Comptonization

Introduction

Comptonization: Upscattering of low-energy photons by inverse Compton collisions in a hot electron gas.

Astronomically important in

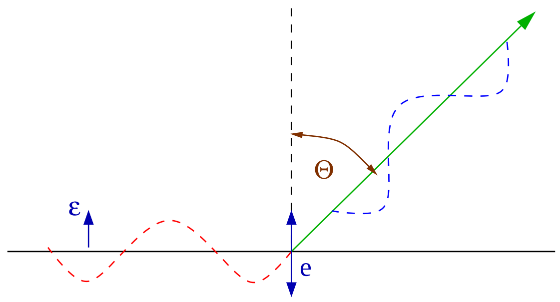
- galactic black hole candidates
- active galactic nuclei

Strategy: First look at classical Thomson scattering, then look at quantum mechanical analogue (Compton scattering).

Literature:

Blumenthal & Gould 1970, RMP 42, 237
Górecki & Wilczewski 1984, Acta Astron. 34, 141
Hua & Titarchuk 1995, ApJ 449, 188
Pozdnyakov et al. 1983, Astrophys. Rep. 2, 189
Sunyaev & Titarchuk 1980, A&A 86, 121

Polarized Radiation, I



As we had seen on Worksheet 4, Thomson scattering can be derived by looking at the radiation from a free electron in response to *linearly polarized* electromagnetic wave.

after Rybicki&Lightman, Fig. 3.6

Force on charge

$$\boldsymbol{F} = m_e \ddot{\boldsymbol{r}} = e\boldsymbol{\epsilon} E_0 \sin\omega_0 t \qquad (7.1)$$

This approach neglects the $\boldsymbol{B}$-field, i.e., assumes $v \ll c$.

Therefore, dipole moment $\boldsymbol{d} = e\boldsymbol{r}$ is

$$\ddot{\boldsymbol{d}} = \frac{e^2 E_0}{m_e}\boldsymbol{\epsilon}\sin\omega_0 t \quad\Longrightarrow\quad \boldsymbol{d} = -\frac{e^2 E_0}{m_e \omega_0^2}\boldsymbol{\epsilon}\sin\omega_0 t \qquad (7.2)$$

Polarized Radiation, II

The dipole moment was

$$\ddot{\boldsymbol{d}} = \frac{e^2 E_0}{m_e}\boldsymbol{\epsilon}\sin\omega_0 t \quad\Longrightarrow\quad \boldsymbol{d} = -\frac{e^2 E_0}{m_e \omega_0^2}\boldsymbol{\epsilon}\sin\omega_0 t \qquad (7.2)$$

Using the dipole approximation,

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{\ddot{\boldsymbol{d}}^2}{4\pi c^3}\sin^2\Theta \quad\text{and}\quad P = \frac{2\ddot{\boldsymbol{d}}^2}{3c^3} \qquad (4.92)$$

we obtain after time averaging ($\langle \sin^2\omega_0 t\rangle = 1/2$)

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{e^4 E_0^2}{8\pi m_e^2 c^3}\sin^2\Theta \quad\text{and}\quad P = \frac{e^4 E_0^2}{3 m_e^2 c^3} \qquad (7.3)$$

Note that the scattering angle is Θ, not θ. The cause for this will become clear shortly.

Polarized Radiation, III

Incident radiation flux on electron is

$$\langle \boldsymbol{S} \rangle = \frac{c}{8\pi} E_0^2 \quad (7.4)$$

The differential cross section, $\mathrm{d}\sigma / \mathrm{d}\Omega$, is defined by

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \langle \boldsymbol{S} \rangle \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{cE_0^2}{8\pi} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \quad (7.5)$$

such that

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right|_{\text{polarized}} = \frac{e^4}{m_e^2 c^4} \sin^2\Theta = r_0^2 \sin^2\Theta \quad (7.6)$$

where the classical electron radius is

$$r_0 = \frac{e^2}{m_e c^2} = 2.82 \times 10^{-13}\,\text{cm} \quad (7.7)$$

Visualization: $\mathrm{d}\sigma$ is the area presented by the electron to a photon that is going to get scattered in direction $\mathrm{d}\Omega$.

Polarized Radiation, IV

The total cross section is obtained from integrating over Ω or immediately from

$$P = \langle S \rangle \sigma \quad (7.8)$$

to obtain

$$\sigma = \frac{8\pi}{3} r_0^2 =: \sigma_T \quad (7.9)$$

where

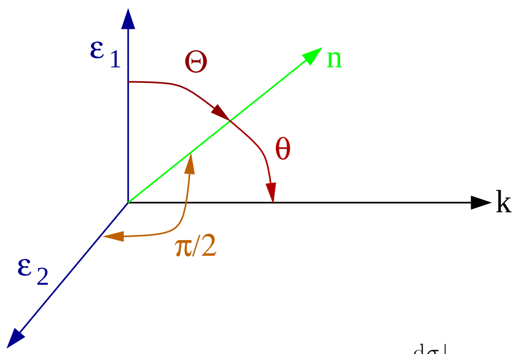
$$\sigma_T = 6.652 \times 10^{-25}\,\text{cm}^2 \quad (7.10)$$

(Thomson cross section)

This is valid for nonrelativistic energies, for higher energies, the Klein-Nishina cross section, Eq. (7.28) has to be used (see below).

