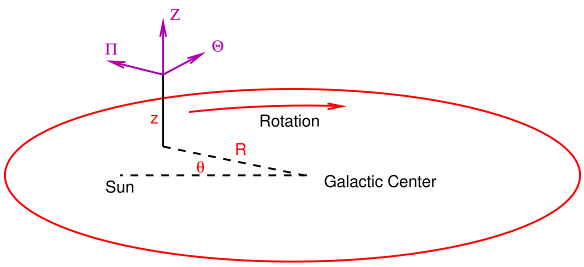


Figure 6: LSR (after Carroll, Ostlie, Fig. 22.21)

3.2.2. Rotation Curve

The rotation curve of the Milky Way can be observed even though we are part of it: From the radial and transversal velocities of a star, the angular velocity can be calculated.
Another method is the 21 cm line emitted from hydrogen, as the electron flips from a spin parallel to that of the proton to an antiparallel spin. This flip releases energy in form of a photon with $\lambda = 21\,\mathrm{cm}$. Because of the ubiquity of hydrogen, this line traces gas extremely well and it is visible from everywhere except for the most dense regions. Observations of this line show that the rotation curve of the Milky Way is approximately flat.

3.2.3. Spiral Arms

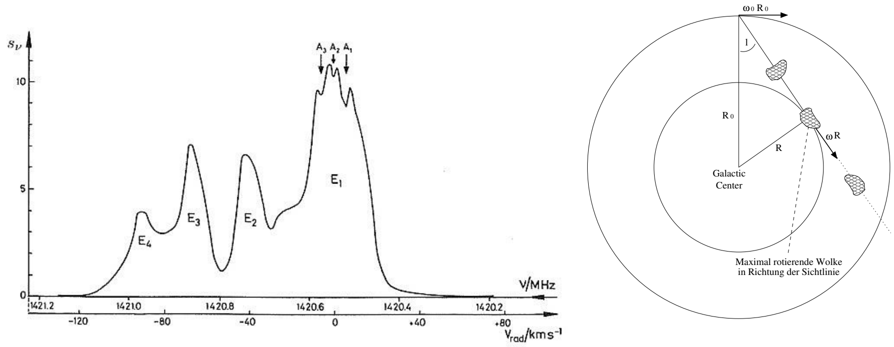


Figure 7: Sketch of a typical H I emission line profile. ν-axis has wrong sign!

The spiral structure of the Galaxy is now rather well understood. The spiral arms are believed to have formed due to density waves (Stars move on nested ovals). If then a cloud of gas passes through a density wave, the compression induces a collapse and stars of all masses form. The more massive ones then dissipate the cloud by their strong UV radiation.

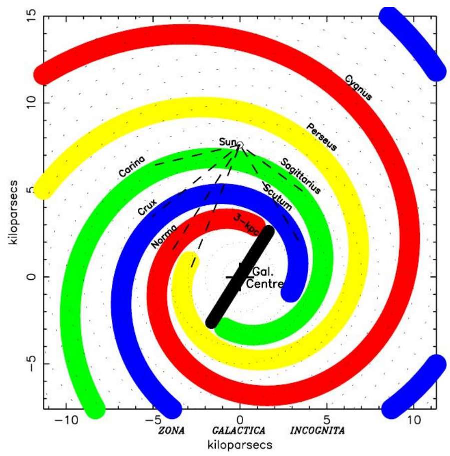


Figure 8: Spiral Structure of the Milky Way (Vallee, 2008)

3.3. The Galactic Center

As globular clusters have a random distribution in the Galactic halo, measuring their distance and calculating their center of mass gives us our distance to the Galactic center: 8 kpc.
Observing the Galactic center however is rather difficult: strong extinction in the direction due to dust reduces the optical wavelengths and makes multiwavelength astronomy necessary. In the infra red, dust becomes transparent and we see the stellar density rise towards a sharp central peak. The Galactic gas disk has a central hole of 3 kpc radius. Within this hole there is a dense and very hot central gas disk, the central 10 pc are dominated by radio source Sgr A. It divides in Sgr A West (radio filaments, massive and dense cluster of early-type stars), and Sgr A* (unresolved point source).
Adaptive optics are able to correct for the astronomical seeing and allow us to observe the trajectories of the stars close to the Galactic Center. Thus, the mass of the central object can be determined via Kepler 3 to $3.7 \pm 1.0 \cdot 10^6\, M_\odot$.

4. Galaxies

4.1. Classification of Galaxies

Since 1920, galaxies are classified in the Hubble Classification Scheme (“tuning fork diagram”), a morphological classification. Elliptical galaxies are called “early types”, spiral galaxies are called “late types”, although this is not an evolutionary sequence!
However, this classification includes also selection effects:

- Diameter: Edges of galaxies are not well defined as they are very dim
- Angular diameters depend on sky brightness
- Small galaxies cannot be destinguished from stars
- Low surface brightness galaxies cannot be seen against sky background

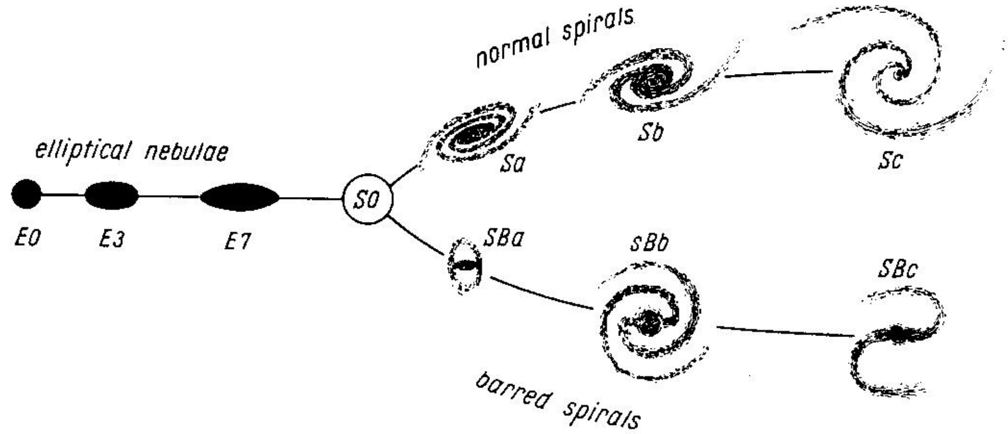


Figure 9: Hubble's classification scheme

4.1.1. Elliptical Galaxies

Elliptical galaxies are classified as Ex where $x = 10(a - b)/a$ (integer part, between 0 and 7).
Characteristics:

- low on dust and gas
- reddish color $\rightarrow$ old stars
- low luminosity
- low mass ($10^6\,M_\odot$)
- radial brightness distribution: de Vaucouleur's law

There are also monster elliptical galaxies up to $12\,M_\odot$ from galaxy mergers in clusters.

