AGN Evolution

Introduction

Result of previous lectures:

AGN produce large amounts of energy over timescales of $\gtrsim 10^8$ years and they strongly interact with their environment.

Questions:

- What galaxies harbor AGN?
- Are these galaxies different from others?
- How do galaxies with AGN evolve?
- How do AGN form?

To answer these questions, we need to study statistical properties of AGN and their hosts, both among morphological type and with time: AGN surveys

But first, we need to talk about the basics of doing science in an expanding universe.

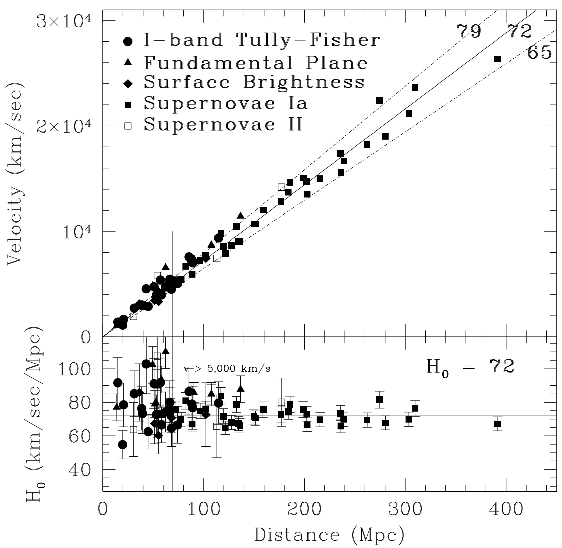
Basic Facts

Observations show that there are *four major facts* about the universe as a whole:

The universe is:
- expanding,
- isotropic,
- and homogeneous.

That the universe is isotropic and homogeneous is called the *cosmological principle*.

Expansion



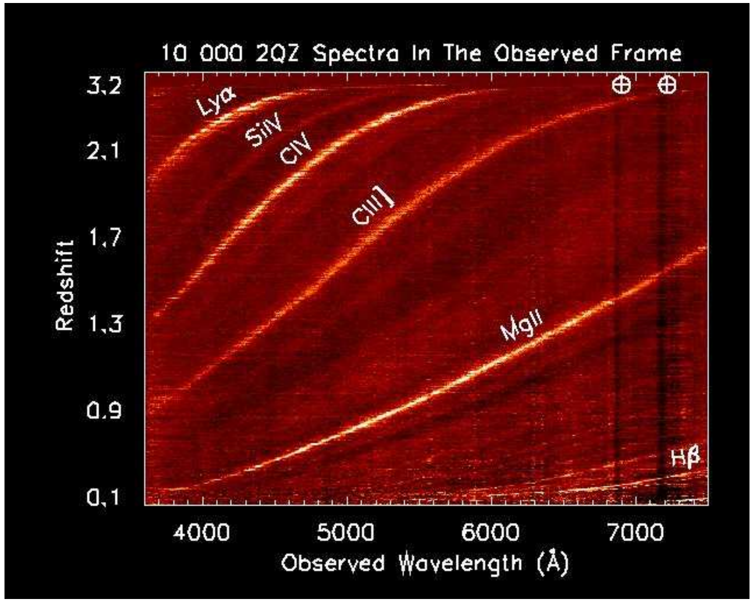
Hubble (1929): The "velocity", v, of a galaxy depends linearly from its distance, d: $v(r) = H_0 d$

where $v/c = \Delta\lambda/\lambda$ and where H_0: Hubble constant or Hubble parameter.

Currently accepted value:

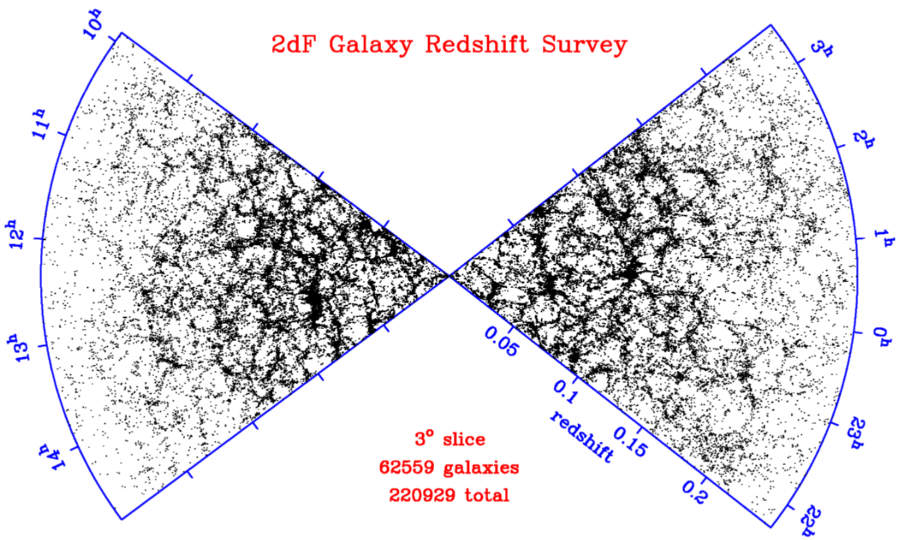
$$H_0 = 72 \pm 8\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \quad (10.1)$$

Freedman et al. (2001, Fig. 4)



courtesy 2dF QSO Redshift survey

As a consequence of the cosmological redshift, for different z different parts of the spectrum of a distant source are visible.



2dF Survey, ~220000 galaxies total

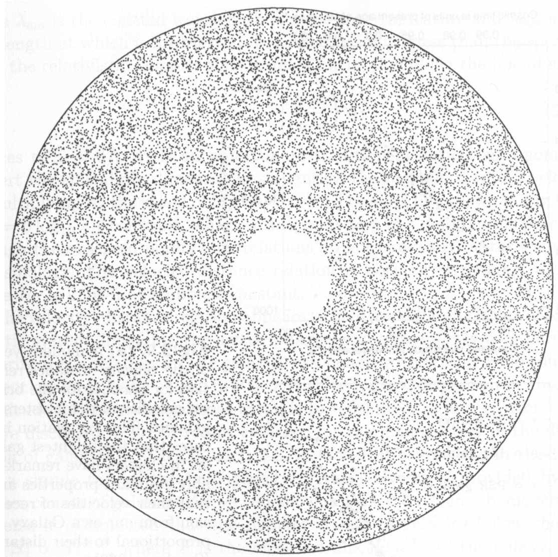
The universe is homogeneous $\Longleftrightarrow$ The universe looks the same everywhere in space

Testable by observing spatial distribution of galaxies.

On scales $\gg$ 100 Mpc the universe looks indeed the same. Below that: structure.

Structures seen are galaxy clusters (gravitationally bound) and superclusters (larger structures, not [yet] gravitationally bound).

Isotropy



The universe is isotropic $\Longleftrightarrow$ The universe looks the same in all directions

Radio galaxies are mainly quasars $\Longrightarrow$ Sample large space volume ($z \gtrsim 1$) $\Longrightarrow$ Clear isotropy.

Anisotropy in the image: galactic plane, exclusion region around Cyg A, Cas A, and the north celestial pole.

Peebles (1993): Distribution of 31000 objects at λ =6 cm from the Greenbank Catalogue.

World Models

World Model: theoretical framework describing a world governed by the cosmological principle.

Use combination of

- General Relativity
- Thermodynamics
- Quantum Mechanics

$\Longrightarrow$ Complicated!

For 99% of the work, the above points can be dealt with separately:

1. Define metric obeying cosmological principle.
2. Obtain equation for evolution of universe using Einstein field equations.
3. Use thermo/QM to obtain equation of state.
4. Solve equations.

World Models

Before we can start to think about universe: Brief introduction to assumptions of general relativity.

$\Longrightarrow$ See theory lectures for the gory details, or check with the literature (Weinberg or MTW).

Assumptions of GRT:

- Space is 4-dimensional, might be curved
- Matter (=Energy) modifies space (Einstein field equation).
- Covariance: physical laws must be formulated in a coordinate-system independent way.
- Strong equivalence principle: There is no experiment by which one can distinguish between free falling coordinate systems and inertial systems.
- At each point, space is locally Minkowski (i.e., locally, SRT holds).

$\Longrightarrow$ Understanding of geometry of space necessary to understand physics.

RW Metric

- Cosmological principle $+$ expansion $\Longrightarrow$ $\exists$ freely expanding cosmical coordinate system.
  - Observers $=:$ fundamental observers
  - Time $=:$ cosmic time

  This is the coordinate system in which the 3K radiation is isotropic and in which clocks can be synchronized, e.g., by adjusting time to the local density of the universe.

  $\Longrightarrow$ Metric has a temporal and a spatial part.

  This also follows directly from the equivalence principle.
- *Homogeneity and isotropy* $\Longrightarrow$ spatial part is spherically symmetric:

$$\mathrm{d}\psi^2 := \mathrm{d}\theta^2 + \sin^2\theta \;\mathrm{d}\phi^2 \tag{10.2}$$

- *Expansion:* $\exists$ scale factor, $R(t)$ $\Longrightarrow$ measure distances using comoving coordinates.

RW Metric

A metric based on these points looks like

$$\mathrm{d}s^2 = c^2 \;\mathrm{d}t^2 - R^2(t)\left[f^2(r)\;\mathrm{d}r^2 + g^2(r)\;\mathrm{d}\psi^2\right] \tag{10.3}$$

where $f(r)$ and $g(r)$ are arbitrary.

Metrics of the form of eq. (10.3) are called Robertson-Walker (RW) metrics (1935), but have been previously also studied by Friedmann and Lemaître.