Note that scattered radiation is linearly polarized in direction of incident polarization vector, $\boldsymbol{\epsilon}$, and direction of scattering, $\boldsymbol{n}$.

Unpolarized Radiation



after Rybicki & Lightman, Fig. 3.7

To calculate σ for nonpolarized radiation, note:

nonpolarized $=$

$$\sum \text{polarized beams at} \angle(90^\circ) \quad (7.11)$$

Thus, to scatter nonpolarized radiation propagating in direction $\boldsymbol{k}$ into direction $\boldsymbol{n}$, we need to average two scatterings:

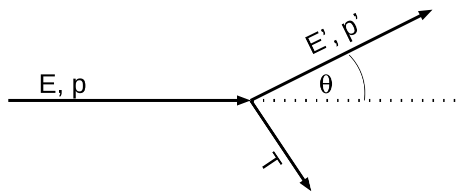
$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right|_{\text{unpol}} = \frac{1}{2}\left(\left.\frac{\mathrm{d}\sigma(\Theta)}{\mathrm{d}\Omega}\right|_{\text{pol}} + \left.\frac{\mathrm{d}\sigma(\pi/2)}{\mathrm{d}\Omega}\right|_{\text{pol}} \right) \quad (7.12)$$

Let $\theta = \angle(\boldsymbol{k}, \boldsymbol{n})$ to obtain

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right|_{\text{unpol}} = \frac{r_0^2}{2}(1 + \cos^2\theta) = \frac{3\sigma_T}{16\pi}(1 + \cos^2\theta) \quad (7.13)$$

The total cross section is again $\sigma = \sigma_T$.

Compton Scattering



Thomson scattering: initial and final wavelength are identical.
But: in reality: light consists of photons
$\Longrightarrow$ Scattering: photon changes direction
$\Longrightarrow$ Momentum change
$\Longrightarrow$ Energy change!
This is a quantum picture
$\Longrightarrow$ Compton scattering.

Dynamics of scattering gives energy/wavelength change:

$$E' = \frac{E}{1 + \frac{E}{m_e c^2}(1 - \cos\theta)} \sim E\left(1 - \frac{E}{m_e c^2}(1 - \cos\theta)\right) \quad (7.14)$$

and

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta) \quad (7.15)$$

where $h/m_e c = 2.426 \times 10^{-10}$ cm (Compton wavelength).

Averaging over θ, for $E \ll m_e c^2$:

$$\frac{\Delta E}{E} \approx -\frac{E}{m_e c^2} \quad (7.16)$$

E.g., at 6.4 keV, $\Delta E \approx 0.2$ keV.

The derivation of Eq. (7.14) is most simply done using four-vectors. In the following, we will use capital letters for four-vectors and small letters for three-vectors. Furthermore, we will adopt the convention

$$\boldsymbol{P}\cdot\boldsymbol{Q} = P_0Q_0 - P_1Q_1 - P_2Q_2 - P_3Q_3 \qquad (7.17)$$

for the product of two four vectors, following, e.g., the convention of Rindler (1991, Introduction to Special Relativity). Note that this convention differs from that of Rybicki & Lightman!

The four-momentum of a particle with non-zero rest-mass, m_0, e.g., an electron, is

$$\boldsymbol{Q} = m_0\gamma\begin{pmatrix} c \\ \boldsymbol{v}\end{pmatrix} = \begin{pmatrix} m_0\gamma c \\ \boldsymbol{q}\end{pmatrix} \qquad (7.18)$$

where $\boldsymbol{v}$ is the velocity of the particle and $\boldsymbol{q}$ its momentum. As usual, $\gamma = (1-(v/c)^2)^{-1/2}$. The square of $\boldsymbol{Q}$ is

$$\boldsymbol{Q}^2 = m_0^2\gamma^2c^2 - m_0^2\gamma^2v^2 = m_0^2c^2\gamma^2\left(1-\left(\frac{v^2}{c^2}\right)\right) = m_0^2c^2 \qquad (7.19)$$

Obviously, $\boldsymbol{Q}^2$ is relativistically invariant.

Analoguously to the above equations, the four-momentum of a photon is given by

$$\boldsymbol{P} = \frac{E}{c}\begin{pmatrix} 1 \\ \hat{\boldsymbol{u}}\end{pmatrix} \qquad (7.20)$$

where $\hat{\boldsymbol{u}}$ is an unit-vector pointing into the direction of motion of the photon. Note that for photons

$$\boldsymbol{P}^2 = 0 \qquad (7.21)$$

as the photon rest-mass is zero.

We will now look at the collision between a photon and an electron. We will denote the four-momenta after the collision with primed quantities.

