



### Question 1: *Fourier Transform of Time Derivatives*

The Fourier transform pair was defined in the lectures via

$$f(\omega) = \int f(t)e^{i\omega t} dt \iff f(t) = \frac{1}{2\pi} \int d(\omega)e^{-i\omega t} d\omega \quad (\text{w1.1})$$

Show that the Fourier transform of the time derivative of  $f(t)$ ,  $\dot{f}(\omega)$ , can be calculated from the very useful equation

$$\dot{f}(\omega) = -i\omega f(\omega) \quad (\text{w1.2})$$

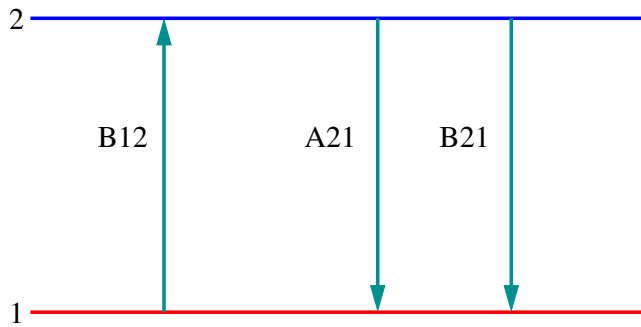
and therefore

$$\ddot{f}(\omega) = \omega^2 f(\omega) \quad (\text{w1.3})$$

*Hint:* Partial integration!

*Hint 2:* A necessary condition for a Fourier transform to exist is  $\lim_{t \rightarrow \pm\infty} f(t) = 0$

## Question 2: Self-Absorption and the Principle of Detailed Balance



The emission and absorption of a 2-level system can be described with the Einstein-coefficients, where  $A_{21}$  is the transition probability per unit time for spontaneous emission, and where  $B_{12}I_\nu$  and  $B_{21}I_\nu$  are the transition probabilities for absorption and for stimulated emission per unit time.

As shown in the lectures, these coefficients are not independent of each other, but obey the Einstein relations:

$$g_1 B_{12} = g_2 B_{21} \quad \text{and} \quad A_{21} = \frac{2h\nu_{21}^3}{c^2} B_{21} \quad (\text{s2.1})$$

- a) Let  $n_1$  and  $n_2$  be the population densities of level 1 and 2. Convince yourself that the equation of radiative transfer,

$$\frac{dI_\nu}{ds} = j_\nu - \alpha_\nu I_\nu \quad (\text{s2.2})$$

can be written in terms of Einstein coefficients as

$$\frac{dI_\nu}{ds} = \frac{h\nu_{21}n_2A_{21}}{4\pi} - n_1B_{12}h\nu_{21}I_\nu + n_2B_{21}h\nu_{21}I_\nu \quad (\text{s2.3})$$

- b) Show from the above equation that

$$j_\nu = \frac{h\nu_{21}n_2A_{21}}{4\pi} \quad (\text{s2.1})$$

- c) Noting that in thermodynamic equilibrium

$$\frac{n_2}{n_1} = \frac{g_2}{g_1} \exp\left(-\frac{h\nu_{21}}{kT}\right) \quad (\text{s2.1})$$

show that

$$\alpha_\nu = h\nu_{21}n_1B_{12} \left(1 - \exp\left(-\frac{h\nu_{21}}{kT}\right)\right) \quad (\text{s2.2})$$