One common choice is

$$\mathrm{d}s^2 = c^2 \;\mathrm{d}t^2 - R^2(t)\left[\mathrm{d}r^2 + S_k^2(r)\;\mathrm{d}\psi^2\right] \tag{10.4}$$

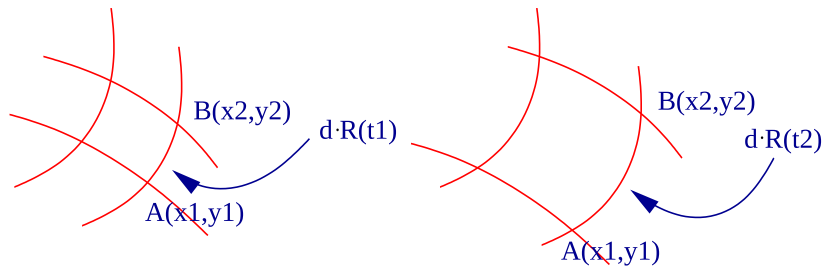
where $R(t)$: scale factor, containing the physics, t: cosmic time, r, θ, ϕ: comoving coordinates, and where

$$S_k(\theta) = \begin{cases} \sin\theta & \text{for } k = +1 \\ \theta & \text{for } k = 0 \\ \sinh\theta & \text{for } k = -1 \end{cases} \tag{10.5}$$

Remark: θ and ϕ describe directions on sky, as seen from the arbitrary center of the coordinate system (=us), r can be interpreted as a radial coordinate.

RW Metric

RW metric: defines universal coordinate system tied to the expansion of space:



Scale factor $R(t)$ describes evolution of universe.

- d is called the comoving distance.
- $D(t) := d \cdot R(t)$ is called the proper distance.

(note that R is unitless, i.e., d and $d \cdot R(t)$ are measured in Mpc)

Hubble's Law

Hubble's Law follows from the variation of $R(t)$:

Small scales $\Longrightarrow$ Euclidean geometry. Proper distance between two observers:

$$D(t) = d \cdot R(t) \quad (10.6)$$

Expansion $\Longrightarrow$ proper separation changes:

$$\frac{\Delta D}{\Delta t} = \frac{R(t+\Delta t)d - R(t)d}{\Delta t} \quad \text{with} \lim_{\Delta t \to 0}: \quad v = \frac{\mathrm{d}D}{\mathrm{d}t} = \dot{R}\, d = \frac{\dot{R}}{R} D =: H\, D \quad (10.7)$$

$\Longrightarrow$ Identify local Hubble "constant" with

$$H = \dot{R}/R = \dot{a}(t) \quad \text{where} \quad a(t) = R(t)/R(\text{today}) \quad (10.8)$$

Note that $R = R(t) \Longrightarrow H$ is time-dependent!

Hubble's Law

The cosmological redshift is a consequence of the expansion of the universe:

Since the comoving distance is constant:

$$d = \frac{D(t = \text{today})}{R(t = \text{today})} = \frac{D(t)}{R(t)} = \text{const.} \quad (10.9)$$

Set $a(t) = R(t)/R(t = \text{today})$, then Eq. (10.9) implies

$$\lambda_{\text{obs}} = \frac{\lambda_{\text{emit}}}{a_{\text{emit}}} \iff z = \frac{\lambda_{\text{obs}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}} = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}} - 1 \quad (10.10)$$

(z: observed redshift, λ_{obs}: observed wavelength, λ_{emit}: emitted wavelength)

$$1 + z = \frac{1}{a_{\text{emit}}} = \frac{R(t = \text{today})}{R(t)} \quad (10.11)$$

Light emitted at $z = 1$ was emitted when the universe was half as big as today!

z: measure for *relative size* of universe at time the observed light was emitted.

Hubble's Law

For light, $d = c\Delta t$. Therefore

$$\frac{c\,\Delta t_{\text{emit}}}{R(t_{\text{emit}})} = \frac{c\,\Delta t_{\text{obs}}}{R(t_{\text{obs}})} \quad \text{such that} \quad \frac{\mathrm{d}t}{R(t)} = \text{const.} \quad (10.12)$$

This means that

$$\frac{\mathrm{d}t_{\text{obs}}}{\mathrm{d}t_{\text{emit}}} = \frac{R(t_{\text{obs}})}{R(t_{\text{emit}})} = 1 + z \quad (10.13)$$

$\Longrightarrow$ Time dilatation of events at large z.

This cosmological time dilatation has been observed in the light curves of supernova outbursts.

Expansion and Spectra

The total number of photons in a box $\mathrm{d}A \cdot c\,\mathrm{d}t$ and in a frequency range ν to $\nu + \mathrm{d}\nu$ is

$$N = n_\nu(\nu)\ \mathrm{d}A\ \mathrm{d}\nu\, c\ \mathrm{d}t \quad (10.14)$$

This number is conserved during the expansion of the universe:

$$n_\nu(\nu_{\text{emit}})\ \mathrm{d}A\ \mathrm{d}\nu_{\text{emit}}\, c\ \mathrm{d}t_{\text{emit}} = n_\nu(\nu_{\text{obs}}) \frac{d\nu_{\text{emit}}}{1+z}\ \mathrm{d}A\ c\ \mathrm{d}t_{\text{emit}}(1+z) \quad (10.15)$$

$$= n_\nu(\nu_{\text{obs}})\ \mathrm{d}A\ \mathrm{d}\nu_{\text{obs}}\, c\ \mathrm{d}t_{\text{obs}} \quad (10.16)$$

but: arrival time differs $\Longrightarrow$ energy flux density changes:

$$F_\nu(\nu_{\text{obs}}) = h\nu_{\text{obs}} n_\nu(\nu_{\text{obs}}) = \frac{h\nu_{\text{emit}}}{1+z} n_\nu(\nu_{\text{emit}}) = \frac{F_\nu(\nu_{\text{emit}})}{1+z} \quad (10.17)$$

and consequently the total flux in a certain energy band changes as well:

$$F_{\text{obs}} = \int F_\nu(\nu_{\text{obs}})\ \mathrm{d}\nu_{\text{obs}} = \int \frac{F_\nu(\nu_{\text{emit}})}{1+z} \cdot \frac{\mathrm{d}\nu_{\text{emit}}}{1+z} = \frac{F_{\text{emit}}}{(1+z)^2} \quad (10.18)$$

One power of $1 + z$ from decreased photon energy, one from decreased arrival rate.

For wavelength based flux densities, since $F_\lambda = F_\nu c/\lambda^2$ one finds $F_\lambda(\lambda_{\text{obs}}) = F_\lambda(\lambda_{\text{emit}})/(1+z)^3$.

Luminosity Distance

For AGN studies at high z, we need to take into account cosmological effects: How to convert an observed flux into luminosity.

Assume source with luminosity L at comoving coordinate r. When its light has reached us, it has spread over a sphere of area

$$A = 4\pi(R_0 r)^2 \tag{10.19}$$

R_0: today's scale factor

such that the flux measured in the same reference frame is

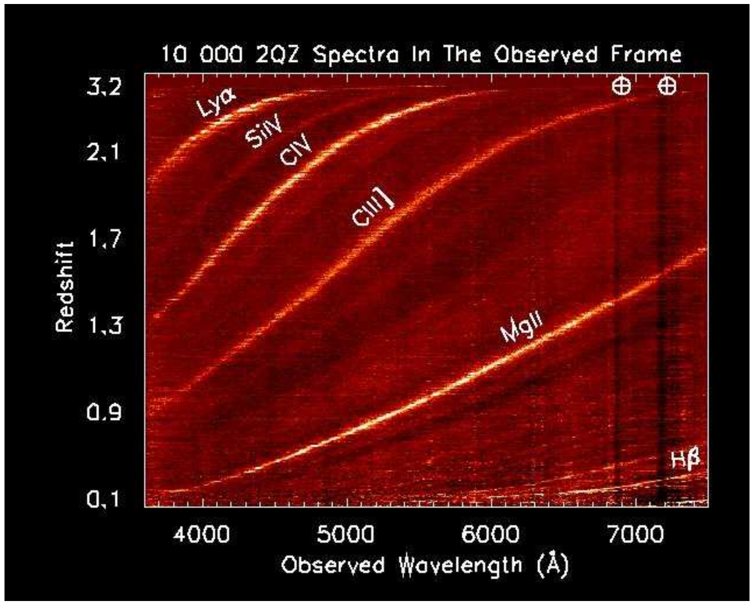
$$F_{\text{obs, ref}} = \frac{L}{4\pi(R_0 r)^2} \tag{10.20}$$

and the observed bolometric flux is (correcting for Doppler effect):

$$F_{\text{obs}} = \frac{F_{\text{ref}}}{(1+z)^2} = \frac{L}{4\pi(1+z)^2(R_0 r)^2} = \frac{L}{4\pi D_{\text{L}}^2} \tag{10.21}$$

where the luminosity distance, D_{L} is defined by

$$D_{\text{L}} = (1+z)R_0 r \tag{10.22}$$



courtesy 2dF QSO Redshift survey

As a consequence of the cosmological redshift, for different z different parts of the spectrum of a distant source are visible.

K-correction

As shown on previous slide, in practice, we do not measure bolometric fluxes, but we measure data in some frequency band.

$\Longrightarrow$ Gives problems in studies of z-dependence of luminosity

Analoguous to the derivation of Eq. 10.21 one finds

$$F_{\text{obs},\nu} = \frac{(1+z)L_{\text{em},\nu'}}{4\pi D_{\text{L}}^2} \tag{10.23}$$

where $F_{\text{obs},\nu}$ is flux observed at frequency ν and $L_{\text{em},\nu'}$ is emitted flux emitted at frequency $\nu' = (1+z)\nu$. This equation can be written as:

$$F_{\text{obs},\nu} = \frac{L_{\text{em},\nu}}{4\pi D_{\text{L}}^2}\left((1+z)\frac{L_{\text{em},\nu'}}{L_{\text{em},\nu}}\right) = \frac{L_{\text{em},\nu}}{4\pi D_{\text{L}}} \cdot K \tag{10.24}$$

$\Longrightarrow$ where K is the K-correction, i.e., a fudge factor correcting for the fact that source spectra change with frequency

introduces many uncertainties...