4.1.2. Spiral Galaxies

A spiral galaxy consists of an elliptical nucleus ("bulge") and a disk with spiral arms designated as Sa, Sb, Sc depending on the opening angle of the spiral, or SBa, SBb, SBc if there is an additional bar present in the galaxy center.
Characteristics:

- bluer than ellipticals
- mass content $\sim 10^11\,M_\odot$, $M/L \sim 20$
- gas content increases from Sa (1%) to Sc (8%)
- spiral arms probably due to density wave
- radial brightness distribution
  - bulge: de Vaucouleur's law (as ellipticals)
  - disk: exponential law

$$I(R) = I_0 \exp(-R/R_0) \tag{10}$$

where the scale length R_0 has a typical value of 3 kpc for thin disk like in the Milky Way

4.1.3. Irregular Galaxies

Irregular galaxies are those that cannot be classified as either elliptical or spiral galaxies:

- no symmetry or spiral arms
- bright knots of O- and B-Type stars
- very blue
- high dust content ($\sim 16\%$)
- varying masses ($10^{6\ldots10}\,M_\odot$), $M/L \sim 3$

The Magellanic clouds are the most prominent example for irregular galaxies. There are also starburst galaxies, interacting galaxies, Seyfert galaxies, ...

4.2. Galaxy Masses

There are several methods to determine the mass of a galaxy:

4.2.1. Galaxy Rotation Curves

Spectra of spiral galaxies are a sum of all constituent spectra (stars plus some nebulae). Absorption lines in these spectra are doppler shifted due to the motion of the stars around the center:

$$\frac{\Delta\lambda}{\lambda} = \frac{v_r}{c} = \frac{v}{c}\sin i \qquad (11)$$

with the radial velocity v_r and the inclination i (angle measured with respect to the plane of sky).
Measurements show that spiral galaxy rotation curves are flat. The Newtonian laws state that the rotation velocity $v_{\mathrm{rot}}(r)$ at a certain radius r is proportional to the mass within that radius, divided by the radius:

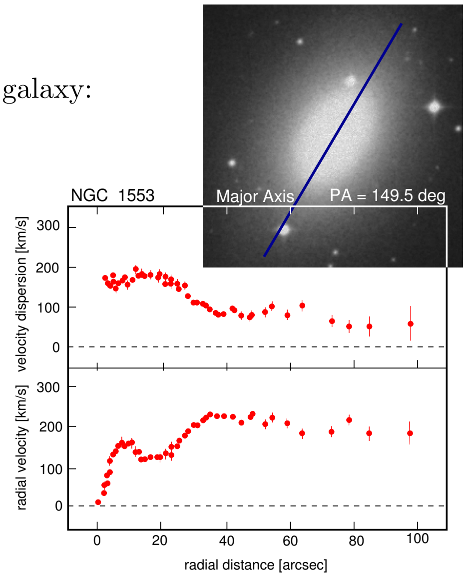


Figure 10: NGC 1553 (S0) (after Kormendy, 1884)

$$v^2_{\mathrm{rot}}(r) \propto \frac{M(\le r}{r} \qquad (12)$$

This implies that the mass has to be proportional to the radius, whereas the luminous mass decreases toward the edge of the disk ($v_{\mathrm{rot}} \propto r^{-1/2}$). This is seen as an evidence for a Dark Matter halo around galaxies!
Although the velocity dispersion of stars in elliptical galaxies is large and the stars show statistical motion, there is still a correlation of the central velocity dispersion with the absolute brightness of the galaxy: $L \propto \sigma^4$ (Faber-Jackson-relation). Estimating their mass from the virial theorem (assuming the stars are in statistical equilibrium) however show, that Dark Matter is also present in elliptical galaxies.
For more information about the possible nature of Dark matter see Section 14.

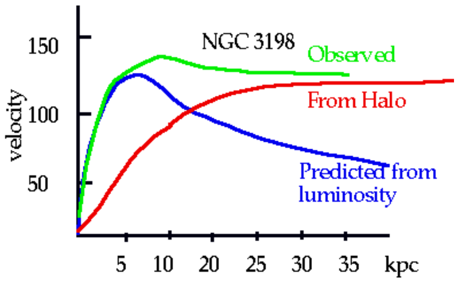


Figure 11: Rotation curve of NGC 3196

5. Galaxy Clusters

The analysis of clusters finds that galaxies have a wide distribution of absolute magnitudes. Generally, this is described by the so called Schlechter function. The masses of clusters derived from the virial theorem ($10^{12\ldots15}\,M_\odot$) are an order of magnitude larger than their luminous mass $\rightarrow$ there have to be Dark Matter haloes in clusters of galaxies! Dark Matter is also required to keep the observed diffuse X-ray intracluster gas bound to the cluster.
Simulations of interacting galaxies in clusters show that elliptical galaxis are results from a merger of two disk galaxies.

6. Active Galaxies

6.1. Phenomenology

Active Galactic Nuclei (AGN), the cores of active galaxies, are supermassive black holes ($M \sim 10^{6\ldots8}\,M_\odot$), accreting $1\ldots2\,M_\odot$ per year (see Section 2.1 for more information on accretion). Thus, the cores themselves have luminosities of $\sim 10^{10}\,L_\odot$, comparable to the luminosities of whole galaxies. There are several kinds of active galaxies:

- Seyfert 1 galaxies:
  - broad dipole allowed lines
  - narrow dipole forbidden lines
- Seyfert 2 galaxies:
  - weak continuum compared to Seyfert 1 galaxies
  - no broad lines
  - narrow forbidden lines
  - absorption lines from underlaying galaxy
- BL Lac Objects / Quasars
  - (variable) bright point sources
  - mostly radio loud

The Seyfert 1/2 classification applies to radio quiet (radio quiet $\neq$ radio silent!) sources. AGN that are showing a jet however are radio loud. Those objects then are named Broad Line Radio Galaxies and Narrow Line Radio Galaxies, respectively.