Conservation of four-momentum requires

$$\boldsymbol{P}+\boldsymbol{Q} = \boldsymbol{P}'+\boldsymbol{Q}' \qquad (7.22)$$

We now use a trick from Lightman et al. (1975, Problem Book in Relativity and Gravitation), solving this equation for $\boldsymbol{Q}'$ and squaring the resulting expression:

$$(\boldsymbol{P}+\boldsymbol{Q}-\boldsymbol{P}')^2 = (\boldsymbol{Q}')^2 \qquad (7.23)$$

Since the collision is elastic, i.e., the rest mass of the electron is not changed by the collision,

$$\boldsymbol{Q}^2 = (\boldsymbol{Q}')^2 \qquad (7.24)$$

furthermore, $\boldsymbol{P}^2 = (\boldsymbol{P}')^2 = 0$, such that

$$\boldsymbol{P}\cdot\boldsymbol{Q} - \boldsymbol{P}\cdot\boldsymbol{P}' - \boldsymbol{Q}\cdot\boldsymbol{P}' = 0 \quad\Longleftrightarrow\quad \boldsymbol{P}\cdot\boldsymbol{P}' = \boldsymbol{Q}\cdot(\boldsymbol{P}-\boldsymbol{P}') \qquad (7.25)$$

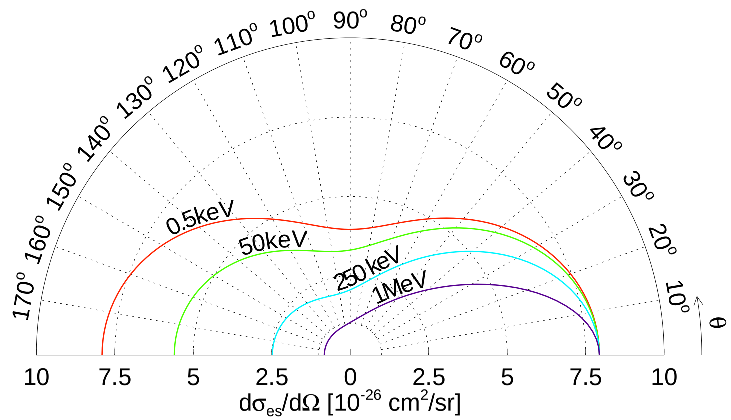
But in the frame where the electron is initially at rest,

$$\boldsymbol{Q}\cdot(\boldsymbol{P}-\boldsymbol{P}') = m_\mathrm{e}c\left(\frac{E}{c}-\frac{E'}{c}\right) = m(E-E') \qquad (7.26)$$

$$\boldsymbol{P}\cdot\boldsymbol{P}' = \frac{E}{c}\frac{E'}{c}(1-\hat{\boldsymbol{u}}\cdot\hat{\boldsymbol{u}}') = \frac{EE'}{c^2}(1-\cos\theta) \qquad (7.27)$$

where $\theta = \angle(\hat{\boldsymbol{u}},\hat{\boldsymbol{u}}')$. Inserting into Eq. (7.25) and solving for E' gives Eq. (7.14).

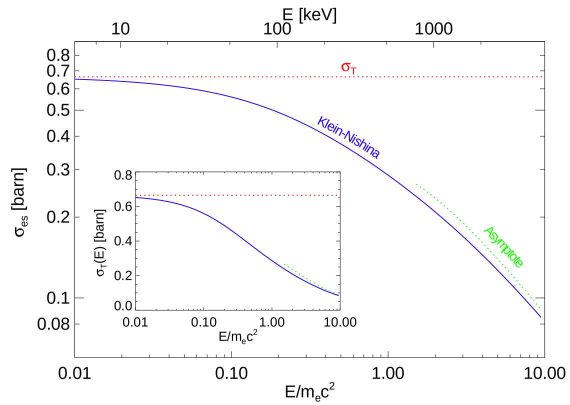
Compton Scattering



At low energies, Thomson (eq. 7.13) holds. For higher energies, the Thomson formula breaks down. For unpolarized radiation, quantum electro-dynamics yields the Klein-Nishina formula:

$$\frac{\mathrm{d}\sigma_\mathrm{es}}{\mathrm{d}\Omega} = \frac{3}{16\pi}\sigma_\mathrm{T}\left(\frac{E'}{E}\right)^2\left(\frac{E}{E'}+\frac{E'}{E}-\sin^2\theta\right) \qquad (7.28)$$

Compton Scattering



Integrating over Klein-Nishina gives the total cross-section:

$$\sigma_\mathrm{es} = \frac{3}{4}\sigma_\mathrm{T}\left[\frac{1+x}{x^3}\cdot\left\{\frac{2x(1+x)}{1+2x}-\ln(1+2x)\right\}+\frac{1}{2x}\ln(1+2x)-\frac{1+3x}{(1+2x)^2}\right] \qquad (7.29)$$

where $x = E/m_\mathrm{e}c^2$.