Friedmann Equations

General relativistic approach: Insert metric into Einstein equation to obtain differential equation for $R(t)$:

Einstein equation:

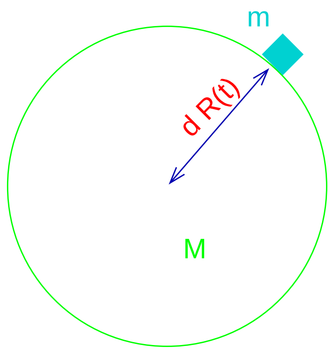
$$\underbrace{R_{\mu\nu} - \frac{1}{2}\mathscr{R}g_{\mu\nu}}_{G_{\mu\nu}} = \frac{8\pi G}{c^4}T_{\mu\nu} + \Lambda g_{\mu\nu} \tag{10.25}$$

where

$g_{\mu\nu}$: Metric tensor ($\mathrm{d}s^2 = g_{\mu\nu}\ \mathrm{d}x^\mu\ \mathrm{d}x^\nu$)
$R_{\mu\nu}$: Ricci tensor (function of $g_{\mu\nu}$)
$\mathscr{R}$: Ricci scalar (function of $g_{\mu\nu}$)
$G_{\mu\nu}$: Einstein tensor (function of $g_{\mu\nu}$)
$T_{\mu\nu}$: Stress-energy tensor, describing curvature of space due to fields present (matter, radiation,...)
Λ: Cosmological constant

$\Longrightarrow$ Messy, but doable

Friedmann Equations



Here, Newtonian derivation of Friedmann equations: Dynamics of a mass element on the surface of sphere of density $\rho(t)$ and comoving radius d, i.e., proper radius $d \cdot R(t)$ (after McCrea & Milne, 1934).

Mass of sphere:

$$M = \frac{4\pi}{3}(d\,R)^3\rho(t) = \frac{4\pi}{3}d^3\rho_0 \quad \text{where} \quad \rho(t) = \frac{\rho_0}{R(t)^3} \qquad (10.26)$$

Force on mass element:

$$m\frac{\mathrm{d}^2}{\mathrm{d}t^2}(d\,R(t)) = -\frac{GMm}{(dR(t))^2} = -\frac{4\pi G}{3}\frac{d\rho_0}{R^2(t)}m \qquad (10.27)$$

Canceling $m \cdot d$ gives the momentum equation:

$$\ddot{R} = -\frac{4\pi G}{3}\frac{\rho_0}{R^2} = -\frac{4\pi G}{3}\rho(t)R(t) \qquad (10.28)$$

Friedmann Equations

Multiplying

$$\ddot{R} = -\frac{4\pi G}{3}\frac{\rho_0}{R^2} = -\frac{4\pi G}{3}\rho(t)R(t) \qquad (10.28)$$

with $\dot{R}$ and integrating, or alternatively considering energy conservation yields the energy equation,

$$\begin{aligned}\frac{1}{2}\dot{R}^2 &= +\frac{4\pi G}{3}\frac{\rho_0}{R(t)} + \text{const.}\\ &= +\frac{4\pi G}{3}\rho(t)R^2(t) + \text{const.}\end{aligned} \qquad (10.29)$$

where the constant can only be obtained from GR.

[redacted]

Note: derivation implicitly assumes $r_{\text{cloud}} < \infty$, which violates the cosmological principle, and assumes that the particle moves through space, which violates SRT. However, since GR $\sim$ Newton on small scales and mass densities, there is a scale invariance on Mpc scales and Newton is valid in the classical limit of GR.

Friedmann Equations

The exact GR derivation of Friedmanns equation gives:

$$\begin{aligned}\ddot{R} &= -\frac{4\pi G}{3}R\left(\rho + \frac{3p}{c^2}\right) + \left[\frac{1}{3}\Lambda R\right]\\ \dot{R}^2 &= +\frac{8\pi G\rho}{3}R^2 - kc^2 + \left[\frac{1}{3}\Lambda c^2 R^2\right]\end{aligned} \qquad (10.30)$$

Notes:

1. For $k = 0$: Eq. (10.30) $\longrightarrow$ Eq. (10.29).
2. k determines curvature of space.
3. The density, ρ, includes the contribution of all different kinds of energy (remember mass-energy equivalence!).
4. There is energy associated with the vacuum, parameterized by the parameter Λ.

The evolution equation of the Hubble parameter is ($\Lambda = 0$):

$$\left(\frac{\dot{R}}{R}\right)^2 = H^2(t) = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2} \qquad (10.31)$$

Ω

Solving Eq. (10.31) for k:

$$\left(\frac{\dot{R}}{R}\right)^2 = H^2(t) = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2} \iff \frac{R^2}{c}\left(\frac{8\pi G}{3}\rho - H^2\right) = k \qquad (10.32)$$

$\Longrightarrow$ Sign of curvature parameter k only depends on density, ρ:

Defining

$$\rho_c = \frac{3H^2}{8\pi G} \quad \text{and} \quad \Omega = \frac{\rho}{\rho_c} \qquad (10.33)$$

it is easy to see that:

$$\begin{aligned}\Omega > 1 &\Longrightarrow k > 0 \quad \text{closed}\\ \Omega = 1 &\Longrightarrow k = 0 \quad \text{flat}\\ \Omega < 1 &\Longrightarrow k < 0 \quad \text{open}\end{aligned}$$

thus ρ_c is called the critical density.

For $\Omega \leq 1$ the universe will expand until ∞

For $\Omega > 1$ we will see the “big crunch”

Current value of ρ_c: $\sim 1.67 \times 10^{-24}\,\text{g}\,\text{cm}^3$, (3…10 H-atoms m^{-3}).

Current Scale Factor

The current scale factor is determined by H_0 and Ω_0:

Friedmann for $t = t_0$:

$$\dot{R}_0^2 - \frac{8\pi G}{3}\rho R_0^2 = -kc^2 \qquad (10.34)$$

Insert Ω and note $H_0 = \dot{R}_0/R_0$

$$\Longleftrightarrow H_0^2 R_0^2 - H_0^2 \Omega_0 R_0^2 = -kc^2 \qquad (10.35)$$

And therefore

$$R_0 = \frac{c}{H_0}\sqrt{\frac{k}{\Omega - 1}} \qquad (10.36)$$

For $\Omega \longrightarrow 0$, $R_0 \longrightarrow c/H_0$, the Hubble length, for $\Omega = 1$, R_0 is arbitrary.

We now have everything we need to solve the Friedmann equation and determine the evolution of the universe.

Current Scale Factor

To study the evolution of the universe, we need to know the equation of state of the stuff filling the universe:

- Matter: Normal particles get diluted by expansion of the universe:

$$\rho_\mathrm{m} \propto R^{-3} \qquad (10.37)$$

- Radiation: The energy density of radiation decreases because of volume expansion and because of the cosmological redshift ($\lambda_\mathrm{o}/\lambda_\mathrm{e} = \nu_\mathrm{e}/\nu_\mathrm{o} = R(t_\mathrm{o})/R(t_\mathrm{e})$):

$$\rho_\mathrm{r} \propto R^{-4} \qquad (10.38)$$

- Vacuum: The vacuum energy density (=Λ) is independent of R:

$$\rho_\mathrm{v} = \text{const.} \qquad (10.39)$$

Inserting these equations of state into the Friedmann equation and solving with the boundary condition $R(t = 0) = 0$ then gives a specific world model.

Example: Matter Dominated Universes

For the matter dominated, flat case (the Einstein-de Sitter case), the Friedmann equation is

$$\dot{R}^2 - \frac{8\pi G}{3}\frac{\rho_0 R_0^3}{R^3} R^2 = 0 \qquad (10.40)$$

Use Eq. 10.32 and use that for $\Omega = 1$

$$\frac{8\pi G \rho_0}{3} = \Omega_0 H_0^2 R_0^3 = H_0^2 R_0^3 \qquad (10.41)$$

Therefore, Friedmann's Eq. is

$$\dot{R}^2 - \frac{H_0^2 R_0^3}{R} = 0 \quad \Longrightarrow \quad \frac{\mathrm{d}R}{\mathrm{d}t} = H_0 R_0^{3/2} R^{-1/2} \qquad (10.42)$$

Separation of variables and the boundary condition $R(t = 0) = 0$ gives

$$\int_0^{R(t)} R^{1/2}\,\mathrm{d}R = H_0 R_0^{3/2} t \quad \Longleftrightarrow \quad \frac{2}{3} R^{3/2}(t) = H_0 R_0^{3/2} t \quad \Longrightarrow R(t) = R_0 \left(\frac{3H_0}{2}\,t\right)^{2/3} \qquad (10.43)$$

$\Longrightarrow$ For $k = 0$, the universe expands until ∞. Its current age is given by ($R(t_0) = R_0$)

$$t_0 = \frac{2}{3H_0} \quad \text{where the Hubble-Time is} \quad H_0^{-1} = \frac{9.78\,\text{Gyr}}{H_0/100\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}} \qquad (10.44)$$

For the other two types of universes dominated purely by matter, the Friedmann equations can be solved in a way similar to that used for the Einstein-de Sitter case:

For the matter dominated, closed case, Friedmann's equation is

$$\dot{R}^2 - \frac{8\pi G}{3}\frac{\rho_0 R_0^3}{R} = -c^2 \quad \Longleftrightarrow \quad \dot{R}^2 - \frac{H_0^2 R_0^3 \Omega_0}{R} = -c^2 \qquad (10.45)$$

Inserting R_0 from Eq. (10.36) gives

$$\dot{R}^2 - \frac{H_0^2 c^3 \Omega_0}{H_0^3 (\Omega - 1)^{3/2}}\frac{1}{R} = -c^2 \qquad (10.46)$$

which is equivalent to

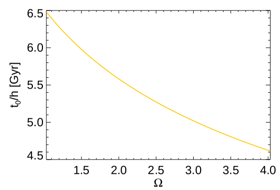
$$\frac{\mathrm{d}R}{\mathrm{d}t} = c\left(\frac{\xi}{R} - 1\right)^{1/2} \quad \text{with} \quad \xi = \frac{c}{H_0}\frac{\Omega_0}{(\Omega_0 - 1)^{3/2}} \qquad (10.47)$$