6.2. Apparent Superluminal Motion

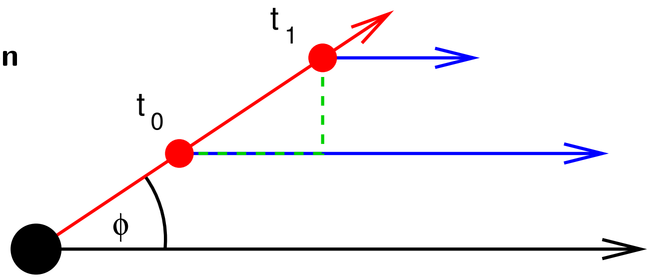


Figure 12: Apparent superluminal motion

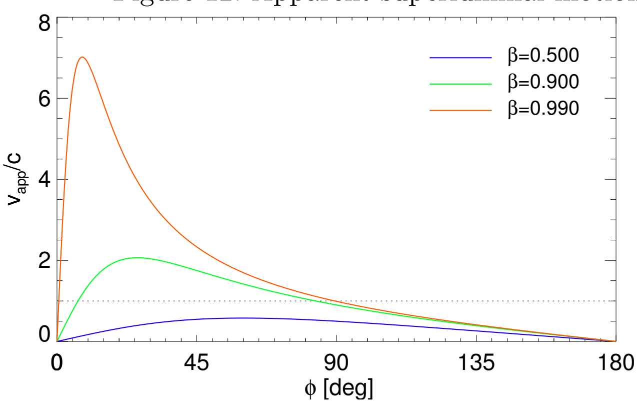


Figure 13: Apparent speed for different Φ and $\beta = \frac{v}{c}$

Observing the structure of AGN jets change in time leads to a puzzling conclusion: The apparent velocities measured in many AGN jets are $v > c$! But this is only due to projection effects: Consider a blob moving towars us with the speed v at an angle of Φ, emitting light signals at t_0 and $t_1 = t_0 + \Delta t_e$. Due to light travel time, the observer sees the signals separated by

$$\begin{aligned}\Delta t_0 &= \Delta t_e - \Delta t_e \frac{v}{c} \cos \Phi \\ &= \left(1 - \frac{v}{c} \cos \Phi\right) \Delta t_e \end{aligned} \qquad (13)$$

The observed distance the blob traveled in the plane of the sky is

$$\Delta \ell_\perp = v \Delta t_e \sin \Phi \qquad (14)$$

Therefore, the apparent velocity deduced from observations is

$$v_\text{app} = \frac{\Delta \ell_\perp}{\Delta t_0} = \frac{v \Delta t_e \sin \Phi}{\left(1 - \frac{v}{c} \cos \Phi\right) \Delta t_e} = \frac{v \sin \Phi}{\left(1 - \frac{v}{c} \cos \Phi\right)} \qquad (15)$$

Thus, for large $\frac{v}{c}$ and small Φ, there is $v_\text{app} > c$.

6.3. AGN Standard Model

The Unified Model explains the different phenomenology of all AGN with the same physical model but different viewing angles. Note that there is no galaxy with one jet only, but there are two kinds of galaxies shown in one plot!