For $x \gg 1$,

$$\sigma \sim \frac{3}{8}\sigma_\mathrm{T}\frac{1}{x}\left(\ln 2x + \frac{1}{2}\right) \qquad (7.30)$$

Energy Exchange

For non-stationary electrons, use previous formulae and Lorentz transform photon into electron's frame of rest (FoR):

1. Lab system $\Rightarrow$ electron's frame of rest:
$$E_{\text{FoR}} = E_{\text{Lab}}\gamma(1-\beta\cos\theta) \tag{7.31}$$
2. Scattering occurs, gives E'_{FoR}.
3. Electron's frame of rest $\Rightarrow$ Lab system:
$$E'_{\text{Lab}} = E'_{\text{FoR}}\gamma(1+\beta\cos\theta') \tag{7.32}$$

Therefore, if electron is relativistic:
$$E'_{\text{Lab}} \sim \gamma^2 E_{\text{Lab}} \tag{7.33}$$
since (on average) θ, θ' are $\mathcal{O}(\pi/2)$.

Thus: Energy transfer is *very* efficient.

Single Scattering, I

To derive the approximate energy gain of photons, look at single scattering first (optically thin case).

Total power *emitted* in electron frame of rest:
$$\left.\frac{\mathrm{d}E'_{\text{FoR}}}{\mathrm{d}t_{\text{FoR}}}\right|_{\text{em}} = \int c\sigma_{\text{T}} E'_{\text{FoR}} V'(E'_{\text{FoR}})\,\mathrm{d}E'_{\text{FoR}} \tag{7.34}$$
where $V'(E')$: photon energy density distribution.

$V(E)$ is related to phase space density $n(p)$ by
$$V(E)\,\mathrm{d}E = n(p)\,\mathrm{d}^3p \tag{7.35}$$
But $V(E)$ is Lorentz invariant:
$$\frac{V_{\text{Lab}}(E_{\text{Lab}})\,\mathrm{d}E_{\text{Lab}}}{E_{\text{Lab}}} = \frac{V_{\text{FoR}}(E_{\text{FoR}})\,\mathrm{d}E_{\text{FoR}}}{E_{\text{FoR}}} \tag{7.36}$$

Single Scattering, II

Assume energy change in rest frame is small, $E'_{\text{FoR}} = E_{\text{FoR}}$ (Thomson limit).

Power is Lorentz invariant:
$$\frac{\mathrm{d}E_{\text{FoR}}}{\mathrm{d}t_{\text{FoR}}} = \frac{\mathrm{d}E_{\text{Lab}}}{\mathrm{d}t_{\text{Lab}}} \tag{7.37}$$
Therefore
$$\left.\frac{\mathrm{d}E_{\text{Lab}}}{\mathrm{d}t_{\text{Lab}}}\right|_{\text{em}} = c\sigma_{\text{T}}\int E^2_{\text{FoR}}\frac{V_{\text{FoR}}\,\mathrm{d}E_{\text{FoR}}}{E_{\text{FoR}}} = c\sigma_{\text{T}}\int E^2_{\text{FoR}}\frac{V_{\text{Lab}}\,\mathrm{d}E_{\text{Lab}}}{E_{\text{Lab}}} \tag{7.38}$$
...Lorentz transform: $E_{\text{FoR}} = (1-\beta\cos\theta)E_{\text{Lab}}$ such that
$$= c\sigma_{\text{T}}\gamma^2\int(1-\beta\cos\theta)^2 E_{\text{Lab}}V_{\text{Lab}}\,\mathrm{d}E_{\text{Lab}} \tag{7.39}$$
...averaging over angles ($\langle\cos\theta\rangle = 0$, $\langle\cos^2\theta\rangle = 1/3$)
$$= c\sigma_{\text{T}}\gamma^2\left(1+\frac{\beta^2}{3}\right)U_{\text{rad}} \tag{7.40}$$
where
$$U_{\text{rad}} = \int EV(E)\,\mathrm{d}E \tag{7.41}$$
(initial photon energy density).

Single Scattering, III

To obtain the *net* power gain of photon field, we have to subtract the power irradiated onto the electron,
$$\left.\frac{\mathrm{d}E_{\text{Lab}}}{\mathrm{d}t_{\text{Lab}}}\right|_{\text{inc}} = c\sigma_{\text{T}}\int EV(E)\,\mathrm{d}E = \sigma_{\text{T}}cU_{\text{rad}} \tag{7.42}$$
Therefore, since
$$\gamma^2 - 1 = \gamma^2\beta^2 \tag{7.43}$$
the net power gain of the photon field is
$$P_{\text{compt}} = \left.\frac{\mathrm{d}E_{\text{Lab}}}{\mathrm{d}t}\right|_{\text{em}} - \left.\frac{\mathrm{d}E_{\text{Lab}}}{\mathrm{d}t}\right|_{\text{inc}} \tag{7.44}$$
$$= \frac{4}{3}\sigma_{\text{T}}c\gamma^2\beta^2U_{\text{rad}} \tag{7.45}$$

Compton catastrophe

Power emitted by synchrotron radiation in a B-field of energy density U_B was

$$P_{synch} = \frac{4}{3}\sigma_T c\gamma^2\beta^2 U_B \qquad (6.10)$$

Magnetized plasma: synchrotron photons are inverse Compton scattered by the electrons. Ratio of emitted powers:

$$\frac{P_{compt}}{P_{synch}} = \frac{U_{rad}}{U_B} \qquad (7.46)$$

Consequence of the fact that (in QED) synchrotron radiation is inverse Compton scattering off virtual photons of the B-field.