With the boundary condition $R(0) = 0$, separation of variables gives

$$ct = \int_0^{R(t)} \frac{\mathrm{d}R}{(\xi/R - 1)^{1/2}} = \int_0^{R(t)} \frac{\sqrt{R}\,\mathrm{d}R}{(\xi - R)^{1/2}} \qquad (10.48)$$

Integration by substitution gives

$$R = \xi \sin^2\frac{\theta}{2} = \frac{\xi}{2}(1 - \cos\theta) \quad \Longrightarrow \quad ct = \frac{\xi}{2}(\theta - \sin\theta) \qquad (10.49)$$

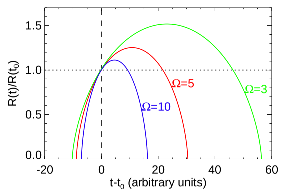


In this case the age of the universe, t_0, is obtained by solving

$$R_0 = \frac{c}{H_0(\Omega_0 - 1)^{1/2}} \qquad (10.50)$$

and can be shown to be

$$t_0 = \frac{1}{2H_0}\frac{\Omega_0}{(\Omega_0 - 1)^{3/2}}\left[\arccos\left(\frac{2 - \Omega_0}{\Omega_0}\right) - \frac{2}{\Omega_0}\sqrt{\Omega_0 - 1}\right] \qquad (10.51)$$



Since R is a cyclic function, the closed universe has a finite lifetime. The max. expansion is reached for $\theta = \pi$, with a maximum scale factor of

$$R_{\max} = \xi = \frac{c}{H_0}\frac{\Omega_0}{(\Omega_0 - 1)^{3/2}} \quad (10.52)$$

After that the universe contracts to the big crunch at $\theta = 2\pi$. The lifetime of the closed universe is

$$t = \frac{\pi}{H_0}\frac{\Omega_0}{(\Omega_0 - 1)^{3/2}} \quad (10.53)$$

Finally, the matter dominated, open case. This case is very similar to the case of $k = +1$:

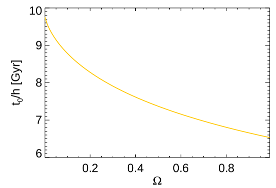
For $k = -1$, the Friedmann equation becomes

$$\frac{dR}{dt} = c\left(\frac{\zeta}{R} + 1\right)^{1/2} \quad \text{where} \quad \zeta = \frac{c}{H_0}\frac{\Omega_0}{(1 - \Omega_0)^{3/2}} \quad (10.54)$$

Separation of variables gives after a little bit of algebra

$$R = \frac{\zeta}{2}(\cosh\theta - 1) \quad \text{and} \quad ct = \frac{\zeta}{2}(\sinh\theta - 1) \quad (10.55)$$

where the integration was again performed by substitution.

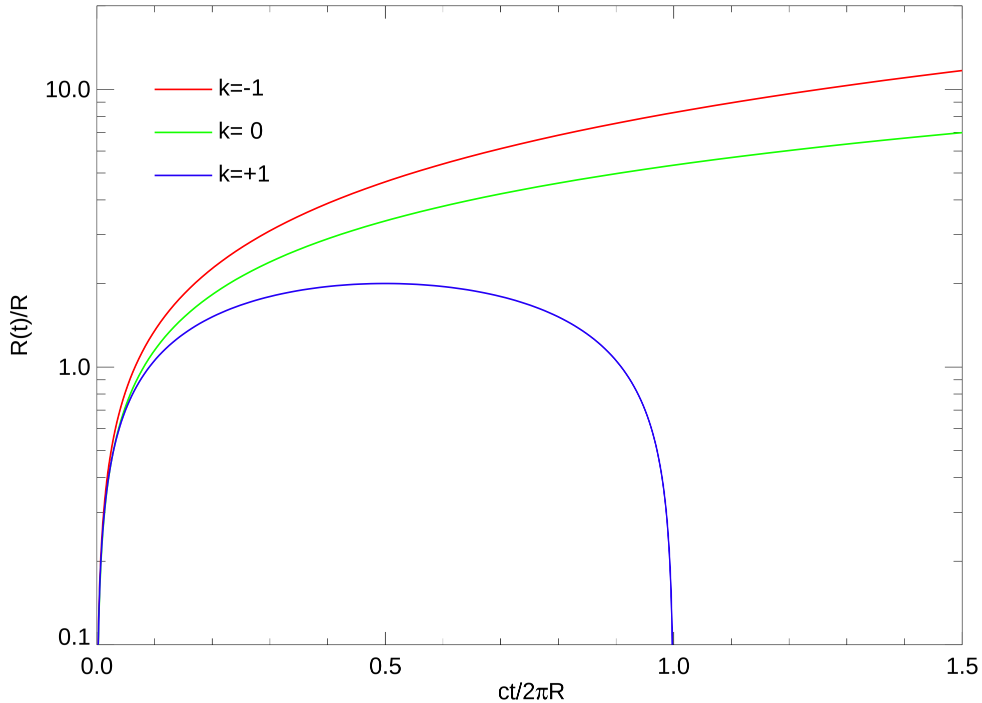


To obtain the age of the universe, note that at the present time,

$$\cosh\theta_0 = \frac{2 - \Omega_0}{\Omega_0}$$
$$\sinh\theta_0 = \frac{2}{\Omega_0}\sqrt{1 - \Omega_0} \quad (10.56)$$

(derived using a derivation identical to that leading to Eq. 10.50) such that

$$t_0 = \frac{1}{2H_0}\frac{\Omega_0}{(1 - \Omega_0)^{3/2}} \cdot \left\{\frac{2}{\Omega_0}\sqrt{1 - \Omega_0} - \ln\left(\frac{2 - \Omega_0 + 2\sqrt{1 - \Omega_0}}{\Omega_0}\right)\right\} \quad (10.57)$$



Example: Matter Dominated Universes

For the matter dominated case, our results from Eqs. (10.49), and (10.55) can be written in form of the cycloid solution

$$R = k\mathscr{R}(1 - C_k(\theta))$$
$$ct = k\mathscr{R}(\theta - S_k(\theta)) \quad (10.58)$$

where θ is called the development angle and where

$$S_k(\theta) = \begin{cases} \sin\theta \\ \theta \\ \sinh\theta \end{cases} \quad \text{and} \quad C_k(\theta) = \begin{cases} \cos\theta & \text{for } k = +1 \\ 1 & \text{for } k = 0 \\ \cosh\theta & \text{for } k = -1 \end{cases} \quad (10.59)$$

The characteristic radius, $\mathscr{R}$, is given by

$$\mathscr{R} = \frac{c}{H_0}\frac{\Omega_0/2}{(k(\Omega_0 - 1))^{3/2}} \quad (10.60)$$

(note typo in Eq. 3.42 of Peacock, 1999).

AGN Statistics

Because the universe evolves in time, its density and therefore the physical conditions in the universe change in time as well.

Black Holes were formed early on, start accreting as structures form

$\Longrightarrow$ We expect the AGN population to develop in time

Studying this evolution gives insight into evolution of structure in the universe.

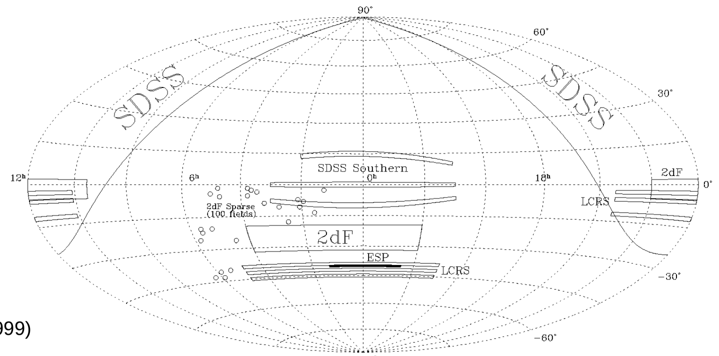
Observational tools: Perform AGN surveys according to well understood criteria:

- Redshift limited samples
- Luminosity limited samples

to study their evolution.

First look at some surveys, then at their results.

Surveys



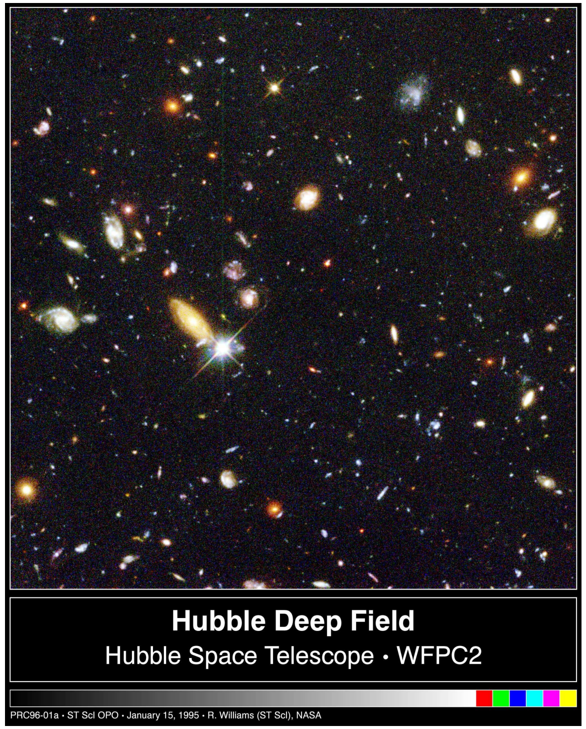
(Strauss, 1999)