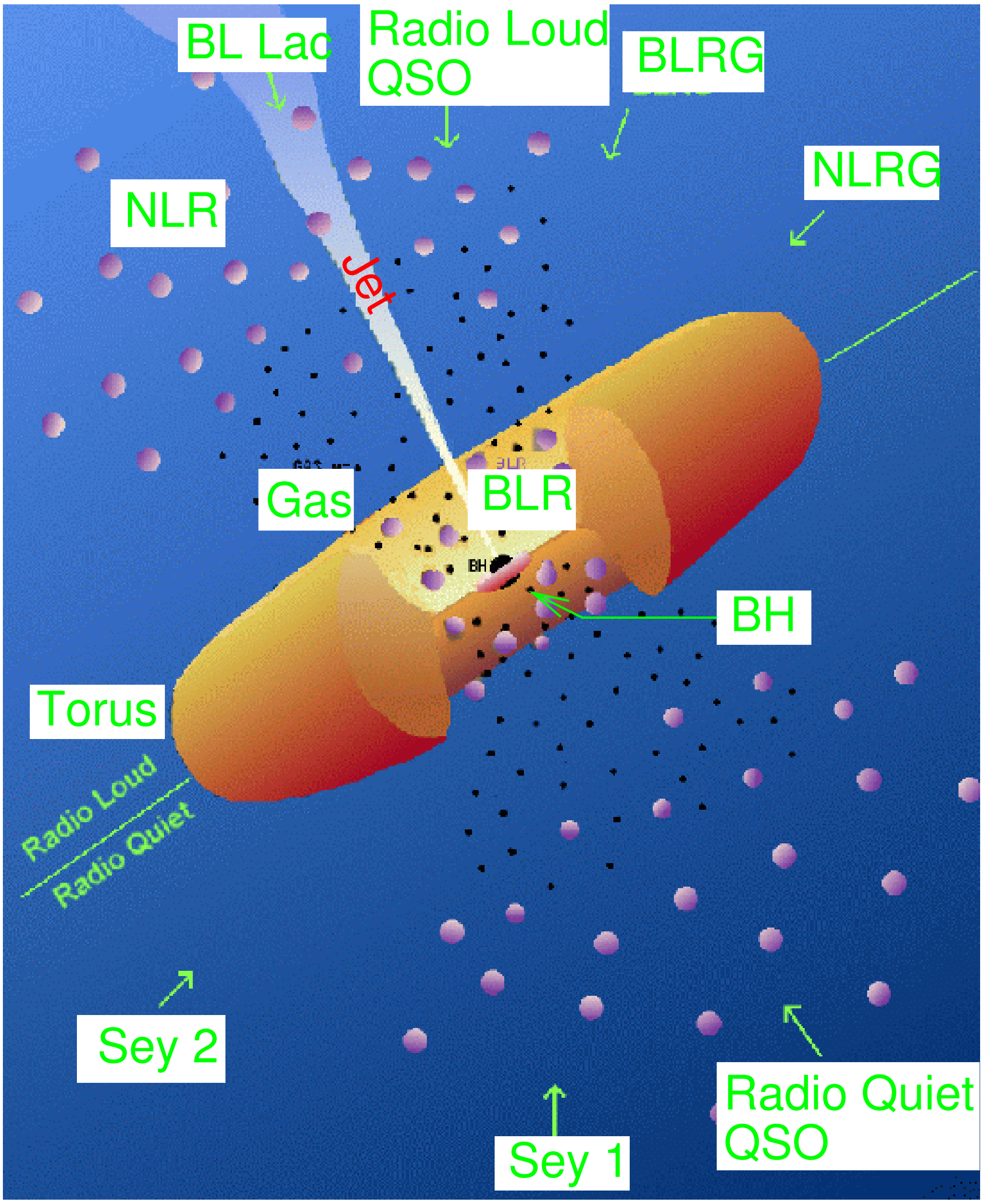


Figure 14: Urry & Padovani, 1995; logarithmic length scale!!

Part III.
Distance Estimation – The Astonomical Distance Ladder

7. Direct Method: Trigonometric Parallax

The motion of the Earth around the Sun within a year leads to apparent motion of stars on the sky (Fig. 15). The projected angular motion has an opening angle of

$$\tan p \sim p = \frac{1\,\mathrm{AU}}{d} \tag{16}$$

where p is called the trigonometric parallax. The determination of the distance of an object thus requires several measurements during the year.
The distance unit parsec is derived from this measurement: A parsec is the distance, where 1 AU has a parallax of $p = 1''$.
The parallax measurement is the only method where a distance can be directly derived. All other methods depend on calibrations in close-by sources.

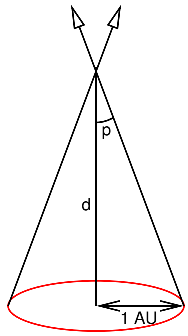


Figure 15: Trigonometric Parallax

8. Indirect Methods

8.1. Standard Candles

A standard candle is an object, for which the absolute magnitude is known. Thus, the physics of standard candles have to be well understood and the absolute magnitude of the object has to be calibrated by measuring the distance to the objects also via another method (which is difficult).

8.2. Reminder: Distance Modulus

Assuming isotropic emission, the flux of a source with the luminosity L measured at a distance d is given by the inverse square law

$$f(d) = \frac{L}{4\pi d^2} \tag{17}$$

The apparent magnitude is defined through comparison of two fluxes

$$m_2 - m_1 = -2.5\log_{10}\left(\frac{f_2}{f_1}\right) \tag{18}$$

The absolute magnitude M is defined as the magnitude a source would have if it were at a distance of 10 pc.

$$M - m = -2.5\log_{10}\left(\frac{f(10\,\mathrm{pc})}{f(d)}\right) = -2.5\log_{10}\left(\frac{L/(4\pi(10\,\mathrm{pc})^2)}{L/(4\pi d^2)}\right) = -2.5\log_{10}\left(\frac{d}{10\,\mathrm{pc}}\right)^2 \tag{19}$$

and therefore the distance modulus is

$$m - M = 5\log_{10}\left(\frac{d}{10\,\mathrm{pc}}\right) = 5\log_{10}(d) - 5 \tag{20}$$

Often the distance modulus of a source is given instead of the actual distance.

8.3. Main Sequence Fitting

The distance of stellar clusters can be determined using their HRD: A shift is observed compared to the stars in the solar vicinity. This shift can be used to calculate the distance modulus for the cluster. This method works up to $\sim 7\,\mathrm{kpc}$.

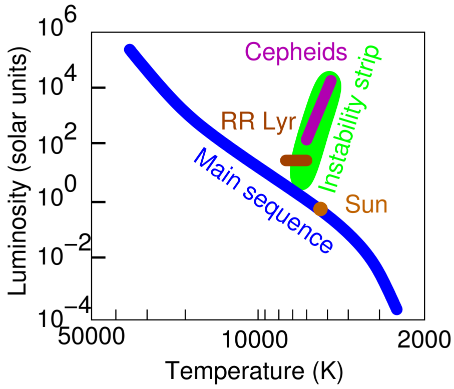


Figure 16: Instability strip in the Hertzsprung-Russell Diagram

8.4. Variable Stars

Certain regions of the Hertzsprung-Russell Diagram contain instable stars. The ionization of Helium leads to change in the transparency of the outer parts of the star. Thus, the size of the star changes as well as its surface temperature and luminosity.

8.4.1. RR Lyr Stars

RR Lyrae stars are mainly located in globular clusters (population II stars, lower metallicity). They vary mainly in temperature on a timescale of $0.2 \ldots 1\,\mathrm{d}$. The RR Lyr gap (Fig. 17) appears in globular cluster HRDs at an absolute magnitude of $M_V = 0.6\,\mathrm{mag}$ ($\rightarrow L_{\mathrm{RR}} \sim 50 L_\odot$). Distance is then calculated via the distance modulus. This method works up to $\sim 50\,\mathrm{kpc}$, i.e. out to the LMC.

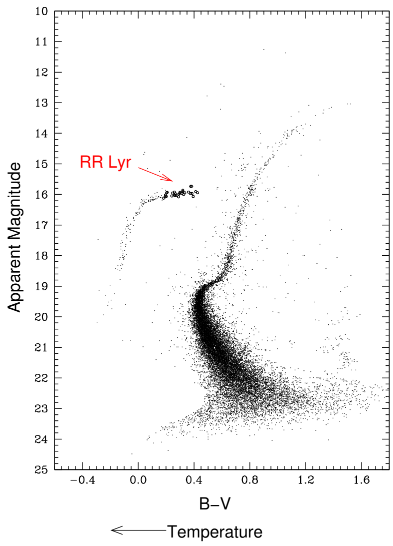


Figure 17: HRD of Globular Cluster M2 (after Lee et al., 1999, Fig. 2

8.4.2. δ Cepheids

Cepheids have a period luminosity relationship:

$$M \propto -\log P \tag{21}$$

Observations find

$$\langle M \rangle = -2.76 \log P - 1.40 \tag{22}$$

where P is measured in days. Using the HST, this method works out to the Virgo Cluster (18.5 Mpc).