For $U_{rad} > U_B$, $P_{compt} > P_{synch}$
$\Longrightarrow$ (synchrotron) photon field will undergo dramatic amplification
$\Longrightarrow$ very efficient cooling of electrons by inverse Compton losses (Compton catastrophe).

As a result, the brightness temperature of radio sources is limited to 10^{12} K.

See worksheet 7 for a proof.

Amplification factor

In electron frame of rest,

$$\frac{\Delta E}{E} = -\frac{E}{m_e c^2} \qquad (7.16)$$

For Maxwellian electrons, a similar relation must hold:

$$\frac{\Delta E}{E} = -\frac{E}{m_e c^2} + \frac{\alpha k T_e}{m_e c^2} \qquad (7.47)$$

where α is so far unknown.

In complete thermodynamical equilibrium, photons and electrons interact only through scattering
$\Longrightarrow$ Photons have Bose-Einstein distribution,

$$N(E) = KE^2 \exp\left(-\frac{E}{kT_e}\right) \qquad (7.48)$$

with

$$\langle E\rangle = 3kT_e \quad \text{and} \quad \left\langle E^2\right\rangle = 12(kT_e)^2 \qquad (7.49)$$

In equilibrium, $\Delta E = 0 \Longrightarrow$

$$\langle \Delta E\rangle = 0 = \frac{\alpha k T_e}{m_e c^2}\langle E\rangle - \frac{\langle E^2\rangle}{m_e c^2} = (\alpha - 4)\frac{3(kT_e)^2}{m_e c^2} \qquad (7.50)$$

such that $\alpha = 4$.

Compton y

We have seen that

$$\frac{\Delta E}{E} \approx \frac{4kT_e - E}{m_e c^2} =: A \qquad (7.51)$$

where A is the Compton amplification factor. Thus:

$E \lesssim 4kT_e \Longrightarrow$ Photons gain energy, gas cools down.
$E \gtrsim 4kT_e \Longrightarrow$ Photons loose energy, gas heats up.

A generalization of the Compton amplification factor for relativistic energies is

$$A = 1 + 4\Theta\frac{K_3(1/\Theta)}{K_2(1/\Theta)} \approx 4\Theta + 16\Theta^2 \qquad (7.52)$$

where $K_i(x)$: modified Bessel functions of 2nd kind (Zdziarski 1985).

Total relative energy change by traversal of hot ($E \ll kT_e$) medium with optical depth $\tau_e = n_e\sigma_T l$:

$$(\text{rel. energy change}) = \frac{\text{rel. energy change}}{\text{scattering}} \times (\#\text{ scatterings}) \qquad (7.53)$$

$$y = \frac{4kT_e}{m_e c^2}\max(\tau_e, \tau_e^2) \qquad (7.54)$$

"Compton y-Parameter"

Spectral shape

The exact equation describing thermal Comptonization is a non-relativistic diffusion equation for the motion of photons through phase-space first derived by Kompaneets (1957):

$$\frac{\partial n}{\partial y} = \frac{1}{x^2}\frac{\partial}{\partial x}x^4\left(n + n^2 + \frac{\partial n}{\partial x}\right) \qquad (7.55)$$

(Kompaneets equation) where

$n = I(E)\frac{(hc)^2}{8\pi E^3}$: Photon Occupation Number
$x = E/kT_e$: Photon energy
$y = \frac{4kT_e}{m_e c^2}\sigma_T N_e ct$: Kompaneets parameter

Interpretation:

$\partial n/\partial x$: Doppler-Motion
n : Recoil-Effect
n^2 : Induced/Stimulated emission

Approximate solutions pf the Kompaneets equation can be obtained from the theory of random walks. See worksheet 7.

The derivation of the Kompaneets equation is relatively complicated, so only an outline can be given here.

Since Comptonization is caused by the scattering of photons by electrons, it is a good idea to describe Comptonization by means of a diffusion process, where photons are diffusing in phase space until a steady state situation is reached. In statistical mechanics, processes of this kind are treated using the *Master Equation*. This equation is obtained by equating the (energy) losses and gains for each energy-cell $\mathrm{d}E'$ in phase space. Denote by $N(E,t)\,\mathrm{d}E$ the probability that an arbitrary particle in a statistical ensemble has the energy E at time t, it is easy to see (Pathria, Statistical Mechanics, eq. 13.5.1)

$$\frac{\partial N(E,t)}{\partial t} = \int_{-\infty}^{+\infty} \left(-N(E,t)W(E,E') + N(E',t)W(E',E)\right)\,\mathrm{d}E' \qquad (7.56)$$

where $W(E,E')\,\mathrm{d}E'\,\mathrm{d}t$ is the probability that a particle is making a transition from energy E to energy E' in time interval $\mathrm{d}t$. Under the assumption that the only transitions that are important are those where E' is very close to E, $W(E,E')$ is a function that is strongly peaked around (E',E'). In this case it makes sense to transform to a new variable $\xi = E' - E$ and to use a function $W_E(\xi)$ which is strongly peaked around $\xi = 0$. Expanding Eq. (7.56) in a Taylor series in ξ to second order yields

$$\frac{\partial N(E,t)}{\partial t} = -\frac{\partial}{\partial E}\left(\mu_1(E)N(E,t)\right) + \frac{1}{2}\frac{\partial^2}{\partial E^2}\left(\mu_2(E)N(E,t)\right) \qquad (7.57)$$