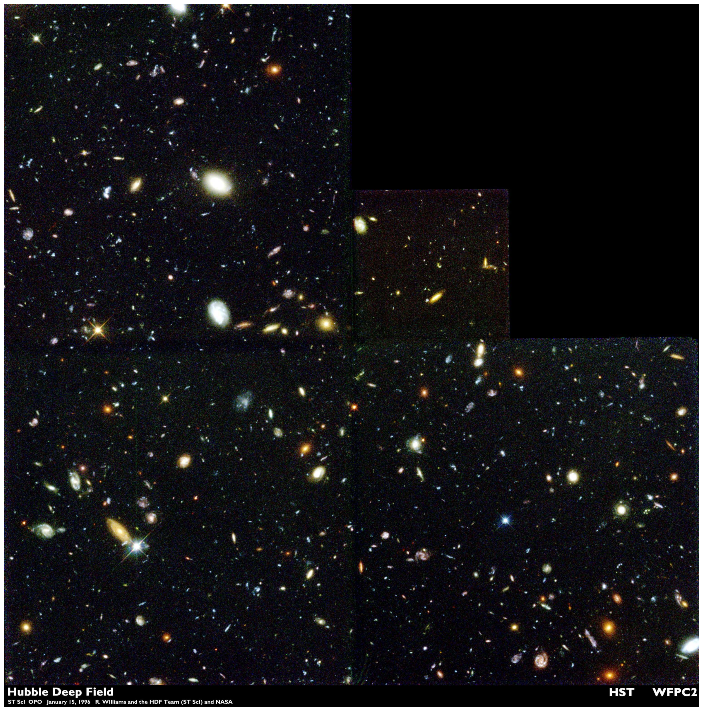
1D-surveys: very deep exposures of small patch of sky, e.g. HST Deep Field, Lockman Hole Survey, Marano Field.

2D-surveys: cover long strip of sky, e.g., CfA-Survey ($1.5 \times 100^\circ$), 2dF-Survey ("2 degree Field").

3D-surveys: cover part or all of the sky, e.g., Sloan Digital Sky Survey, *ROSAT*-survey, or *eROSITA*-survey.

These surveys attempt to go to certain limit in z or m.



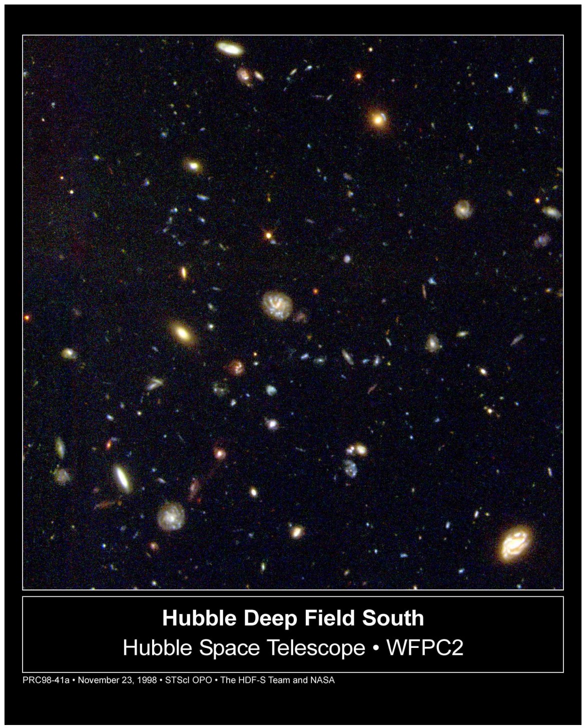


HDF: $\sim$ 150 ksec/Filter for 4 HST Filters made in 1995 December.

Many galaxies with weird shapes $\Longrightarrow$ protogalaxies!

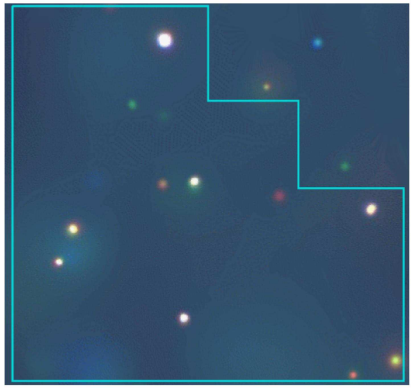
Redshifts: $z \in [0.5, 5.3]$ (Fernández-Soto et al., 1999)

Hubble Deep Field, courtesy STScI



1998: Hubble Deep Field South, 10 d of total observing time!

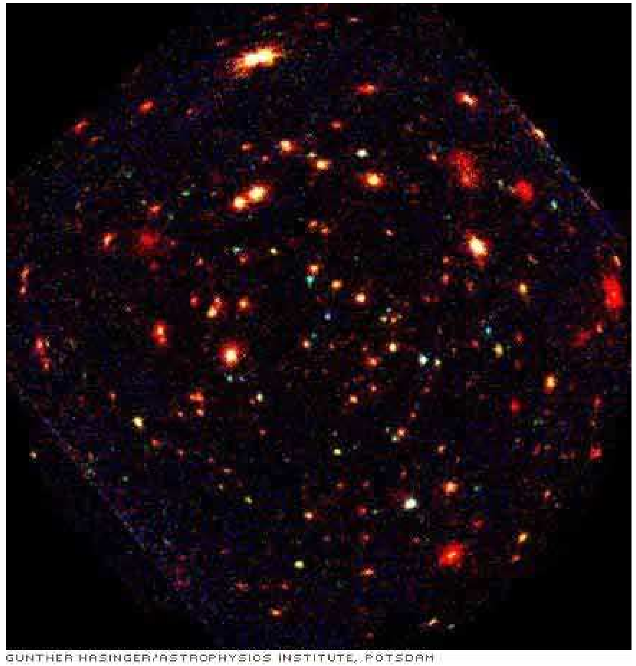
Deep X-ray Surveys



Chandra/HST Image of Hubble Deep Field North; 500 ksec

Problem of optical surveys: many sources are not AGN

Joint multi-wavelength campaigns allow the measurement of broad-band spectra of sources in the early universe!



Lockman Hole: Northern Sky region with very low N_{H}
$\Longrightarrow$ low interstellar absorption
$\Longrightarrow$ "Window in the sky"
$\Longrightarrow$ X-rays: evolution of active galaxies with z!

XMM-Newton, Hasinger et al., 2001,
blue: hard X-ray spectrum,
red: soft X-ray spectrum

Deep X-ray Surveys

Deep optical surveys: Many foreground objects

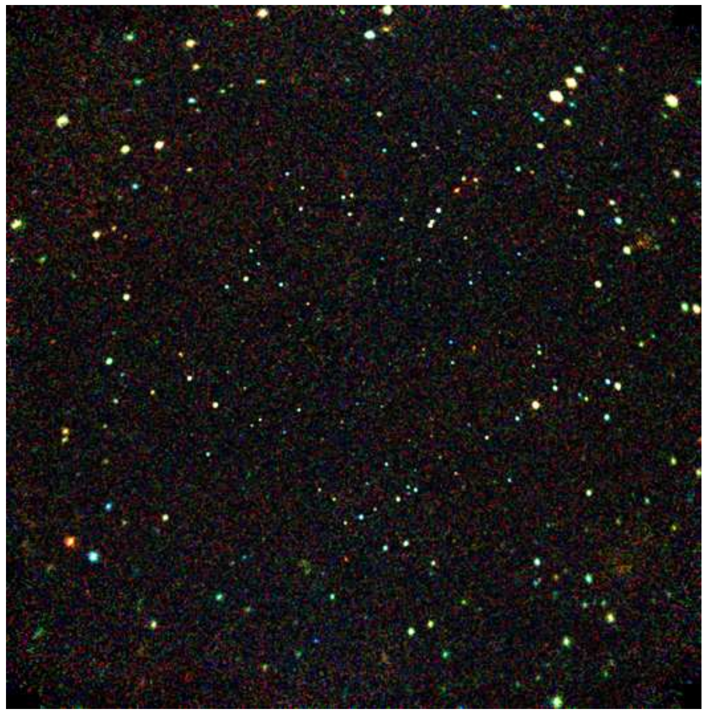
$\Longrightarrow$ go into the X-rays, where AGN dominate

$\Longrightarrow$ Deep X-ray Surveys

Review: Brandt & Hasinger (2005)

History:

- Early 1970s: *Uhuru* and *Ariel*: strong cosmic X-ray background (CXRB)
- Early 1980s: *Einstein* satellite (Wolter telescope): 25% of the 1–3% CXRB resolved into discrete sources, mainly AGN
  Sensitivity limit: $3 \times 10^{-14}\,\mathrm{erg\,cm^{-2}\,s^{-1}}$
- Early 1990s: *ROSAT* resolves ∼75% of CXRB into discrete sources
  Sensitivity limit: $10^{-15}\,\mathrm{erg\,cm^{-2}\,s^{-1}}$, AGN density: 780–870 per square degree
- Late 1990s: surveys with *ASCA* and *BeppoSAX*
- State of the art: *Chandra* and *XMM-Newton* Deep Fields.



Chandra Deep Field South: 1 Msec (10.8 days) on one region in Fornax $\Longrightarrow$ Deepest X-ray field ever...

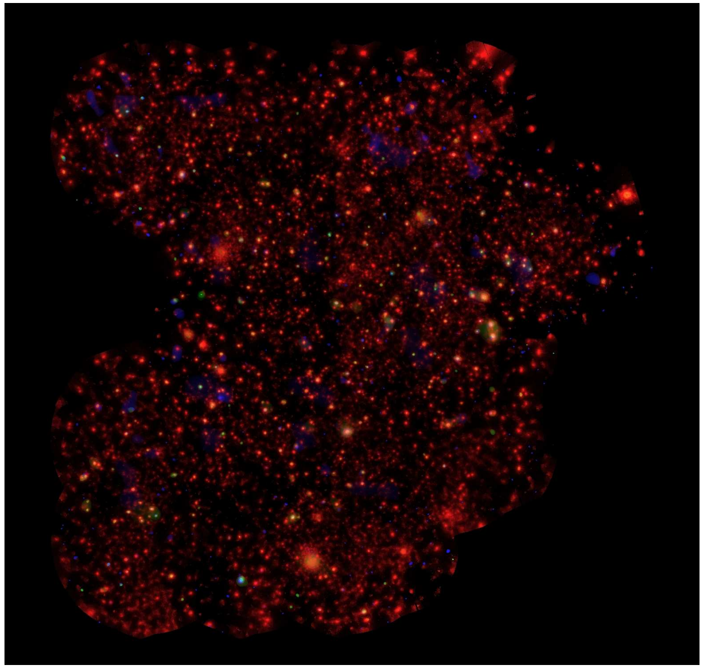
color code: spectral hardness

$\gtrsim 70\%$ of sources in deep X-ray surveys are AGN

in deepest *Chandra* fields, AGN density is $\approx 7200\,\mathrm{deg}^{-2}$ (Bauer et al., 2004)

scale: $15' \times 15'$; courtesy NASA/JHU/AUI/R.Giacconi et al.