8.5. Type Ia Supernovae

After correction of systematic effects and time dilatation (expansion of the universe), SN Ia light curves all look the same. They are used as standard candles. A SN Ia is the explosion of a white dwarf when pushed over the Chandrasekhar limit of $1.4\,M_\odot$. The process is always similar. Supernovae show very characeristic light curves of a fast rise, a rapid fall and an exponential decay with a half-time of 60 days. They are calibrated in nearby galaxies where Cepheids are known to an absolute magnitude of $M_B = -19.3 \pm 0.1\,\mathrm{mag}$ ($\rightarrow L \sim 10^{9\ldots10} L_\odot$. As they are observable out to more than 1 Gpc, they cover almost the whole universe.

8.6. Expansion of the Universe

The expansion of the Universe leads to a redshift

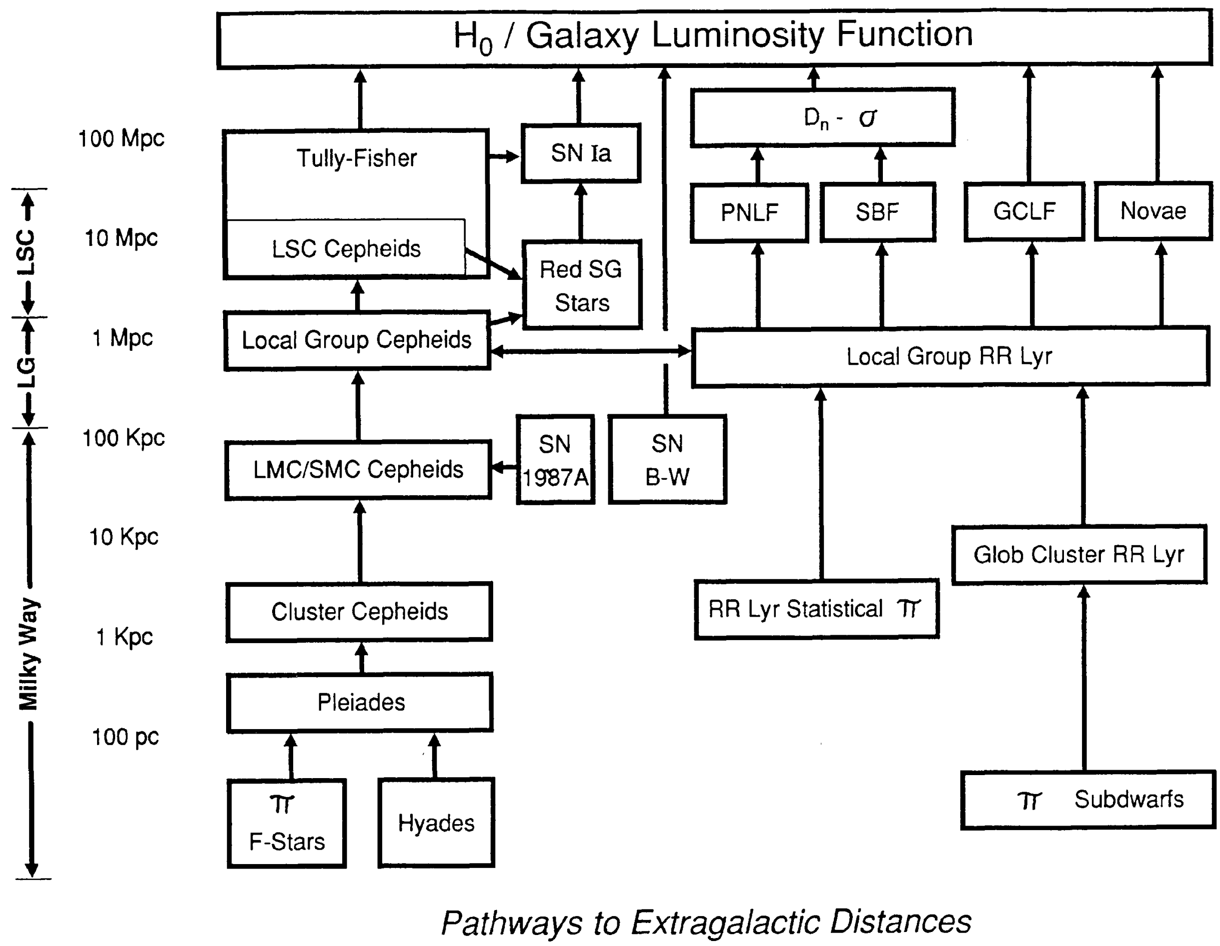
$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}} = \frac{v}{c} \tag{23}$$

of spectral lines of galaxies. This redshift is proportional to the distance of the galaxy:

$$v = cz = H_0 d \tag{24}$$

where $H_0 = 72 \pm 8\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ is the Hubble constant (see Section 11 for more information).

9. Summary



Pathways to Extragalactic Distances

Figure 18: Jacoby (1992, Fig. 1)

Part IV. Cosmology

10. Cosmological Principle

Cosmology is the science of a universe as a whole. Four basic facts about the universe:

- it is expanding
- it is homogeneous: looks the same, regardless from where it is observed
- it is isotropic: looks the same in every direction
- it is habitable

Isotropy and homogenity of the universe are called the “Cosmological Principle”. The expansion law is unchanged under rotation and translation (isomorphism). Despite everything receding from us, we are NOT at the center of the universe. Copernicus’ principle is still valid!!