This equation is the *Fokker-Planck equation*. The Fokker-Planck coefficients μ_1 and μ_2 are given by

$$\mu_1(E) = \int_{-\infty}^{+\infty} \xi W_E(\xi)\,\mathrm{d}\xi \qquad (7.58)$$

$$\mu_2(E) = \int_{-\infty}^{+\infty} \xi^2 W_E(\xi)\,\mathrm{d}\xi \qquad (7.59)$$

i.e., by the first and second moment of $W_E(\xi)$.

If the scattering of photons is to be described, then $N(E,t)$ is given by the spectral photon number density, and μ_1 gives the average change of the photon energy in a unit time, $\mu_1 = \langle \Delta E/\Delta t\rangle$, while μ_2 is the mean change of the energy-change squared, $\mu_2 = \mathrm{d}\left\langle \Delta E^2\right\rangle/\mathrm{d}t$ which we have computed on the previous slides. Therefore, after multiplication with $N_\mathrm{e}\sigma_\mathrm{T}c$ the moments are

$$\frac{\mu_1}{E} = N_\mathrm{e}\sigma_\mathrm{T}c\left(-\frac{E}{m_\mathrm{e}c^2} + \frac{4kT_\mathrm{e}}{m_\mathrm{e}c^2}\right) \qquad (7.60)$$

$$\frac{\mu_2}{E^2} = N_\mathrm{e}\sigma_\mathrm{T}c\frac{2kT_\mathrm{e}}{m_\mathrm{e}c^2} \qquad (7.61)$$

The evaluation of μ_1 and μ_2 in the case of relativistic Compton scattering is much more involved because the Klein-Nishina cross-section has to be used. For cold gas the moments μ_1 and μ_2 have been given to high precision by Xu et al. (1991).

Instead of using the Fokker-Planck equation and the phase space density it is usually more convenient to use the photon occupation number n. The photon occupation number is related to the spectral radiative intensity by $n = I(E)\,(hc)^2/8\pi E^3$. Substituting this into Eq. (7.57) and using the nonrelativistic moments one obtains the classical version of the Kompaneets equation, i.e., the Kompaneets equation without the n^2 term. This latter term comes from stimulated scattering and can only be derived using quantum-mechanical arguments. For a derivation of the Kompaneets equation from the Boltzmann equation, see Chapter 7.6 of Rybicki & Lightman.

Approximate spectral shape, I

Photon spectra can be found by analytically solving the Kompaneets equation. See Sunyaev & Titarchuk (1980) for examples. Solution is only possible for special cases and simple geometries.

For the most common case, unsaturated Comptonization, one obtains

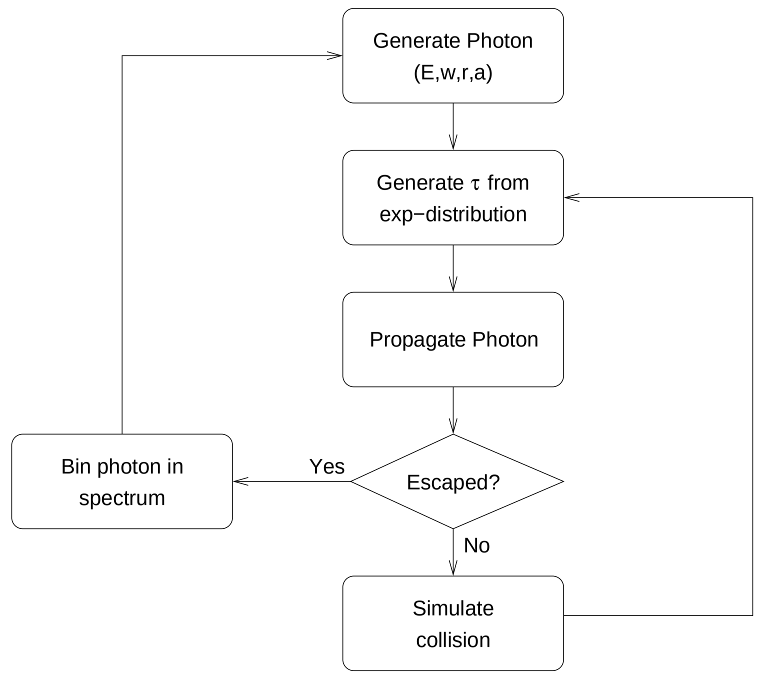
$$I(x) \propto \begin{cases} x^3\exp(-x) & \text{for } x \gg 1 \\ x^{3-\Gamma} & \text{for } x \ll 1 \end{cases} \qquad (7.62)$$