Deep X-ray Surveys

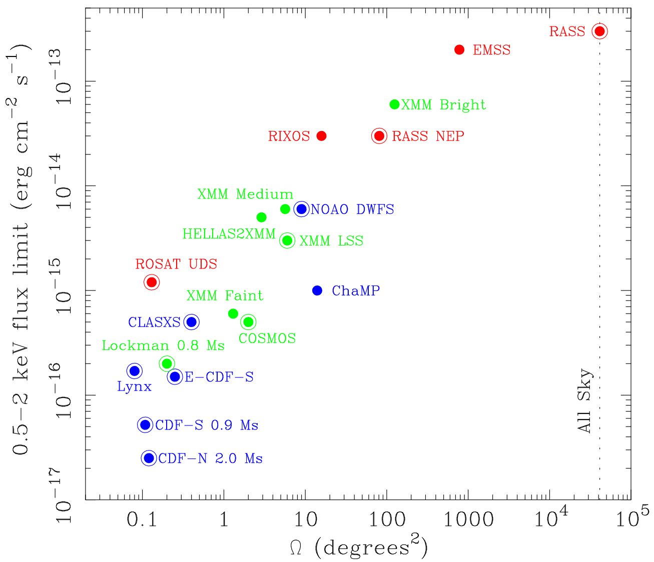


COSMOS field: 1.4 Msec (16.4 days) with *XMM-Newton*, observations from the IR to the X-rays are available

color code: spectral hardness

682 sources detected

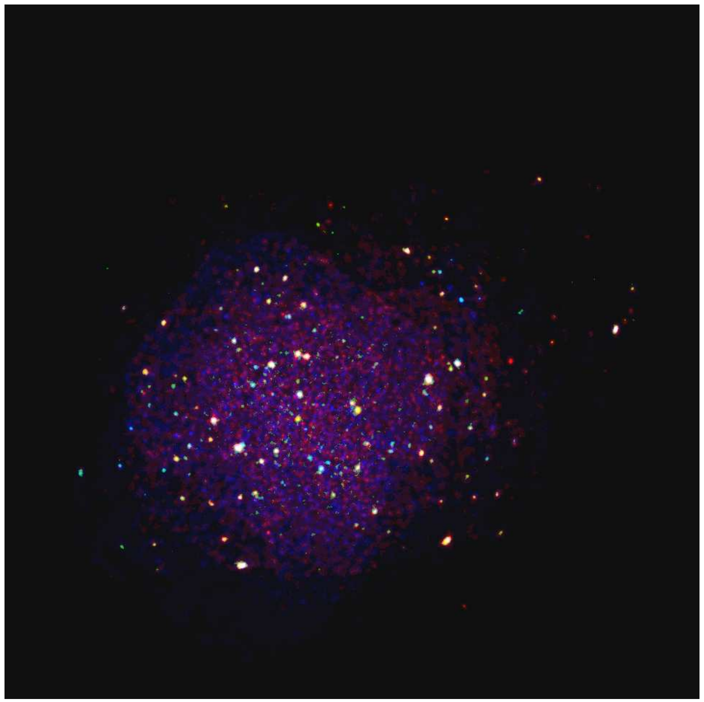
courtesy MPE



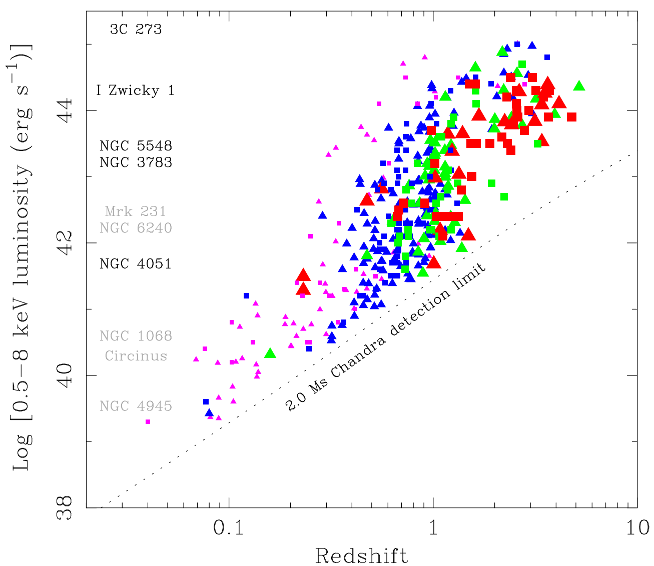
Sensitivity limits of the most prominent X-ray AGN surveys.

Brandt & Hasinger (2005, Fig. 1)

Deep X-ray Surveys



Deep *XMM-Newton* image of the Marano Field (IAAT/AIP/MPE)



Distant AGN have to be very luminous to be detectable!

Brandt & Hasinger (2005, Fig. 3)

Statistics

Most important statistics: number counts

Assume AGN have a space density $n(r)$ as a function of distance.

To illustrate, first look at the number of objects of same luminosity, L, ("δ-function luminosity function") in an Euclidean space:

$$\mathrm{d}N(r) = n(r)\,\mathrm{d}V = n(r)r^2\,\mathrm{d}r\,\mathrm{d}\Omega \tag{10.61}$$

such that surface density (AGN at distance r per square degree):

$$\frac{\mathrm{d}N(r)}{\mathrm{d}\Omega} = n(r)r^2\,\mathrm{d}r \tag{10.62}$$

Often: flux limited sample: count all sources with $F > S$, i.e., out to distance

$$r_{\max} = \left(\frac{L}{4\pi S}\right)^{1/2} \tag{10.63}$$

Number of sources detected *above* a flux S:

$$N(>S) = \int_0^{r_{\max}} n(r)r^2\,\mathrm{d}r \tag{10.64}$$

cumulative source distribution as a function of flux

Statistics

As an example, consider $N(>S)$ for an uniform space density, $n(r) = n_0$:

$$N(>S) = \int_0^{r_{\max}} n(r)r^2\,\mathrm{d}r = \int_0^{r_{\max}} n_0 r^2\,\mathrm{d}r = \frac{n_0 r_{\max}^3}{3} = \frac{n_0}{3}\left(\frac{L}{4\pi S}\right)^{3/2} \tag{10.65}$$

or

$$\log(N(>S)) = \log\left(\frac{n_0 L^{3/2}}{3(4\pi)^{3/2}}\right) - \frac{3}{2}\log S \tag{10.66}$$

For a constant source population, the slope in a $\log N$–$\log S$ diagram is $-3/2$.

. . . disregarding cosmological effects.

When working in magnitudes: $m \propto -2.5\log S \Longrightarrow \log S \propto -0.4m$, such that

$$\log N(m) \propto 0.6m \tag{10.67}$$

So for a constant space density, the number of objects detected increases by a factor $10^{0.6} = 4$ per optical magnitude.

What this means practically: In an optical flux limited sample, 80% of all sources are within 1 mag of the detection limit. . .

Statistics

The slope of the $\log N(>S)$–$\log S$-relationship for constant density and δ-function luminosity function is

$$\beta = -\frac{\mathrm{d}\log N}{\mathrm{d}\log S} = \frac{3}{2} \tag{10.68}$$

Now include cosmology. Again, for sources with a δ-function luminosity function (=one-to-one relation between flux and redshift), β can be written

$$\beta = -\frac{\mathrm{d}\log N}{\mathrm{d}\log S} = -\frac{\mathrm{d}\log V}{\mathrm{d}\log z}\cdot\frac{\mathrm{d}\log z}{\mathrm{d}\log S} \tag{10.69}$$

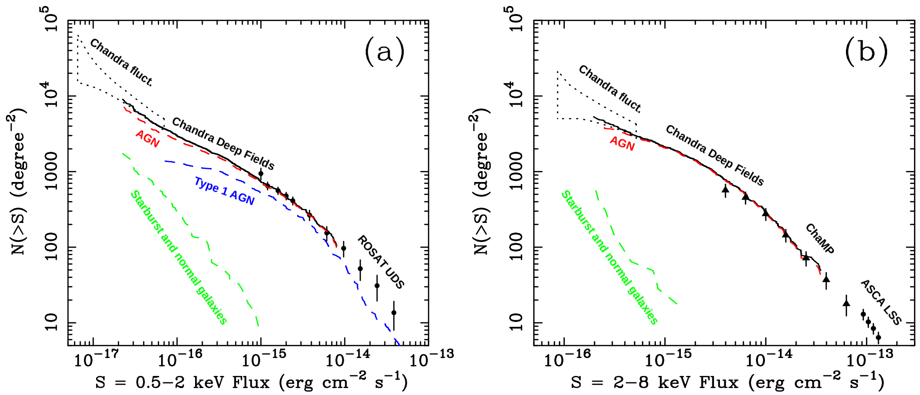
For $\Omega = 1$ and $z \gg 1$, Peacock (1999) shows:

$$\frac{\mathrm{d}\log V}{\mathrm{d}\log z} \sim \frac{1.5}{\sqrt{z}} \quad\text{and}\quad \frac{\mathrm{d}\log S}{\mathrm{d}\log z} \sim -(1+\alpha) \tag{10.70}$$

for power law source spectra, $F_\nu \propto \nu^{-\alpha}$, such that

$$\beta = \frac{3}{2}\cdot\frac{1}{(1+\alpha)\sqrt{z}} < \frac{3}{2} \tag{10.71}$$

Statistics



Brandt & Hasinger (2005, Fig. 3)

Contributions of different identified types of AGN to total $\log N$–$\log S$: AGN dominate

The problem: Measurements show $\beta \gtrsim 1.5$, i.e., AGN population is evolving.