11. Timescales

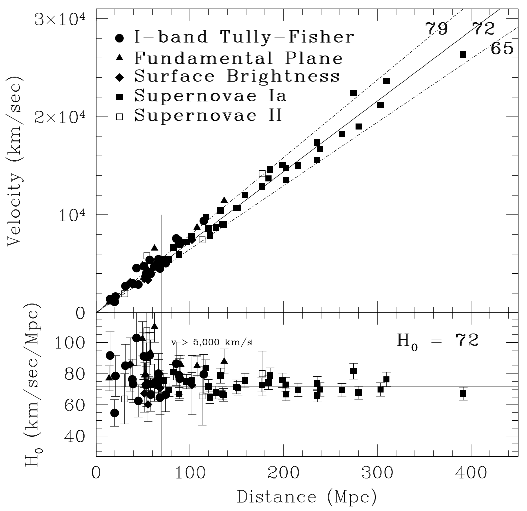


Figure 19: Freedman 2001, Fig. 4

The redshift is

$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}} \tag{25}$$

Therefore, the Hubble relation follows: The redshift of a galaxy is proportional to its distance:

$$v = cz = H_0 d \tag{26}$$

with the Hubble constant $H_0 = 72 \pm 8\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. H_0 is a measure of the slope of the line trhough the distance versus recession velocity data (Fig. 19). This line represents our home position and therefore goes through the origin as we are not moving away from ourselves. To determine H_0 one has to measure velocities and distances to many galaxies.
The inverse of the Hubble constant has the unit of time. The Hubble Time is therefore definded as

$$T = \frac{1}{H_0} \tag{27}$$

It gives the time for the universe to run backwards to the Big Bang if the expansion rate (the Hubble “constant”) were constant. Thus, it is a rough measure of the age of the Universe.

12. The Friedmann Equations

Friedmann: Mathematical description of the Universe using classical coordinates (“comoving coordinates”) AND a scale factor $R(t)$ which describes the evolution of the Universe.

12.1. Newtonian Approach

The mass of a sphere in this approach:

$$M = \frac{4\pi}{3}\left[dR(t)\right]^3 \rho(t) = \frac{4\pi}{3} d^3 \rho_0 \tag{28}$$

and therefore

$$\begin{aligned} m\ddot{r} &= m\tfrac{\mathrm{d}^2}{\mathrm{d}t^2}(dR(t)) \\ &= -\tfrac{GmM}{(dR(t))^2} \\ &= -\tfrac{\frac{GmM4\pi}{3} d^3 R(t)^3 \rho(t)}{d^2 R(t)^2} \\ &= -\tfrac{4\pi}{3} GmdR(t)\rho(t) \\ &= -\tfrac{4\pi}{3} Gmd\tfrac{\rho_0}{R(t)^2} \end{aligned} \tag{29}$$

or

$$\ddot{R(t)} = -\frac{4\pi}{3} G \frac{\rho_0}{R(t)^2} = \frac{4\pi}{3} G R(t) \rho(t) \tag{30}$$

which is the momentom equation. Multiplying with $\dot{R}$ and integrating yields the energy equation:

$$\frac{1}{2}\dot{R(t)}^2 = \frac{4\pi G}{3}\frac{\rho_0}{R} + \text{const.} = \frac{4\pi G}{3}\rho(t)R(t)^2 + \text{const.} \tag{31}$$

The integration constants cannot be found using the Newtonian approach.

12.2. ART Approach

Why is the Newtonian approach wrong?

- The Cosmological Principle states that the Universe does not have an edge or center, therefore the sphere model is incorrect
- Because of $E = mc^2$, equation of motion must allow for mass density as well as all other kinds of energy
- Particles are not moving through space but space expands.

Insert metric into Einstein equation to obtain differential equation for $R(t)$ and obtain somewhat different equations:

$$\ddot{R} = -\frac{4\pi G}{3} R \left(\rho + \frac{3p}{c^2}\right) + \left[\frac{1}{3}\Lambda R\right] \tag{32}$$

$$\dot{R}^2 = \frac{8\pi G \rho}{3} R^2 - kc^2 + \left[\frac{1}{3}\Lambda c^2 R^2\right] \tag{33}$$

with the curvature of space, k (Fig. 20):

- $k > 0$: closed universe (finite volume)
- $k = 0$: flat universe
- $k < 0$: open universe (infinite volume)

The density includes all contributions of all different kinds of energy. The physics of the cosmological constant Λ is yet unknown.

12.3. Hubble's Law

The variation of $R(t)$ implies Hubble's law:
The proper distance between two observers with comoving distance d:

$$D(t) = d \cdot R(t) \tag{34}$$

With the expansion of the Universe, D changes:

$$v = \frac{\mathrm{d}D}{\mathrm{d}t} = \dot{R}d = \frac{\dot{R}}{R}D = HD \tag{35}$$

where we can identify the Hubble "constant" to be time-dependent:

$$H = H(t) = \frac{\dot{R}(t)}{R(t)} \tag{36}$$

This implies: The redshift is a consequence of the expansion of the Universe! Calculations show

$$1 + z = \frac{1}{a_{\mathrm{emit}}} = \frac{R(t = \mathrm{today})}{R(t)} = \frac{\nu_{\mathrm{em}}}{\nu_{\mathrm{obs}}} \tag{37}$$

Light emitted at $z = 1$ was emitted when the universe was half as big as it is today.
Now looking at the Friedmann energy equation for $\Lambda = 0$, we find for the Hubble parameter

$$\left(\frac{\dot{R}}{R}\right) = H(t)^2 = \frac{8\pi G\rho(t)}{3} - \frac{kc^2}{R} \tag{38}$$

The critical density then is defined as

$$\Omega = \frac{\rho}{\rho_{\mathrm{crit}}} \tag{39}$$

with $\rho_{\mathrm{crit}} = \frac{3H^2}{8\pi G}$. This critical density is directly related to the curvature parameter k (Fig. 20):

- $\Omega > 1 \rightarrow k > 0$: closed universe, finite life
- $\Omega = 1 \rightarrow k = 0$: flat universe, expands forever
- $\Omega < 1 \rightarrow k < 0$: open universe, expands forever

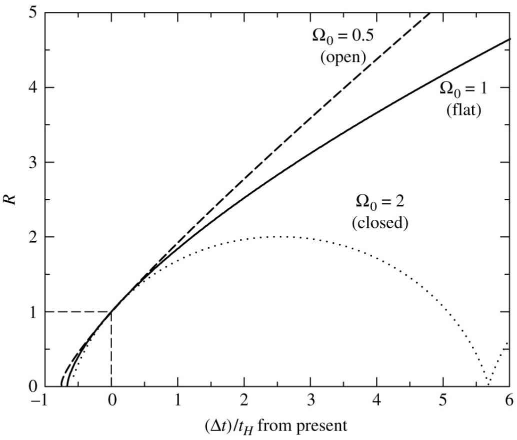


Figure 20: Carroll/Ostlie, Fig. 29.5

The total Ω has two contributions: Matter (Ω_{m}) and the cosmological constant ($\Omega_\Lambda = \Lambda c^2/3H^2$), often called "dark energy".