where

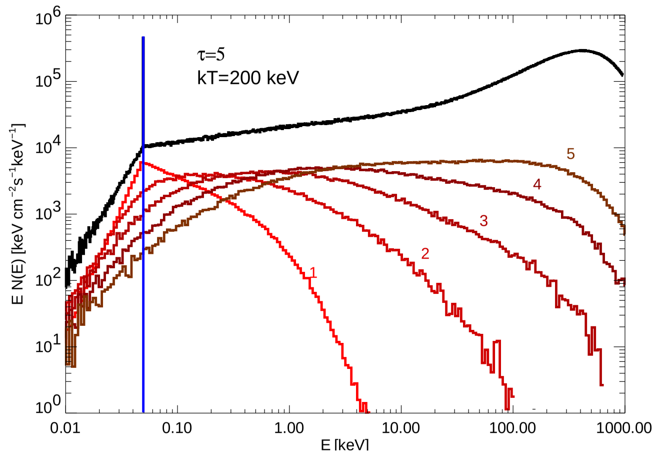
$$\Gamma = \frac{3}{2} \mp \sqrt{\frac{9}{4} + \frac{4}{y}} \qquad (7.63)$$

where $-$-root for $y \gg 1$, $+$-root for $y \ll 1$, and average for $y \sim 1$. Typical sources have $y \sim 1$, i.e., power law with photon index ~ 1.5

General solution: Possible via the Monte Carlo method (Pozdnyakov et al., 1982,...)

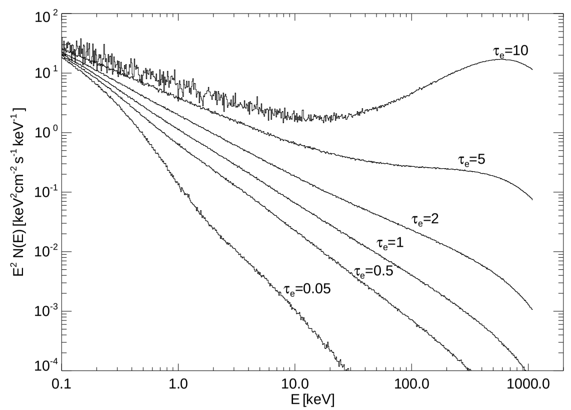


Results: Spectrum



Monte Carlo simulation shows: Spectrum is $\Longrightarrow$ Power law with exponential cutoff (here: with additional “Wien hump”, see next slide)

Results: Spectrum

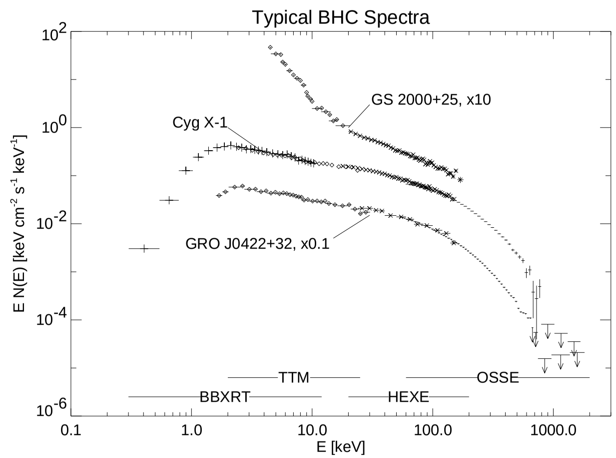


Sphere with $kT_e = 0.7 m_e c^2$ ($\sim$ 360 keV), seed photons come from center of sphere.

$y \ll 1$: pure power-law.
$y < 1$: power-law with exponential cut-off
$y \gg 1$: “Saturated Comptonization”.

Saturated Comptonization has never been observed.

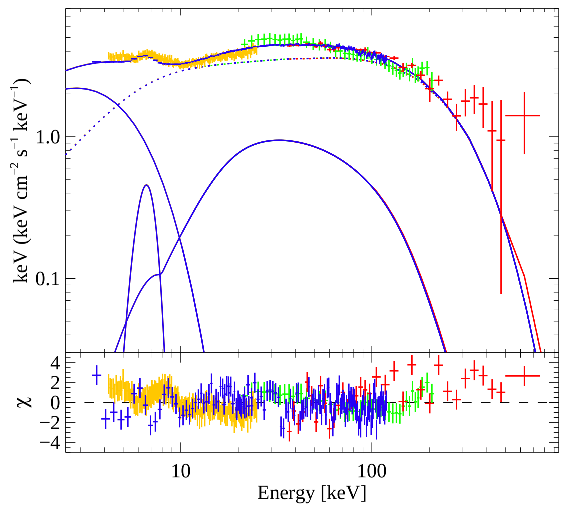
Galactic Black Holes, I



(Cyg X-1: Wilms et al., 1996, ; GRO J0422+32, GS2000+25: Sunyaev et al., 1993, Kroeger [priv. comm.])

X-ray spectra of galactic black hole candidates can be well explained by thermal Comptonization in a plasma with $kT \sim 150$ keV and with $y \sim 1$.