AGN Evolution: Observations

Local distribution of AGN can be parameterized as

$$\rho(L) = \rho_0 \left[\left(\frac{L}{L^*}\right)^{\alpha} + \left(\frac{L}{L^*}\right)^{\beta} \right]^{-1} \qquad (10.72)$$

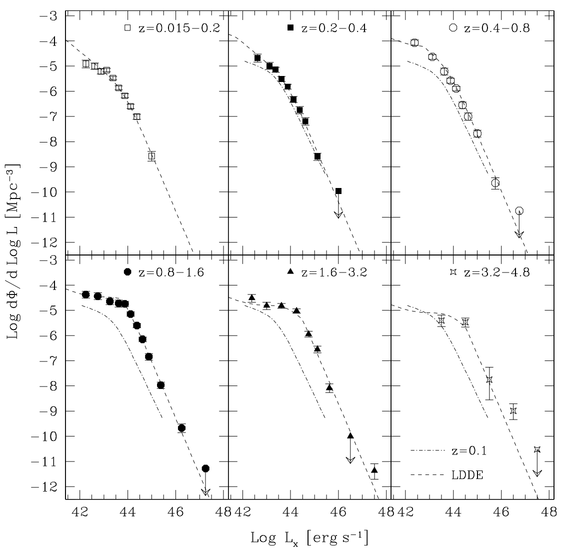
with $\alpha = 0.3$, $\beta = 2.3$, $\rho_0 = 10^{3.6} h^{-3}\,\mathrm{Gpc}^{-3}$ and $L^*_{0.5-2\,\mathrm{keV}} = 10^{42.8}\,\mathrm{erg\,s^{-1}}$ for $z = 0$. At $z \sim 2$: L^* factor 30 larger, find $L \propto (1+z)^3 \Longrightarrow$ AGN Evolution!

General Ansatz: parameterize AGN density, ρ, as function of emitted power L and redshift, z. Two extreme cases

$$\rho(L, z) = \begin{cases} f(z)\rho_0(L) & \text{pure density evolution} \\ \rho_0(L/g(z)) & \text{pure luminosity evolution} \end{cases} \qquad (10.73)$$

The evolution functions $f(z)$ and $g(z)$ are often parameterized as powers of $1+z$.

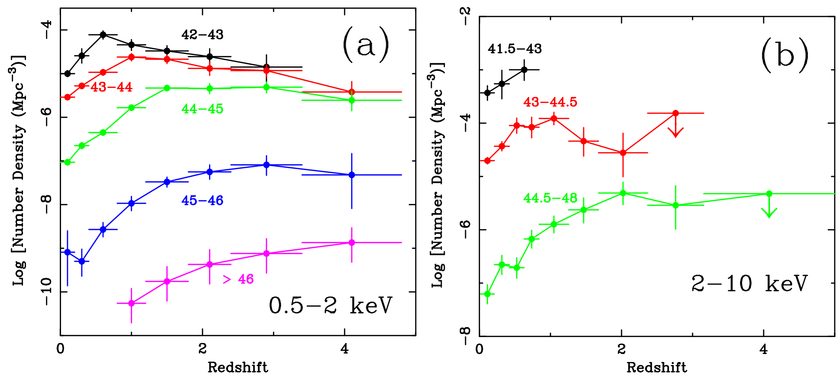
AGN Evolution: Observations



Evolution of $\log N$–$\log S$-with redshift: changes at high L_X!

(Brandt & Hasinger, 2005, Fig. 7)

Observed Space Density



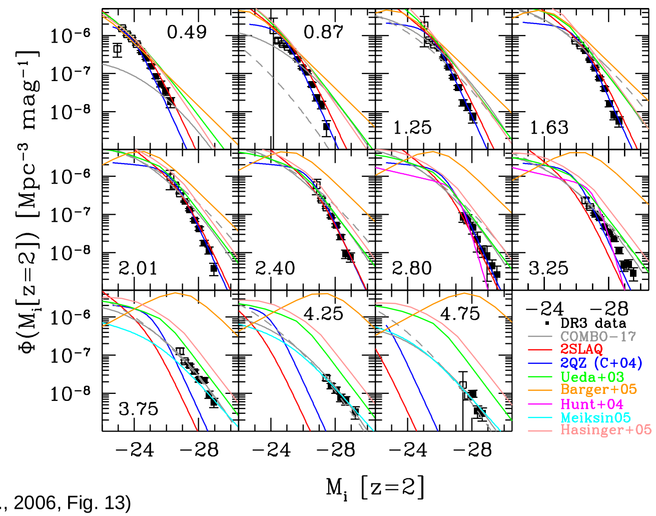
Brandt & Hasinger (comoving AGN space density; 2005, Fig. 8)

X-ray surveys show luminosity evolution:

- peak space density moves to smaller z with smaller L_X
- rate of evolution from now to peak is slower for less luminous AGN: less evolution for low L_X.

$\Longrightarrow$ if L_X traces M_{BH}, then the most massive BH formed first! ("anti-hierarchical AGN evolution")

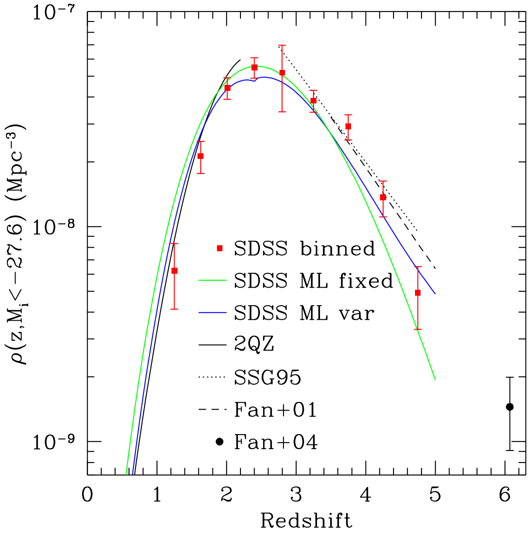
Observed Space Density



(Richards et al., 2006, Fig. 13)

X-ray fields too small to cover high luminosity quasars $\Longrightarrow$ optical surveys

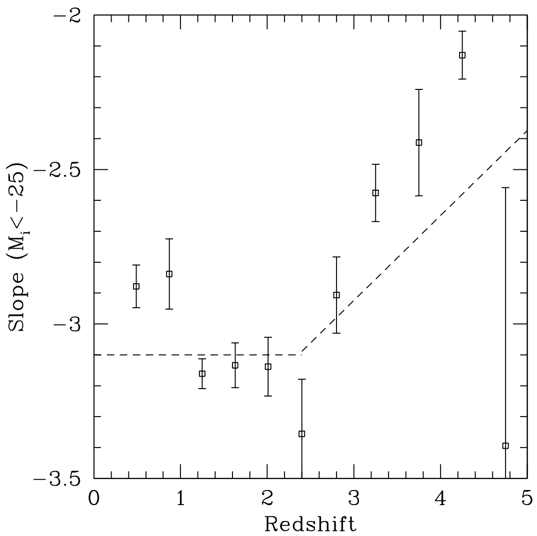
Observed Space Density



Optical surveys such as the Sloan Digital Sky Survey (SDSS) also show quasars to peak at $z \sim 2$.

(Richards et al., 2006, Fig. 20)

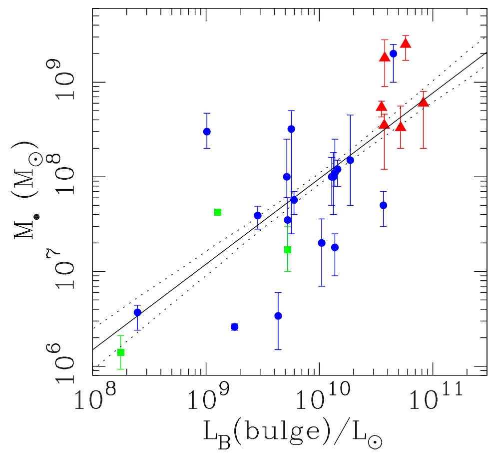
Observed Space Density



SDSS also shows strong quasar evolution, mainly density evolution, but similarly to X-rays data start to hint also at luminosity dependent density evolution.

(Evolution of luminosity function slope, $\Phi \propto L^{-\beta}$, Fig. 21 of Richards et al., 2006)

AGN and Host Galaxies



Evolution models predict large numbers of dormant BH in local galaxies. These are indeed found:

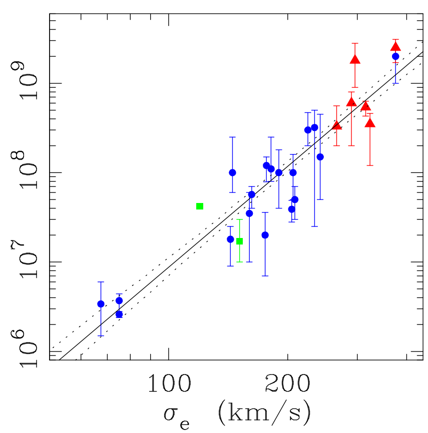
The BH mass scales with the luminosity of host galaxy bulge.

See (Ferrarese & Ford, 2005) for a review.

(Gebhardt et al., 2000, Fig. 2)

$$\log(M_\bullet) = (8.37 \pm 0.11) - (0.419 \pm 0.085)(B_T^0 + 20.0) \quad (10.74)$$

AGN and Host Galaxies



The BH mass scales with the velocity dispersion of the host galaxy bulge.