13. The Big Bang

13.1. Cosmic Microwave Background

The Cosmic Microwave Background (CMB) was discovered in 1965 by Penzias and Wilson (Nobel Prize 1978) as an isotropic, unpolarized, free from seasonal variations excess radiation. The CMB spectrum resembles a perfect blackbody spectrum with a temperature of $T = 2.725 \pm 0.002\,\mathrm{K}$.

13.2. The Hot Big Bang

Initially, the universe was very hot and as it expandet, it cooled down. The behaviour of the universe can be described from $t = 10^{-11}\,\mathrm{s}$ (Planck Time) after the Big Bang:

- $t = 10^{-44}$ s: GUT & gravitation → Theory of Everything (TOE)
- $t = 10^{-34}$ s: electroweak & strong nuclear force → Grand Unifying Theory (GUT)
- $t = 10^{-11}$ s: electroweak & weak nuclear forces → electroweak force
- $t = 10^{-4}$ s: Temperature decreased to $T = 10^{12}$ K
  - Universe consists of photons, electrons, positrons, neutrinos and anti-neutrinos, as well as a few hadrons (protons, neutrons)
  - pair formation and annihilation and transformation of particles leads to thermodynamic equilibrium (neutron-proton-ratio)

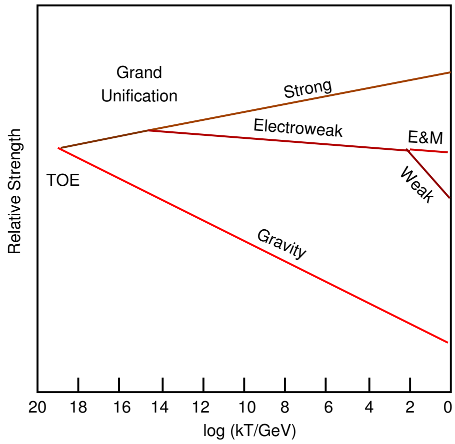


Figure 21: After Carroll/Ostlie, Fig. 30.2

$$\frac{N(\mathrm{n})}{N(\mathrm{p})} = \exp\left(-\frac{(m_\mathrm{p} - m_\mathrm{n})c^2}{kT}\right) = 0.985 \tag{40}$$

- $t \sim 2$ s: $T \sim 10^{10}$ K. Now, the timescale for nuclear reactions is smaller than the expansion time scale, the thermodynamic equilibrium ends and the Universe "freezes out". No neutrons can be formed any more, the remaining ones decay to protons (up to today, $N_\mathrm{n}/N_\mathrm{p}$ decayed from 0.223 to 0.164).
- $t = 230$ s: $T \sim 10^9$ K. Deuterium and Helium are formed.
- later: Primordeal Nucleosynthesis starts with formation of Helium, saving most of the neutrons from decay. Heavier elements are formed up to Lithium, formation of higher elements prevented by a gap of stable nuclei.
- The Hot Big Bang finishes after less than 15 min!

Stars produce much less Helium than what is observed in the Universe → smoking gun for the Hot Big Bang. Deuterium and Lithium abundances show: The density parameter $\Omega_\mathrm{m} = 0.04$: Baryons contribute only 4 % of the critical density.
Remainig problems:

- Horizon Problem: Why is CMB so isotropic?
- Flatness Problem: Why is and was the density so close to the critical density, i.e. $\Omega = 1$?
- Baryogenesis: Why is there virtually no antimatter in the Universe?
- What is the nature of Dark Matter and Dark Energy?
- Stucture formation: Fluctuations of CMB are too small for stars and galaxies to form as early as they are observed.

13.3. Inflation

A possible solution for most of these problems is the inflation. Basic assumption:
During the Big Bang there was a phase where Λ dominated the Friedmann equation.
When and why did inflation happen? → Inflation = phase transition of a scalar field ("inflaton") associated with GUT. Thus, inflation leads to an expansion by a factor of $e^{100} \sim 10^{43}$, corresponding to a volume expansion by a factor of $\sim 10^{130}$. The inflation lasted for ~ 100 Hubble times, therefore the Universe was so small before inflation that all parts of it were in casual contact. Thus, inflation solves the horizon, antimatter and flatness problem.

14. Determination of Ω_m

Constituents of Ω_m:

- Radiation (3K Radiation): $\Omega_\gamma h^2 = 2.48 \cdot 10^{-5}$
- Neutrinos: $\Omega_\nu h^2 = 1.69 \cdot 10^{-5}$
- Baryons (normal matter): $\Omega_\mathrm{b} h^2 = 0.02$; Methods of measurement:
  - Mass of galaxy clusters: Virial Theorem $\rightarrow$ lots of dark matter
  - More precise mass of galaxy clusters: X-ray emission of gas in hydrostatic equilibrium
  - Sunyaev-Zeldovich effect (Compton upscattering of photons from behind cluster)
  - Gravitational lensing (massive clusters act as light benders for photons of objects located behind the cluster in the line of sight)
- Other, non radiating, gravitating material ("dark matter"): $\Omega_\mathrm{d} h^2 = 0.23$;

Together, we get $\Omega_\mathrm{m} = 0.3$, which, under the assumption of $\Omega_\mathrm{total} = 1$, leads to $\Omega_\Lambda = 0.7$.
The large part of Ω_m is contributed by dark matter, but WHAT IS dark matter?