Galactic Black Holes, II



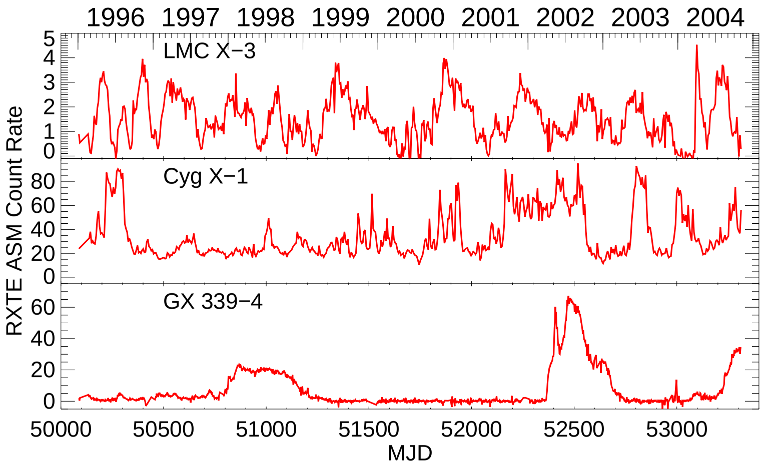
Fritz, et al., 2007

Fit of a `Comptonization` model to *RXTE*/*INTEGRAL* data from the galactic black hole Cygnus X-1

$kT_{\text{soft}} = 1.21$ keV,
$\tau_e = 1.09$,
$kT_e \sim 100$ keV

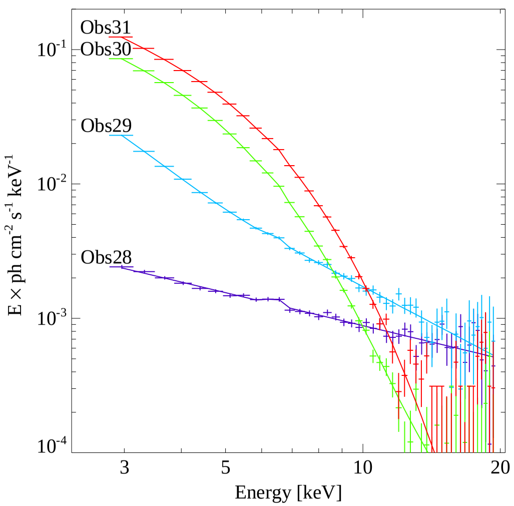
Note presence of a Compton reflection hump (evidence of close vicinity of hot electrons and only mildly ionized material)

Galactic Black Holes, III



Black Holes: Variability on all time scales

Spectral States, I

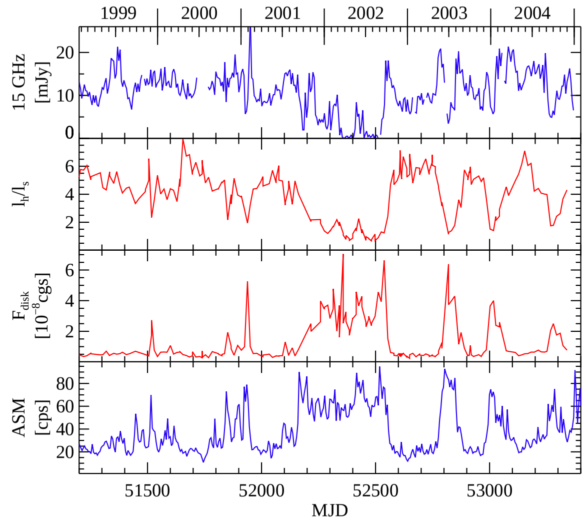


LMC X-3, Wilms et al. (2001)

X-ray States:

- $L_X \gtrsim 0.05\, L_{Edd}$:
  soft state/high state:
  - thermally dominated
  - low variability (few percent rms)
- $L_X \lesssim 0.05\, L_{Edd}$:
  hard state/low state:
  - power law spectrum,
  - high variability (few 10% rms)

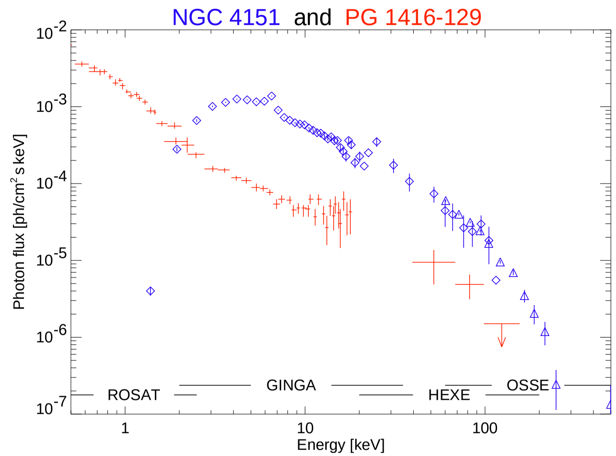
Spectral States, II



Cyg X-1 (Wilms et al., 2006)

New satellites allow detailed study of evolution of Comptonizing plasma over timescales of years.

AGN



(PG 1416−129: de Kool et al., 1994, Williams et al., 1992, Staubert & Maisack, 1996; NGC 4151: Maisack 1991, 1993)

Note: NGC 4151 not corrected for interstellar absorption.

Spectral shape of AGN very similar to galactic Black Holes ⟹ Same physical mechanism (=Comptonization) responsible!