See Ferrarese & Ford (2005) for a review.

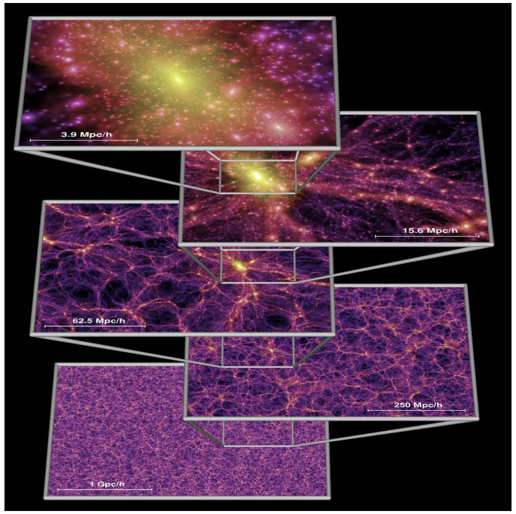
Consequence: Black Hole formation and galaxy bulge formation are closely related to each other

... even though AGN exist in bulge-less galaxies.

(Gebhardt et al., 2000, Fig. 2)

$$\frac{M_\bullet}{10^8 M_\odot} = (1.66 \pm 0.24)\left(\frac{\sigma}{200\,\mathrm{km\,s^{-1}}}\right)^{4.86\pm0.43} \text{ to } \sim 30\% \quad (10.75)$$

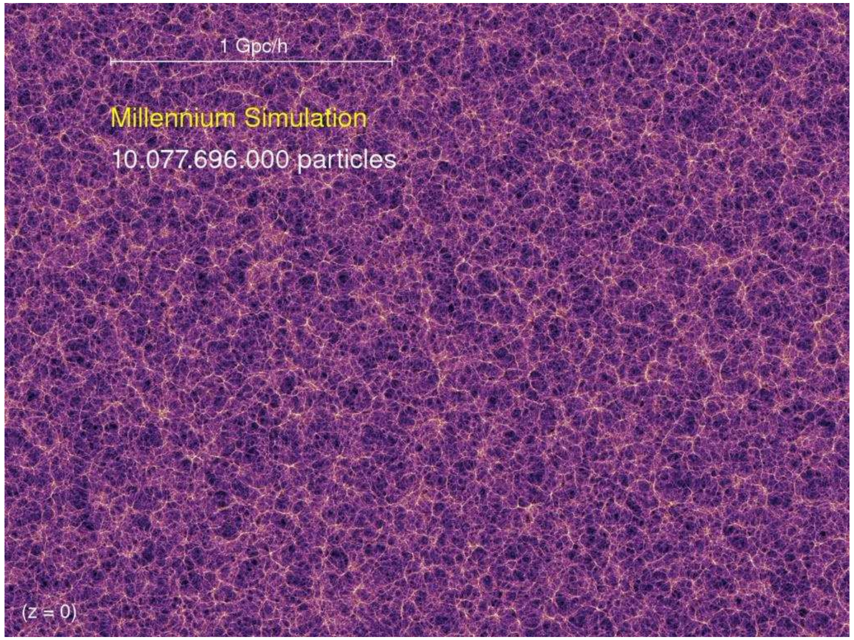
AGN Formation



Contrast AGN-evolution with theory: Millenium simulation: numerical simulation of galaxy evolution in a ΛCDM universe, 10$\times$ larger than anything previously done.

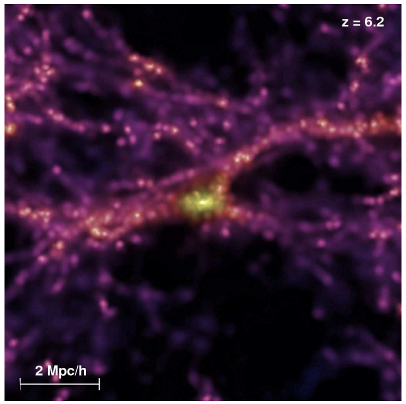
Baseline: semi-analytical evolution formalism adjusted to yield galaxy parameters (luminosity-color evolution, morphology, gas content, BH mass) consistent with observations. Covers galaxies down to SMC size, includes AGN formation and growth.

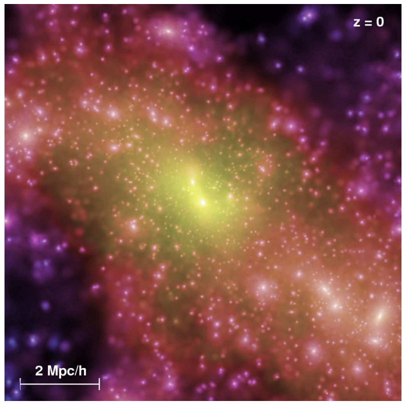
See Springel et al. (2005) for details.



Movie Time: The Millenium Simulation, `formationmovies/millenium_sim_1024x768.avi` (10^{10} particles; 512 processors, 350000 h (28 clock days) of CPU time, see Springel et al. 2005)

AGN Formation

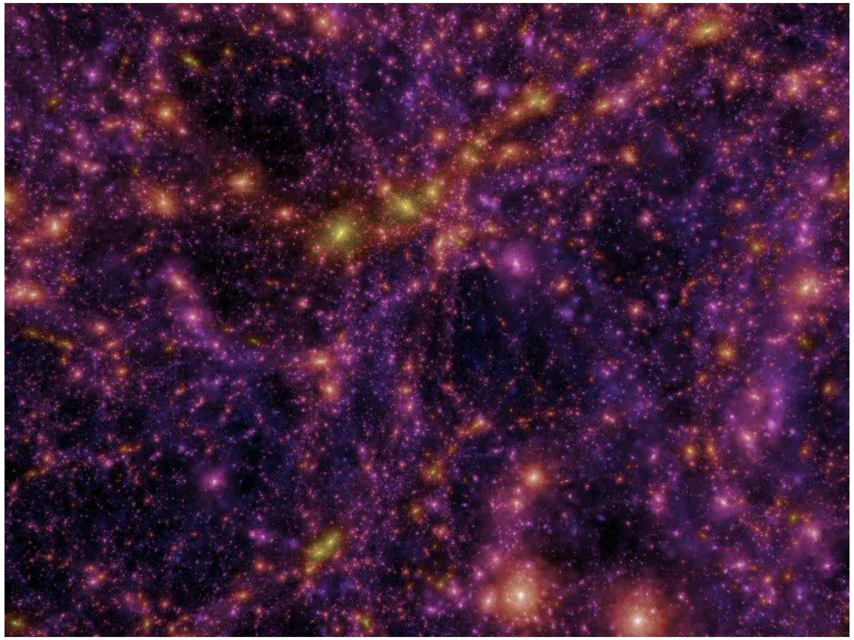




(Springel et al., 2005)

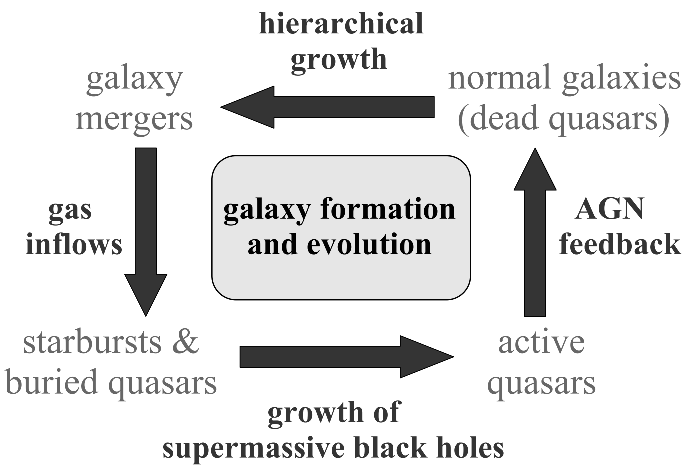
Volume of Millenium simulation is too small to contain more than a few quasar candidates.

Here: Evolution of largest mass object, from halo dark matter mass $1.8 \times 10^{10}\,M_\odot$ at $z = 16.7$ to now $3.9 \times 10^{12}\,M_\odot$ in DM, $6.8 \times 10^{10}\,M_\odot$ normal matter, and a star formation rate of $235\,M_\odot\,\mathrm{year}^{-1}$.



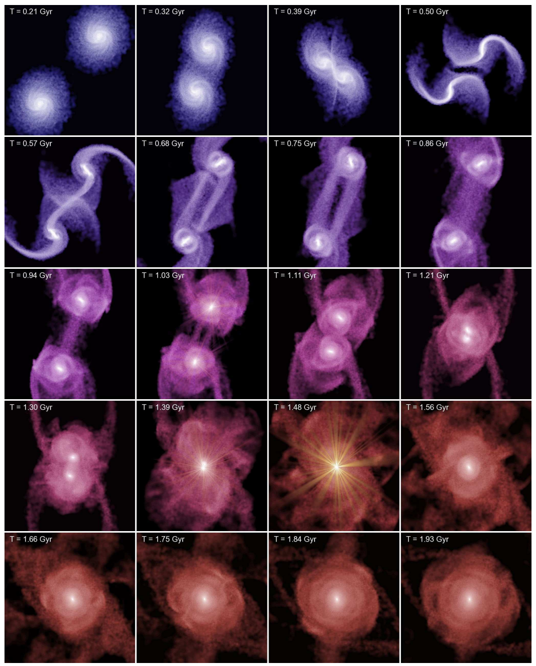
Movie Time: Fly through the Millenium Simulation, `formationmovies/millenium_flythru.avi`
2.5 billion light years; see Springel et al. 2005

AGN Formation



(Hopkins et al., 2006, Fig. 1)

AGN formation and evolution are probably linked to galaxy mergers.



Evolution of a merger in a $80h^{-1}$ kpc wide box: blue: baryonic mass fraction 20%, red: $<5\%$.

Point sources shown when quasar activity would be observable.

(Hopkins et al., 2006, Fig. 2)

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