- MACHOs (Massive Compact Halo Objects, white dwarfs in galaxy halos)
  - Pro: low luminosity, very difficult to detect, but already seen in microlensing events towards Magellanic clouds; MW halo consists of 50% white dwarfs
  - Contra: possible self-lensing (confirmed for a few causes), inferred white dwarf formation rate would be much too high
- Neutrinos
  - Pro: Exist, mass limits few eV (which is enough)
  - Contra: Relativistic $\rightarrow$ galaxy formation would not have worked as it did
- WIMPs (Weakly Interacting Massive Particles)
  - Pro: May be identified with particles of Supersymmetry theory in Particle Physics, masses of few GeV possible
  - Contra: We do not know they exist (but they may soon be detected by LHC)

As WIMPS are heavy, they are non-relativistic and can help to explain formation of stars and galaxies. WIMPS are Cold Dark Matter.

15. Determination of Ω_Λ

As we now know that there has to be an Ω_Λ, we have to pay attention to the Λ term in the Friedmann equations. Thus, many different kinds of world models are possible, depending on Ω and Λ. Universes with $\Omega_\Lambda > 0$ are older than those with $\Omega_\Lambda < 0$. However, most of all models can be excluded by observations. For $\Omega_\Lambda < 1$, at first, the Universe is dominated by matter and expansion is slowed, whereas later, Λ dominates and $a(\tau)$ rises exponentially.
To determine exact Λ: Observe objects at high redshifts. There we get for the observed source flux

$$F = \frac{L}{4\pi R_0^2 r^2 (1+z)^2} = \frac{L}{4\pi d_L^2} \tag{41}$$

with the luminosity distance

$$d_L = R_0 r (1+z) \tag{42}$$

The best objects to observe are Type 1a Supernovae. As they are standard candles, the luminosity distance can be determined via the distance modulus. For the analysis of these data, many corrections have to be applied before one gets results. Each world model now gives a certain dependence of the apparent magnitude on the measured redshifts. Comparing these models to the measured data gives best values for Ω_m and Ω_Λ to

$$\Omega_\mathrm{m} = 0.3 \tag{43}$$

and

$$\Omega_\Lambda = 0.7. \tag{44}$$

Even if the total $\Omega \neq 1$, there has to be $\Omega_\Lambda \neq 0$. Baryons are an energetically unimportant constituent of the Universe.
But what is the physical meaning of Dark Energy? A possible candidate is vacuum energy. However: quantum field theories and observations mismatch by 120 orders of magnitude. The current (still very speculative!) discussion to solve the vacuum energy problem is the introduction of new scalar field: "quintessence".

16. Evolution of the Universe

16.1. Structure formation

Structure formation: Study of the formation of density perturbations in an initially approximately smooth universe and of their evolution.
To study the structure of the Universe, large redshift surveys have been started (difficult: the farer away, the more observing time needed):

- 1D-surveys: very deep exposures of small patches of the sky (pencil beam survey): observations only possible via satellites (e.g. HST)
- 2D-surveys: cover long strip of sky
- 3D-surveys: cover large parts of the sky (Sloan Digital Sky Survey, SDSS)

Caution: Selection effect!!! The galaxy density is not smaller the farer we look out, but we just do not see the dimmer objects.
How do such structures form? Consider a collapsing sphere of gas in a non-expanding universe. The sphere collapses for a potential energy larger than the kinetic energy content. Calculations show that this is the case for objects with the Jeans Mass

$$M_\mathrm{J} = \frac{\pi}{6}\rho\lambda_\mathrm{J}^3 \tag{45}$$

with the Jeans length $\lambda_\mathrm{J} = c_\mathrm{s}\sqrt{\frac{\pi}{G\rho}}$ and the speed of sound c_s. Structures with masses smaller than the Jeans Mass cannot grow (but c_s and therefore also M_J is time dependent!).
The general idea is

1. Big Bang generates initial density perturbations (Dark Matter is important here)
2. Those density fluctuations that can grow
3. Those density fluctuations that cannot grow get smoothed out by expanding and disappear

Numerical simulations show that hot dark matter (relativistic particles) leads to "top down structure formation" which is not observed. Only cold dark matter (slow particles) is able to explain the observed "bottom up structure formation" where first galaxies are formed, then galaxy clusters. Thus, luminous baryonic mass traces dark matter.
As soon as the universe becomes transparent, photons are able to leave the potential wells. Doing this they loose energy (gravitational redshift), which is now visible in temperature fluctuations in the cosmic microwave background (Sachs-Wolfe-Effect). Measurements of the power spectrum of the cosmical microwave background radiation (acoustic peak) give an independent evidence for the Hubble parameter and Ω_m consistent with earlier observed values.

As a conclusion we can now say that, as the universe expanded, its energetic contents changed dramatically. Physics as of today can only describe 4% out of it, the other 96% are yet to be explored.

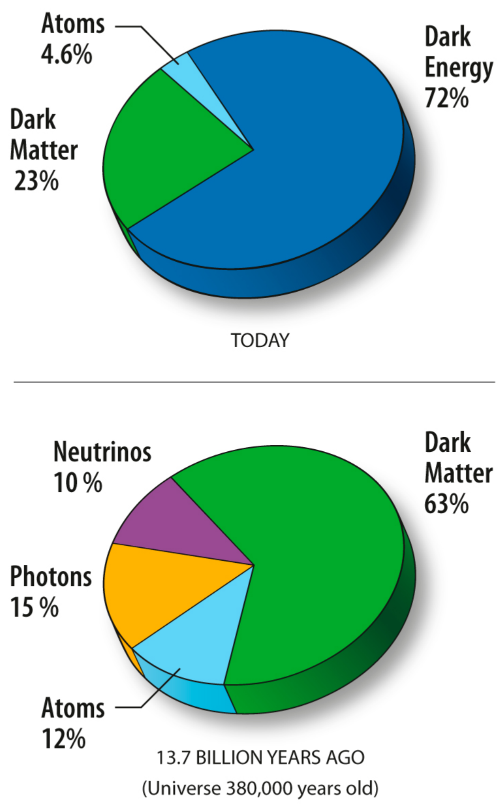


Figure 22: WMAP Science Team